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Seismic response of arch dams considering infinite radiation damping and joint opening effects

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Abstract: Effects of two important factors on earthquake response of high arch dams are considered and combined into one program. These factors are: effects of radiation damping of the infinite canyon and local non-linearity of the contraction joint opening between the dam monoliths. For modeling of rock canyon, the discrete parameters are obtained based on a curve fitting, thus allowing the nonlinear dam system to be solved in the time domain. The earthquake uniform free-field input at the dam-canyon interface is used. An engineering example is given to demonstrate the significant effects of the radiation damping on the structure response.

Keywords: arch dams; dam-canyon interaction; radiation damping; joint opening; local nonlinearity

1 Introduction

Starting from the end of the last century, a series of high arch dams up to 250-300m in height has been constructed and some are being planned to construct. For seismic analysis of such high arch dams to resist strong earthquakes, it is necessary to consider some important factors. These factors include (1) complete interaction effects between the dam and the rock foundation; (2) non-linearity of the dam with contraction joint opening during the extreme ground motions.

State-of-the-art procedures dealing with seismic analysis of arch dams assume a truncated massless rock foundation and apply the design earthquake input at the rigid base beneath the truncated rock foundation. These assumptions neglect the interaction effects due to radiation damping of the infinite mass rock and the non-uniform input motions along the canyon. Recent studies(Nowak and Hall, 1990; Kojic and Trifunac, 1991;Dominguez and Maeso, 1992; Chopra and Tan,1992; Du *et al*, 1996; Lin *et al*, 1997) have revealed that these interaction effects are important and should be included in the analysis. With this objective, a time domain procedure of coupling finite elements (FEs), boundary elements (BEs) and infinite boundary elements (IBEs) has been developed by Zhang *et al*.(1993,1995). Studies on the effects of interaction on the response of the dam have been conducted in detail for some arch dams(Zhang *et*

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al.,1996).

With regard to the nonlinear behaviors of arch dams, the most important nonlinearity is initiated by the contraction joint opening during strong ground motions. This phenomenon often occurs in the upper portion of a dam where the largest tensile stresses up to $5 - 6MPa$ are expected to occur in the arch direction for moderately strong earthquake motion. Thus, the opening of contraction joints is inevitable leading to a substantial reduction of tensile stresses in arch direction, while, on the other hand, a significant increase in the possibility of cracking of vertical cantilevers. Fenves *et al*. (1989) used a 3-D nonlinear joint element and an efficient numerical procedure for solving this problem. The F.E. substructure technique is employed by considering the set of joint elements as a single nonlinear substructure while the cantilevers between joints as linear ones and their degrees of freedom can be condensed out.

To combine the above-mentioned two factors into one program, i.e. the complete interaction effects between arch dam and foundation, and the non-linearity of the contraction joint opening, a valuable study with a simplified earthquake input procedure was presented (Zhang *et al*., 1997). The present study is to go further, incorporating the uniform free-field earthquake input into the program. The development of the time domain BE-IBE coupling model for simulation of rock canyon provides a means for solving problems with structural non-linearity. The joint element and the solution technique for dam substructure developed by Fenves *et al.* are retained in this procedure and the discrete parameters for simulation of infinite rock canyon are incorporated

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into the program. For examination of the effects of these two factors on the dam response, the earthquake analysis of the Xiaowan arch dam is performed.

2 System analyzed

Shown in Fig.1 is a complete dam-canyonreservoir system. Finite elements are used for discretization of the dam and the reservoir, assuming the latter being incompressible. The dam may be viewed to consist of a series of cantilever elements separated by nearly vertical contraction joints. Since the joints can be expected to open and close during an earthquake, it is evident that the dam behaves as a locally nonlinear subsystem, provided the dam body excluding the joints remains linear elastic. Preliminary analysis showed that a significant redistribution of stresses will occur due to this non-linearity, thus raising a concern of the integrity and safety of the dam. The substructure of the canyon is discretized into boundary elements and infinite boundary elements. Frequency-dependent impedance functions are first obtained for all degrees of freedom on the dam canyon interface. By using a curve fitting, these impedance functions are transformed into a mass-spring-dashpot system which is frequency independent. Finally, these discrete parameters together with the linear substructure of cantilevers of the dam body are condensed into the boundaries of the nonlinear substructure—a set of contraction joint elements. The equilibrium iteration during a time step is conducted only for the degrees-of-freedom for the nonlinear substructure.

Fig.1 Schematic layout of dam-canyon system with contraction joints

2.1 Modeling of rock canyon and free field input

bou ndary elements (IBEs) are used for discretization 3-D boundary elements (BEs) and infinite of the canyon, taking into account the irregular geometrical conditions in the near field of the canyon. The detailed formulations of the infinite boundary elements can be found elsewhere(Zhang *et al*.,1991).

equ ation by BE-IBE coupling leads to a series of Numerical integration of the boundary integral linear equations

$$
Hu = Gp \tag{1}
$$

where coefficient matrices assembled from individual elements including the H and *G* contribution due to IBEs to H , with no contribution to *G* from these elements due to the traction-free condition. Eq. (1) can then be used to obtain impedance functions at all defined nodes on canyon surface in frequency domain. It is, therefore, necessary to condense the impedance functions into the dam canyon interface.

Partitioning E q. (1) according to the degrees-offreedom associated with the dam-canyon interface *C* and the remaining portion of the canyon surface r leads to Eq. (1) rewritten as

$$
\begin{bmatrix} H_{\rm cc} & H_{\rm cr} \\ H_{\rm rc} & H_{\rm rr} \end{bmatrix} \begin{Bmatrix} u_{\rm c} \\ u_{\rm r} \end{Bmatrix} = \begin{bmatrix} G_{\rm cc} & G_{\rm cr} \\ G_{\rm rc} & G_{\rm rr} \end{bmatrix} \begin{Bmatrix} p_{\rm c} \\ p_{\rm r} \end{Bmatrix} \qquad (2)
$$

After condensation and recognizing the traction-free condition $p_r = 0$, the impedance matrix of the infinite canyon S_{cc} , which is defined on the dam-canyon interface, can be easily obtained(Zhang *et al*., 1993, 1995 and 1996).

Since $S_{\text{cc}}(\omega)$ is frequency dependent, it is necessary to transform them into discrete parameters so that the entire dam-canyon system can be solved in the time domain for non-linearity of the structure. Applying the following relationship

$$
\boldsymbol{S}_{\rm cc} = -\omega^2 \overline{\boldsymbol{m}}_{\rm cc} + i\omega \, \overline{\boldsymbol{c}}_{\rm cc} + \overline{\boldsymbol{k}}_{\rm cc} \tag{3}
$$

to each coefficient of the impedance matrix yields the discrete parameters \overline{m}_{cc} , \overline{c}_{cc} , $k_{\overline{c}c}$, which are the elements of equivalent mass \boldsymbol{M}_{cc} , damping \boldsymbol{C}_{cc} and stiffness \boldsymbol{K}_{cc} matrices respectively. It should be noted that Eq. (3) can be only used in the case that $S_{\rm cc}(\omega)$ changes monotonously with ω increasing in the concerned range. Such a restricted condition will be easily filled in the application in rock foundation of arch dams due to its high elastic modulus. Therefore a good precision can be ensured.

Assuming that ω_1 and ω_2 represent the prescribed excitation frequency at the two boundary points within the frequency range of interest and that the corresponding impedance functions $S_{cc}(\omega_1)$ and $S_{\text{cc}}(\omega_2)$ are known, the substitution of $\widetilde{S}_{\text{cc}}$ at the above points into Eq. (3) leads to

$$
\overline{\boldsymbol{m}}_{cc} = \frac{1}{\omega_2^2 - \omega_1^2} \{ \text{Re}[\boldsymbol{S}_{cc}(\omega_1)] - \text{Re}[\boldsymbol{S}_{cc}(\omega_2)] \}
$$

$$
\overline{\boldsymbol{c}}_{cc} = \frac{1}{2} \{ \frac{1}{\omega_1} \text{Im}[\boldsymbol{S}_{cc}(\omega_1)] + \frac{1}{\omega_2} \text{Im}[\boldsymbol{S}_{cc}(\omega_2)] \} \quad (4)
$$

$$
\overline{\boldsymbol{k}}_{cc} = \text{Re}[\boldsymbol{S}_{cc}(\omega_1)] + \omega_1^2 \overline{\boldsymbol{m}}_{cc}
$$

in which \overline{m}_{cc} , k_{cc} are obtained by equating the real parts of $S_{\text{cc}}^{\text{ce}}$, and \bar{c}_{cc} is based on averaging the imaginary parts at the boundary points.

The three parameters for all degrees-of-freedom and matrices M_{cc} , C_{cc} and K_{cc} and to form the their coupling terms on the dam-canyon interface can be obtained from Eq. (4) and then assembled to give

corresponding equation of motion for the canyon

$$
\overline{M}_{\rm cc}\ddot{u}_{\rm c} + \overline{C}_{\rm cc}\dot{u}_{\rm c} + \overline{K}_{\rm cc}u_{\rm c} = \overline{F}_{\rm c} \tag{5}
$$

in which \mathbf{u}_c , $\dot{\mathbf{u}}_c$ and $\ddot{\mathbf{u}}_c$ denote the interaction displacement, velocity and acceleration vectors of the canyon respectively, and \boldsymbol{F}_{c} represents the interaction forces acting on the canyon interface.

For coupling the equations of the canyon motion wit h the dam, the total response of the arch dam and its boundary connection with canyon can be written in partitioned form as

$$
\begin{bmatrix}\nM_{dd} & M_{dc} \\
M_{cd} & M_{cc}\n\end{bmatrix}\n\begin{bmatrix}\n\ddot{U}_d \\
\ddot{U}_c\n\end{bmatrix} +\n\begin{bmatrix}\nC_{dd} & C_{dc} \\
C_{cd} & C_{cc}\n\end{bmatrix}\n\begin{bmatrix}\n\dot{U}_d \\
\dot{U}_c\n\end{bmatrix}
$$
\n
$$
+\n\begin{bmatrix}\nK_{dd} & K_{dc} \\
K_{cd} & K_{cc}\n\end{bmatrix}\n\begin{bmatrix}\nU_d \\
U_c\n\end{bmatrix} =\n\begin{bmatrix}\nF_d \\
F_c\n\end{bmatrix}
$$
\n(6)

where M, C and K are the mass, damping and stiffness matrices of the system respectively; \bm{F} denotes the load vector; subscripts d and c refer to the degrees of freedom associated with the internal nodes of the dam and the boundary nodes connecting with the canyon respectively.

Combining equation (5) an d (6), recognizing the force-free condition $F_d = 0$, and noting that

$$
\begin{aligned} \boldsymbol{F}_{\rm c} &= -\overline{\boldsymbol{F}}_{\rm c} \\ \boldsymbol{U}_{\rm c} &= \overline{\boldsymbol{u}}_{\rm c} + \boldsymbol{u}_{\rm c} \end{aligned} \tag{7}
$$

in which \boldsymbol{u}_c denote the free-field displacement input on dam-canyon interface. The equation of motion for dam-canyon interaction are finally obtained as

$$
\begin{bmatrix}\n\boldsymbol{M}_{dd} & \boldsymbol{M}_{dc} \\
\boldsymbol{M}_{cd} & \boldsymbol{M}_{cc} + \overline{\boldsymbol{M}}_{cc}\n\end{bmatrix}\n\begin{bmatrix}\n\ddot{\boldsymbol{U}}_{d} \\
\ddot{\boldsymbol{U}}_{c}\n\end{bmatrix} +\n\begin{bmatrix}\n\boldsymbol{C}_{dd} & \boldsymbol{C}_{dc} \\
\boldsymbol{C}_{cd} & \boldsymbol{C}_{cc} + \overline{\boldsymbol{C}}_{cc}\n\end{bmatrix}\n\begin{bmatrix}\n\dot{\boldsymbol{U}}_{d} \\
\dot{\boldsymbol{U}}_{c}\n\end{bmatrix} +\n\begin{bmatrix}\n\boldsymbol{K}_{dd} & \boldsymbol{K}_{dc} \\
\boldsymbol{K}_{cd} & \boldsymbol{K}_{cc} + \overline{\boldsymbol{K}}_{cc}\n\end{bmatrix}\n\begin{bmatrix}\boldsymbol{U}_{d} \\
\boldsymbol{U}_{c}\n\end{bmatrix} =\n\begin{bmatrix}\n\boldsymbol{0} & \boldsymbol{0} \\
\boldsymbol{M}_{cc}\ddot{\boldsymbol{u}}_{c} + \overline{\boldsymbol{C}}_{cc}\dot{\boldsymbol{u}}_{c} + \overline{\boldsymbol{K}}_{cc}\overline{\boldsymbol{u}}_{c}\n\end{bmatrix}
$$
\n(8)

Solving Eq. (8) and using Eq. (7), the t dis otal placement field $(U_d, U_c)^T$ for the dam can be obtained.

It should be noted that the terms of the right hand sid e of Eq. (8) are now free-field motions acting on the dam-canyon interface. The Eq. (8) can either be linear or nonlinear and be solved by a typical numerical scheme such as the Newmark average acceleration method.

of 2.2 Modeling of nonlinear contraction joints dam

al. ,1973) is shown in Fig.2. The element is of iso parametric and consists of two coincident surfaces A 3-D nonlinear joint element (Ghaboussi *et* each of which is defined by four nodes.

stresses and the relative displacements of the joint can be assumed as follows The constitutive relationship between resisting

$$
q_i = \begin{cases} k_i v_i & v_i \le q_{oi} / k_i \\ 0 & v_i > q_{oi} / k_i \end{cases}
$$
 i=1,2,3 (9)

where q_i denotes the resisting stresses in i direction. k_i is the stiffness of the joint in relationship. It is reasonable to assume that the e direction. q_{oi} is a specified tensile strength of the compression. v_i is the relative displacement in *i* joint. Fig.3 shows the nonlinear constitutiv subsequent tensile strength of the joint will drop down to zero after the first opening of the joint when $v_i > q_{oi}/k_i$.

Fig.2 Nonlinear joint element

Fig.3 Stress-relative displacement relationship

for joint element

2.3 Substructure technique used in coupling model

The substructure technique is used to solve the Equations of motion of the above mentioned coupling model for computational efficiency. All the contraction joint sets can be taken as a nonlinear substructure and the cantilevers between them as linear ones. The canyon substructure is also considered as linear.

For each linear substructure of dam body, its equation of motion is given by

$$
m\ddot{u} + c\dot{u} + ku = q \tag{10}
$$

stiffness matrices, respectively; \boldsymbol{u} is the total displacement; q denotes the forces at the boundary of where M , c and k are the mass, damping and the substructure.

input can be taken into account in the equation as For the linear substructure of canyon, the free-field follows:

$$
\overline{M}_{\rm cc}\ddot{u}_{\rm c} + \overline{C}_{\rm cc}\dot{u}_{\rm c} + \overline{K}_{\rm cc}u_{\rm c} = q_c \qquad (11)
$$

presents the interaction forces acting on the canyon interface; in which M_{cc} , C_{cc} and K_{cc} canyon; are the discrete parameters of the infinite canyon; q_c re \boldsymbol{u}_c refers to the interaction displacement. and

Considering the relationship between the interaction and total displacements

$$
u = u_{\rm c} + \overline{u}_{\rm c}
$$

Eq. (11) can be rewritten as

$$
\overline{M}_{\rm cc}\ddot{\boldsymbol{u}} + \overline{C}_{\rm cc}\dot{\boldsymbol{u}} + \overline{K}_{\rm cc}\boldsymbol{u} = \boldsymbol{q}_{\rm c} + \boldsymbol{f}_{\rm c} \qquad (12)
$$

where f_c is the free-field input load, given by

$$
\boldsymbol{f}_{\rm c} = \overline{\boldsymbol{M}}_{\rm cc} \ddot{\overline{\boldsymbol{u}}}_{\rm c} + \overline{\boldsymbol{C}}_{\rm cc} \dot{\overline{\boldsymbol{u}}}_{\rm c} + \overline{\boldsymbol{K}}_{\rm cc} \overline{\boldsymbol{u}}_{\rm c} \qquad (13)
$$

In this model, non-uniform free-field can be input to consider its effects on nonlinear dynamic responses of arch dams. But at the current stage, only uniform free -field input is applied in the engineering application.

For the nonlinear substructure, its equation of motion can be represented as

$$
M\ddot{U} + P(\dot{U}, U) = Q \tag{14}
$$

in which *M* is the mass matrix; $P(\dot{U}, U)$ is the vector of restoring forces which is a nonlinear function of \dot{U} and U ; \dot{Q} denotes the boundary forces of the nonlinear substructure.

between the nonlinear substructure and the linear ones, the following relation is applied: For equilibrium and compatibility conditions

$$
\sum q + q_{\rm c} + Q = 0 \tag{15}
$$

$$
u = aU \tag{16}
$$

where \boldsymbol{a} is a Boolean matrix. The detail procedure of

the equilib rium iteration can be referred in reference (Fenves et al., 1989; Zhang *et al*., 1991)

3 Engineering applications

The Xiaowan arch dam, located on the Lanchang River, Yunnan Province, China was chosen as an engineering example. The dam has a crest length of 935 m and the maximum dam height is 292m. Fig. 4 shows the simulation of contraction joints (25 joints). The material properties for the concrete are: unit weight=24kN/m³, modulus of elasticity=2.73 $\times 10^4$ MPa, Poisson's ratio=0.217; for the foundation rock: unit weight=25.7kN/m³, modulus of elasticity =5.46× 104 MPa, Poisson's ratio=0.319. Rayleigh damping in the dam is represented by a viscous damping ratio of 0.05 in the first and fifth modes. No material damping in the foundation is assumed for both mass and massless models. The hydrodynamic effect of the reservoir is employed as added mass by FEM.

Fig.4 Simulation of contraction joints (25 joints)

Xiaowan arch dam is used, with the maximum peak acc eleration of 0.308g in two horizontal directions and The designed earthquake ground motion of 2/3 PHA in the vertical direction. Time step Δt of $0.01s$ is used.

Static analysis of the dam-reservoir-foundation system is first performed before a nonlinear dynamic analysis to be carried out. This includes stress analysis under self-weight of concrete and hydrostatic pressure of the reservoir water.

Nonlinear dynamic analysis is performed using massless canyon (without radiation damping) and infinite mass canyon (with radiation damping) for comparison. Fig.5 shows the comparisons of the displacement envelope (upstream direction) distributions along the dam crest and crown cantilever. Fig.6 shows the comparison of displacement time histories at the middle point of the crest. The maximum joint openings for all contraction joints are compared in Fig.7 with different foundation models, the corresponding time histories of the middle joints are shown in Fig.8.

Comparisons of stresses between the two models are shown in Figs. 9 and 10. The comparisons of the maximum stresses, displacements and joint openings between different cases are also listed in Table 1.

The observations from these results are summarized as follows:

(b) Along the crown cantilever

Fig.6 Comparison of displacement (upstream direction) histories at the middle point

of dam crest between the massless and infinite mass canyon

Fug.7 Comparison of joint maximum opening distributions between the massless and infinite mass canyon

Fig.8 Comparison of opening histories (middle point between the massless and infinite mass canyon

(a) Arch stresses in the massless canyon

(b) Arch stresses in the infinite mass canyon

(c) Cantilever stresses in the massless canyon

Fig.9 Maximum stress envelope distributions on the dam upstream face / MPa (d) Cantilever stresses in the infinite mass canyon

(d) Cantilever stresses in the infinite mass canyon Fig.10 Maximum stress envelope distributions on the dam downstream face / Mpa

From Figs.5(a) and $5(b)$, showing the comparisons of the displacement envelopes distributed along the cre st and the crown cantilever, it is shown that due to the infinite canyon radiation damping, the reduction of the dynamic displacement can be obviously observed, by an extent of approximately 20%. Furthermore, as shown in Fig.6, the displacement reduction occurs during the whole time histories.

joint openings is also significant for all joints. The dec rease of the maximum opening is about 20%. The As shown in Fig.7 the drawdown of the maximum same conclusion can be obtained from the comparison of the joint opening time histories of the middle joints, as shown in Fig.8.

openings, we can see with different foundation models, the response distributions exhibit very similar pattern, From the results of displacements and joint only the peak values decrease proportionally due to the effects of the infinite canyon radiation damping.

 and and cantilever components, as shown in Figs. 9 10 , their stress distributions represent very similar Comparing the maximum stress envelops in arch patterns but the obvious reduction of the stress level observed as a whole is only perceivable. The reduction of the maximum stress is about 10%.

4 Conclusions

The paper presents a seismic analysis model considering the contraction joint non-linearity coupled with the infinite mass foundation, which can be applied to evaluate the effects of the infinite canyon radiation damping on the responses of arch dams under uniform free field input motion. Through its application to Xiaowan arch dam, some conclusions can be drawn as follows:

obviously dissipates the vibrating energy, but will not hav e much effect on the vibration modal shapes of the (1) The radiation damping of infinite canyon dam-canyon-reservoir system.

infinite canyon, significant reduction of the dynamic res ponses can be observed in dynamic displacements (2) With the radiation damping considered in the and joint openings. However, the maximum stress envelops behave only a perceivable reduction in general. The reduction extent of the maximum displacements and joint openings is about 20%, meanwhile of stresses, about 10%. This conclusion shows the beneficial effectiveness of considering radiation damping in the aseismic safety evaluation of arch dams.

rating the non-uniform free-field input procedure into eng ineering applications. (3) Further efforts could be devoted to incorpo-

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