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Multi-Waves, Breathers, Periodic and Cross-Kink Solutions to the (2+1)-Dimensional Variable-Coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada Equation

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Abstract The present article deals with multi-waves and breathers solution of the (2+1)-dimensional variable-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation under the Hirota bilinear operator method. The obtained solutions for solving the current equation represent some localized waves including soliton, solitary wave solutions, periodic and cross-kink solutions in which have been investigated by the approach of the bilinear method. Mainly, by choosing specific parameter constraints in the multi-waves and breathers, all cases the periodic and cross-kink solutions can be captured from the 1- and 2-soliton. The obtained solutions are extended with numerical simulation to analyze graphically, which results in 1- and 2-soliton solutions and also periodic and cross-kink solutions profiles. That will be extensively used to report many attractive physical phenomena in the fields of acoustics, heat transfer, fluid dynamics, classical mechanics, and so on. We have shown that the assigned method is further general, efficient, straightforward, and powerful and can be exerted to establish exact solutions of diverse kinds of fractional equations originated in mathematical physics and engineering. We have depicted the figures of the evaluated solutions in order to interpret the physical phenomena.

Key words variable-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation; Hirota bilinear operator method; soliton; multi-waves and breathers; periodic and cross-kink; solitray wave solutions

1 Introduction

Partial differential equations (PDEs) play important roles in the numerous areas such as biology, physics, chemistry, fluid mechanics and many engineering and sciences applications among others (Dai *et al.*, 2008; Dehghan *et al.*, 2011; Ma and Zhu, 2012; Manafian and Lakestani, 2016a; Foroutan *et al.*, 2018). Furthermore, the approaches to solving these types of equations alongside nonlinear PDEs ranging from analytical to numerical methods are very important in many engineering and sciences applications. Some of these methods include finding the exact solutions by using the special techniques in which can be manifested to new works with the vigorous references (Dehghan and Manafian, 2009; Wang *et al.*, 2010; Manafian, 2015; Baskonus and Bulut, 2016; Manafian and Lakestani, 2016b; Tang *et al.*, 2016; Zhou *et al.*, 2016; Gao, 2017; Wang and Liu, 2018; Chen *et al.*, 2019a).

The nonlinear (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada equation is given as

$$36u_t+u_{xxxxx}+15(uu_{xx})_x+45u^2u_x-5u_{xxy}-$$
$$15uu_y-15u_x\partial_x^{-1}u_y-5\partial_x^{-1}u_{yy}=0\,, \qquad (1)$$

in which $u=u(x,y,t)$, and $\partial_x^{-1}=\int(\cdot)\mathrm{d}x$ which was first proposed by Konpelchenko and Dubrovsky by the help of the inverse scattering transform method (Konopelchenko and Dubrovsky, 1984). The nonlinear (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada equation has been investigated for finding the exact solutions in which can be pointed to vigorus works containing the algebraic method with symbolic computation (Yang, 2006), the solitary waves and lump waves with interaction phenomena by the way of vector notations (Peng *et al.*, 2018), the quasi-periodic solutions by the Riemann theta functions (Cao *et al.*, 1999; Geng *et al.*, 2019), some novel group invariant solutions by utilizing the classical symmetry reduction method (Cheng *et al.*, 2019), and in continue we take the (2+1)-dimensional variable coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada (VC CDGKS) equation which reads

$$u_t+b_1u_{xxxxx}+b_2u_xu_{xx}+b_3uu_{xxx}+b_4u^2u_x+b_5u_{xxy}+$$

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$$b_6\partial_x^{-1}u_{yy}+b_7u_x\partial_x^{-1}u_y+b_8uu_y+b_9u=0\,, \quad (2)$$

in which $b_j=b_j(t)$ ($j=1, 2, \cdots, 9$) are functions with respect to t. Cheng *et al.* (2014) obtained the bilinear form, bilinear BT, Lax pair, and infinite conservation law for Eq. (2). Moreover, the VC CDGKS equation with essential applications in the incompressible fluid has been investigated by Wang *et al.* (2019a), and new non-traveling lump solutions, their interaction solutions, and mixed lump-kink solutions for the considered equation have been achieved to explain relative physics or expect some new physical phenomena.

For getting to the lump solutions and their interactions authors have conjugated sufficient time to search the exact rational soliton solutions, for example, the Kadomtsev-Petviashvili (KP) equation (Ma, 2015), the B-Kadomtsev-Petviashvili equation (Yang and Ma, 2016), the reduced p-gKP and p-gbKP equations (Ma *et al.*, 2016), the (2+1)-dimensional KdV equation (Wang, 2016), the (2+1)-dimensional generalized fifth-order KdV equation (Lü *et al.*, 2018), the (2+1)-dimensional Burger equation (Wang *et al.*, 2016), the nonlinear evolution equations (Tang *et al.*, 2016), the generalized (3+1)-dimensional Shallow water-like equation (Zhang *et al.*, 2017), (2+1)-dimensional Sawada-Kotera equation (Huang and Chen, 2017), and (2+1)-dimensional bSK equation (Lu and Bilige, 2017; Manafian and Lakestani, 2019). Various types of work for finding the periodic solitary wave solutions on the (2+1)-dimensional extended Jimbo-Miwa equations (Manafian, 2018), the interaction between lump and other kinds of solitary, periodic and kink solitons for the (2+1)-dimensional Breaking Soliton equation (Manafian *et al.*, 2019), the lump and interaction between different types of those on the variable-coefficient Kadomtsev-Petviashvili equation (Ilhan *et al.*, 2019), and periodic type and periodic cross-kink wave solutions (Ilhan and Manafian, 2019) are achieved through the Hirota bilinear operator.

Due to simplifications obtained by Cheng *et al.* (2019) as the below form, we have

$$\begin{cases} b_2=b_3=\dfrac{15b_1}{\lambda_0}\exp\left(\int b_9\mathrm{d}t\right),\; b_4=\dfrac{45b_1}{\lambda_0^2}\exp\left(2\int b_9\mathrm{d}t\right) \\ b_5=5\lambda_1 b_1,\; b_6=-5\lambda_1^2 b_1,\; b_7=b_8=\dfrac{15\lambda_1 b_1}{\lambda_0}\exp\left(\int b_9\mathrm{d}t\right) \end{cases}, \quad (3)$$

in which λ_0 and λ_1 are the free constants. Through the relation between u and f, one can get to the following conversion as

$$u=2\lambda_0\,\mathrm{e}^{-\int b_9(t)\mathrm{d}t}(\ln f)_{xx}\,. \quad (4)$$

Then, by using Eq. (4) in Eq. (2), the following bilinear model concludes

$$\left(D_xD_t+b_1D_x^6+5\lambda_1b_1D_x^3D_y-5\lambda_1^2b_1D_y^2\right)f\cdot f=0\,. \quad (5)$$

Suppose the Hirota derivatives in terms of the functions f and g can be written as

$$\prod_{i=1}^{4}D_{j_i}^{\beta_i}f\cdot g=\prod_{i=1}^{4}\left(\frac{\partial}{\partial j_i}-\frac{\partial}{\partial j_{i'}}\right)^{\beta_i}f(j)g(j')\Bigg|_{j'=j}\,, \quad (6)$$

where the vectors $j=(j_1, j_2, j_3, j_4)=(x, y, z, t)$, $j'=(j'_1, j'_2, j'_3, j'_4)=(x', y', z', t')$ and β_1, β_2, β_3, β_4 are the arbitrary non-negative integers, and its corresponding bilinear formalism equals as below form

$$\begin{aligned} &2(ff_{xt}-f_xf_t)+\\ &2b_1(ff_{xxxxxx}-6f_xf_{xxxxx}+15f_{xx}f_{xxxx}-10f_{xxx}^2)+\\ &10\lambda_1b_1(ff_{xxxy}-f_yf_{xxx}-3f_{xxy}f_x+3f_{xx}f_{xy})-\\ &10\lambda_1^2b_1(ff_{yy}-f_y^2)=0\,. \end{aligned} \quad (7)$$

The soliton solutions to a few (3+1)-dimensional generalized nonlinear integrable equations have been constructed. Recently, a special kind of reductions of soliton solutions to rational functions that are actively studying is lump solutions to nonlinear partial differential equations by Ma and Zhou (2018), their interactions with solitons to Hirota-Satsuma-Ito equation in (2+1)-dimensions by Ma (2019a), and even for linear PDEs by the same author (Ma, 2019b). Also, Ma (2020) presented the inverse scattering transforms and soliton solutions for nonlocal reverse-time nonlinear Schrodinger equations. The same author offered an application of the nonlinear steepest descent method to a three-component coupled mKdV system associated with a 4×4 matrix spectral problem (Ma, 2019b). Nowadays NLPDEs have been creating a significant opportunity for the researchers to explain the tangible incidents. Therefore, mathematicians and scientists are working tirelessly to bring out different kinds of soliton solutions. As a result, in the past few years several effective, rising and realistic methods have been initiated and dilated to extract closed-form solutions to the NLPDEs, videlicet, observational/experimental consideration on certain (2+1)-dimensional waves in the cosmic/laboratory dusty plasmas (Gao, 2019), a vector nonlinear Schrodinger equation in a birefringent optical fiber (Yin *et al.*, 2020), dark-bright semi-rational solitons and breathers for a higher-order coupled nonlinear Schrodinger system (Du *et al.*, 2020), conservation laws of a (2+1)-dimensional nonlinear Schrodinger equation (Du *et al.*, 2019), rogue waves, and modulation instability for the coherently coupled nonlinear Schrodinger equations (Chen *et al.*, 2019b), the higher-order Boussinesq-Burgers system, auto- and non-auto-Bäcklund transformations (Gao *et al.*, 2020), lump wave-soliton interactions for a (3+1)-dimensional generalized Kadomtsev-Petviashvili equation (Hu *et al.*, 2019), rogue waves for a (2+1)-dimensional reduced Yu-Toda-Sasa-Fukuyama equation (Wang *et al.*, 2019b), and interactions of the couple Fokas-Lenells system (Zhang *et al.*, 2020). We clearly confirm that others' published papers do not cover theirs and made work is really new.

Our purpose here is to discover exact solutions of the VC CDGKS equation under consideration of the Hirota bilinear method for getting the multi-waves, breathers solution, periodic solution, cross-kink solution, and new solitary wave solutions in which can be captured from the 1- and 2-soliton. Discussion about the nonlinear VC CDGKS equation and the Hirota bilinear method is given. In the continuation, we will offer the graphical illustrations of some solutions of the considered model. After that, we will deal with the probe of solutions and we will finish by a conclusion.

2 New Multi-Waves Solutions for VC CDGKS Equation

Here, we will compose multi-waves solutions of the Eq. (2), we choose the three waves hypothesis which can be discovered through employing Hirota operator (Geng and Ma, 2007). The solution can be expressed in the below form as:

$$\begin{cases} f = q_1 H_1 + q_2 H_2 + q_3 H_3 \\ H_1 = \cos(a_1 x + a_2 y + a_3 t + a_4),\ H_2 = \cos(b_1 x + b_2 y + b_3 t + b_4),\ H_3 = \cos(c_1 x + c_2 y + c_3 t + c_4) \end{cases},$$

$$u = 2\lambda_0 e^{\int b_9(t)\mathrm{d}t} \frac{\partial^2}{\partial x^2}\ln(f) = 2\left[\frac{a_1^2 q_1 H_1 - b_1^2 q_2 H_2 + c_1^2 q_3 H_3}{f} - \frac{(a_1 q_1 H_{1'} - b_1 q_2 H_{2'} + c_1 q_3 H_{3'})^2}{f^2}\right], \tag{8}$$

where a_i, b_i, c_i, q_j, $i = 1, \cdots, 4$, $j = 1, 2, 3$ are the free parameters in which are to find later. Plugging relations (8) into the Eq. (7) and then collecting the coefficients, we get to system of the nonlinear algebraic equations.

Solving the obtained equations we achieve to obtained cases:

$$\begin{cases} a_1 = a_1,\ a_2 = -\dfrac{a_1^3}{\lambda_1},\ a_3 = 9a_1^5\delta_1,\ a_4 = a_4,\ b_1 = b_1,\ b_2 = \dfrac{b_1^3}{\lambda_1},\ b_3 = 9b_1^5\delta_1,\ b_4 = b_4 \\ c_1 = c_1,\ c_2 = -\dfrac{c_1^3}{\lambda_1},\ c_3 = 9c_1^5\delta_1,\ c_4 = c_4,\ q_1 = q_1,\ q_2 = q_2,\ q_3 = q_3 \end{cases}. \tag{9}$$

Inserting Eq. (9) into Eq. (8), we get a multi-wave solution of the Eq. (2) as follows:

$$\begin{cases} u_2 = 2\lambda_0 e^{\int b_9(t)\mathrm{d}t} \dfrac{\partial^2}{\partial x^2}\ln(f) = 2\left[\dfrac{a_1^2 q_1 H_1 - b_1^2 q_2 H_2 + c_1^2 q_3 H_3}{f} - \dfrac{(a_1 q_1 H_{1'} - b_1 q_2 H_{2'} + c_1 q_3 H_{3'})^2}{f^2}\right] \\ f = q_1 H_1 + q_2 H_2 + q_3 H_3 \end{cases}, \tag{10}$$

$$H_1 = q_1\cosh\left(a_1 x - \frac{a_1^3}{\lambda_1} y + 9a_1^5\delta_1 t + a_4\right),\ H_2 = q_2\cos\left(b_1 x + \frac{b_1^3}{\lambda_1} y + 9b_1^5\delta_1 t + b_4\right),\ H_3 = q_3\cosh\left(c_1 x - \frac{c_1^3}{\lambda_1} y + 9c_1^5\delta_1 t + c_4\right),$$

where $H_{1'} = q_1\sinh(\cdots)$, $H_{3'} = q_1\sinh(\cdots)$, a_1, a_4, b_1, b_4, c_1, c_4, q_1, q_2 and q_3 are arbitrary values.

Moreover, we obtained five sets of solutions as mentioned above, we neglect to bring those categories of solutions (see Fig.1).

3 New Breather Solutions for VC CDGKS Equation

Here, we will compose breather wave solutions of the

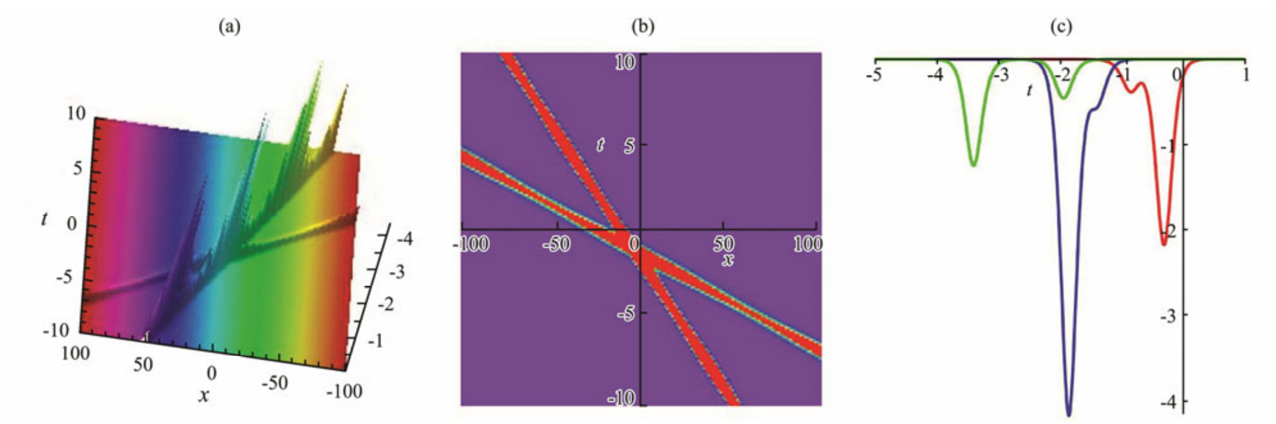

Fig.1 Diagram of multi-waves Eq. (10) using values $a_1=0.6$, $a_4=1$, $b_1=0.5$, $b_4=1$, $c_1=1.2$, $c_4=1$, $q_1=2$, $q_2=0.5$, $q_3=2$, $\delta_1=0.5$, $\lambda_0=-1$, $\lambda_1=1$, $b_9(t)=\cos(t)$, $y=-10$, and (a) 3D plot, (b) density plot, and (c) 2D plot with (red $x=-1$, blue $x=0$, and green $x=1$).

Eq. (2), we choose the following function that can be expressed in the below form as:

$$\begin{cases} f=q_1H_1+q_2H_2+q_3H_3 \\ H_1=\exp(a_1x+a_2y+a_3t+a_4),\ H_2=\cos(b_1x+b_2y+b_3t+b_4),\ H_3=\exp(-a_1x-a_2y-a_3t-a_4) \end{cases},$$

$$u=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\frac{\partial^2}{\partial x^2}\ln(f)=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\frac{a_1^2q_1H_1-b_1^2q_2H_2+a_1^2q_3H_3}{f}-\frac{(a_1q_1H_{1'}-b_1q_2H_{2'}-a_1q_3H_{3'})^2}{f^2}\right], \tag{11}$$

where a_i, b_i, q_j, $i=1, \cdots, 4$, $j=1, 2, 3$ are the free parameters in which are to find later. Plugging Eq. (11) into the Eq. (7) and then collecting the coefficients, we get to system of the nonlinear algebraic equations.

Solving the obtained equations we achieve to obtained cases:

$$\begin{cases} a_1=a_1,\ a_2=a_2 \\ a_3=-\dfrac{\delta_1\left(a_1^7-5\lambda_1^2\left(a_1a_2^2-a_1b_2^2+2a_2b_1b_2\right)-a_1b_1^2\left(9a_1^4+5a_1^2b_1^2-5b_1^4\right)+5\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2a_2-2a_1b_1b_2-a_2b_1^2\right)\right)}{a_1^2+b_1^2} \\ a_4=a_4,\ b_1=b_1,\ b_2=b_2 \\ b_3=-\dfrac{\delta_1\left(b_1^7-5\lambda_1^2\left(2a_1a_2b_2-a_2^2b_1+b_1b_2^2+a_1^2b_1\left(5a_1^4-5a_1^2b_1^2-9b_1^4\right)\right)+5\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2b_2+2a_1a_2b_1-b_1^2b_2\right)\right)}{a_1^2+b_1^2} \\ b_4=b_4,\ q_2=q_2,\ q_3=q_3 \\ q_1=-\dfrac{q_2^2\left(a_1^2b_1^2\left(a_1^4-a_1^2b_1^2-5b_1^4\right)+b_1\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2b_2+2a_1a_2b_1+3b_1^2b_2\right)\lambda_1^2\left(a_1b_2-a_2b_1\right)^2-3b_1^8\right)}{4q_3\left(a_1^2b_1^2\left(5a_1^4+a_1^2b_1^2-b_1^4\right)+a_1\lambda_1\left(a_1^2+b_1^2\right)\left(3a_1^2a_2+2a_1b_1b_2+a_2b_1^2\right)-\lambda_1^2\left(a_1b_2-a_2b_1\right)^2+3a_1^8\right)} \end{cases}. \tag{12}$$

Inserting Eq. (12) into Eq. (11), we get a breather wave solution of the Eq. (2) as follows:

$$\begin{cases} u_2=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\dfrac{\partial^2}{\partial x^2}\ln(f)=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\dfrac{a_1^2q_1H_1-b_1^2q_2H_2+a_1^2q_3H_3}{f}-\dfrac{(a_1q_1H_{1'}-b_1q_2H_{2'}-a_1q_3H_{3'})^2}{f^2}\right] \\ f=q_1H_1+q_2H_2+q_3H_3 \end{cases}, \tag{13}$$

$$\begin{cases} H_1=q_1\exp\left(a_1x+a_2y-\dfrac{\delta_1\left(a_1^7-5\lambda_1^2\left(a_1a_2^2-a_1b_2^2+2a_2b_1b_2\right)-a_1b_1^2\left(9a_1^4+5a_1^2b_1^2-5b_1^4\right)+5\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2a_2-2a_1b_1b_2-a_2b_1^2\right)\right)}{a_1^2+b_1^2}t-a_4\right) \\ H_2=q_2\cos\left(b_1x+b_1y-\dfrac{\delta_1\left(b_1^7-5\lambda_1^2\left(2a_1a_2b_2-a_2^2b_1+b_1b_2^2+a_1^2b_1\left(5a_1^4-5a_1^2b_1^2-9b_1^4\right)\right)+5\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2b_2+2a_1a_2b_1-b_1^2b_2\right)\right)}{a_1^2+b_1^2}t+b_4\right) \\ H_3=q_3\exp\left(-a_1x-a_2y+\dfrac{\delta_1\left(a_1^7-5\lambda_1^2\left(a_1a_2^2-a_1b_2^2+2a_2b_1b_2\right)-a_1b_1^2\left(9a_1^4+5a_1^2b_1^2-5b_1^4\right)+5\lambda_1\left(a_1^2+b_1^2\right)\left(a_1^2a_2-2a_1b_1b_2-a_2b_1^2\right)\right)}{a_1^2+b_1^2}t-a_4\right) \end{cases},$$

where $H_{2'}=\sin(\cdots)$, $H_{1'}=H_{3'}=\exp(\cdots)$, a_4, b_1, b_4, q_2 and q_3 are the arbitrary values (see Fig.2).

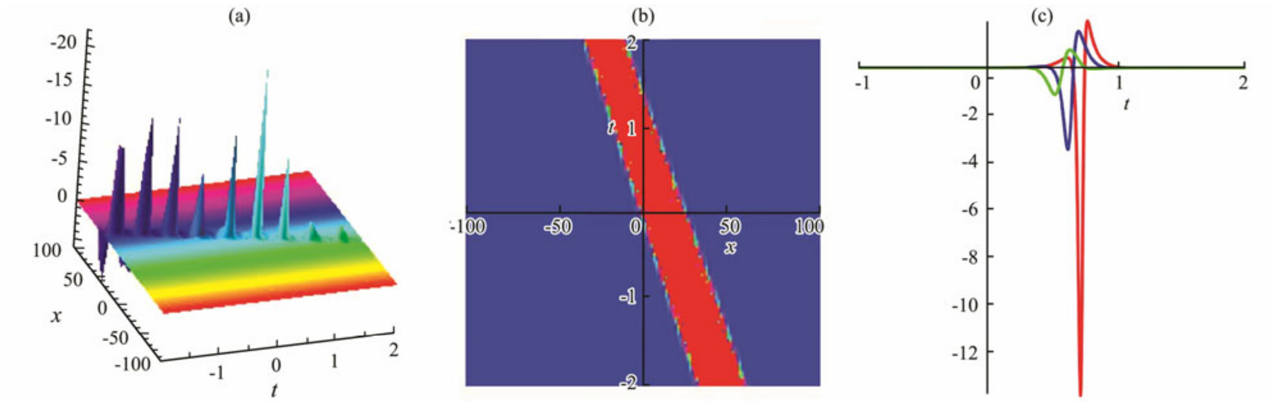

Fig.2 Diagram of breather wave Eq. (13) using values $a_1=1$, $a_2=1.2$, $a_4=2$, $b_1=1.5$, $b_4=1$, $q_2=1$, $q_3=2$, $\delta_1=0.5$, $\lambda_0=-1$, $\lambda_1=1$, $b_9(t)=\cos(t)$, $y=-10$, and (a) 3D plot, (b) density plot, and (c) 2D plot with (red $x=-1$, blue $x=0$, and green $x=1$).

4 New Instanton Wave Solution for VC CDGKS Equation

Here, we will compose a special rogue-wave that is generated by cutting the lump wave through a pair of resonance stripe soliton waves of the Eq. (2), we choose the following function that can be expressed in the below form as:

$$\begin{cases} f=H_1^2+H_2^2+q_1H_3 \\ H_1=a_1x+a_2y+a_3t+a_4 \\ H_2=b_1x+b_2y+b_3t+b_4 \\ H_3=\cosh(c_1x+c_2y+c_3t+c_4) \\ H_{3'}=\sinh(c_1x+c_2y+c_3t+c_4) \end{cases},$$

$$u=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\frac{\partial^2}{\partial x^2}\ln(f)=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\frac{2a_1^2+2b_1^2+c_1^2q_1H_3}{f}-\frac{(2a_1H_1+2b_1H_2+c_1q_1H_{3'})^2}{f^2}\right], \tag{14}$$

where a_i, b_i, c_i $(i=1,\cdots,4)$ and q_1, are the free parameters in which are to find later. Plugging Eq. (14) into the Eq. (7) and then collecting the coefficients, we get to the follow- ing results:

$$\begin{cases} a_1=a_1,\ a_2=\dfrac{3c_1^2\left(a_1^2-3b_1^2\right)\left(a_1^2+b_1^2\right)}{16a_1\lambda_1b_1^2},\ a_3=\dfrac{45\delta_1c_1^4\left(a_1^8-20a_1^6b_1^2+14a_1^4b_1^4+28a_1^2b_1^6-7b_1^8\right)}{256a_1^3b_1^4},\ a_4=\dfrac{a_1b_4}{b_1} \\ b_1=b_1,\ b_2=-\dfrac{3c_1^2\left(3a_1^4+2a_1^2b_1^2-b_1^4\right)}{16a_1^2b_1\lambda_1},\ b_3=-\dfrac{45c_1^4\delta_1\left(7a_1^8-28a_1^6b_1^2-14a_1^4b_1^4+20a_1^2b_1^6-b_1^8\right)}{256b_1^3a_1^4},\ b_4=b_4 \\ c_1=c_1,\ c_2=\dfrac{c_1^3\left(3a_1^4-10a_1^2b_1^2+3b_1^4\right)}{16a_1^2b_1^2\lambda_1},\ c_3=\dfrac{9\delta_1c_1^5\left(5a_1^8-60a_1^6b_1^2+126a_1^4b_1^4-60a_1^2b_1^6+5b_1^8\right)}{256a_1^4b_1^4},\ c_4=c_4 \\ q_1=\dfrac{2}{c_1^2}\sqrt{6a_1^2b_1^2-a_1^4-b_1^4} \end{cases}. \tag{15}$$

Inserting Eq. (15) into Eq. (14), we get a instanton wave solution of the Eq. (2) as follows:

$$\begin{cases} u_3=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\dfrac{2a_1^2+2b_1^2+c_1^2q_1H_3}{f}-\dfrac{(2a_1H_1+2b_1H_2+c_1q_1H_{3'})^2}{f^2}\right] \\ f=H_1^2+H_2^2+\dfrac{2}{c_1^2}\sqrt{6a_1^2b_1^2-a_1^4-b_1^4}\,H_3 \end{cases}, \tag{16}$$

$$\begin{cases} H_1=a_1x+\dfrac{3c_1^2\left(a_1^2-3b_1^2\right)\left(a_1^2+b_1^2\right)}{16a_1\lambda_1b_1^2}y+\dfrac{45\delta_1c_1^4\left(a_1^8-20a_1^6b_1^2+14a_1^4b_1^4+28a_1^2b_1^6-7b_1^8\right)}{256a_1^3b_1^4}t+a_4=\dfrac{a_1b_4}{b_1} \\ H_2=b_1x-\dfrac{3c_1^2\left(3a_1^4+2a_1^2b_1^2-b_1^4\right)}{16a_1^2b_1\lambda_1}y-\dfrac{45c_1^4\delta_1\left(7a_1^8-28a_1^6b_1^2-14a_1^4b_1^4+20a_1^2b_1^6-b_1^8\right)}{256b_1^3a_1^4}t+b_4 \\ H_3=\cosh\left(c_1x+\dfrac{c_1^3\left(3a_1^4-10a_1^2b_1^2+3b_1^4\right)}{16a_1^2b_1^2\lambda_1}y+\dfrac{9\delta_1c_1^5\left(5a_1^8-60a_1^6b_1^2+126a_1^4b_1^4-60a_1^2b_1^6+5b_1^8\right)}{256a_1^4b_1^4}t+c_4\right) \end{cases},$$

where $H_{3'}=\sinh(\cdots)$, a_1, a_4, b_1, b_4, and c_4 are the arbitrary values (see Fig.3).

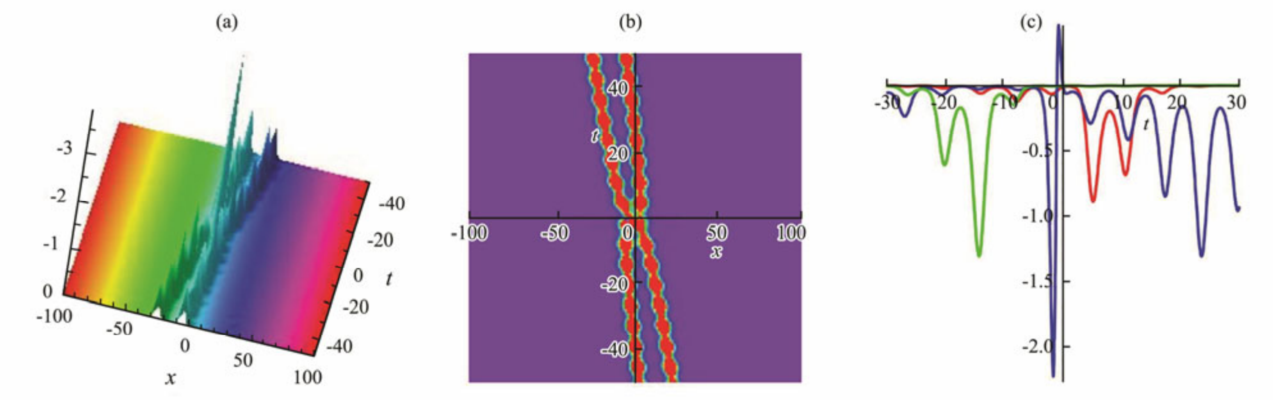

Fig.3 Diagram of instanton wave Eq. (16) using values $b_1=1.5$, $b_4=1$, $c_1=0.5$, $c_2=0.2$, $c_4=1$, $\delta_1=0.5$, $\lambda_0=-1$, $\lambda_1=1$, $b_9(t)=\cos(t)$, $q_1=2$, $y=-4$, and (a) 3D plot, (b) density plot, and (c) 2D plot with (red $x=-1$, blue $x=0$, and green $x=1$).

5 Novel Periodic Wave Solutions of the VC CDGKS Equation

To get for the periodic wave solutions of the VC CDGKS equation, we would like to commence from a function as below form

$$\begin{cases} f=H_1+H_2+H_3+H_4+k_4 \\ H_1=k_1 e^{a_1x+a_2y+a_3t+a_4} \\ H_2=e^{-a_1x-a_2y-a_3t-a_4} \\ H_3=k_2\cos(b_1x+b_2y+b_3t+b_4) \\ H_4=k_3\cosh(c_1x+c_2y+c_3t+c_4) \end{cases},$$

$$u=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\frac{\partial^2}{\partial x^2}\ln(f)=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\frac{a_1^2H_1+a_1^2H_2-b_1^2H_3+c_1^2H_4}{f}-\frac{(a_1H_1-a_1H_2-b_1H_{3'}+c_1H_{4'})^2}{f^2}\right], \tag{17}$$

where a_i, b_i, c_i, k_j ($i=1, \cdots, 4$; $j=1, 2, 3$) are the free parameters in which are to find later. Plugging Eq. (17) into the Eq. (7) and then collecting the coefficients, we get to system of the nonlinear algebraic equations.

Solving the obtained equations we achieve to obtained cases:

$$\begin{cases} a_1=a_1,\ a_2=-\dfrac{\left(a_1^2+3c_1^2\right)a_1}{4\lambda_1},\ a_3=\dfrac{9}{16}\delta_1\left(a_1^4+10a_1^2c_1^2+5c_1^4\right),\ a_4=a_4 \\ b_1=b_1,\ b_2=b_2,\ b_3=b_3,\ b_4=b_4 \\ c_1=c_1,\ c_2=-\dfrac{\left(c_1^2+3a_1^2\right)a_1}{4\lambda_1},\ c_3=\dfrac{9}{16}c_1\delta_1\left(c_1^4+10a_1^2c_1^2+5a_1^4\right),\ c_4=c_4 \\ k_1=\dfrac{c_1^4k_3^2}{4a_1^4},\ k_2=0,\ k_3=k_3,\ k_4=k_4 \end{cases}. \tag{18}$$

Substituting Eq. (18) into Eq. (17), we obtain a periodic solution of the Eq. (2) as follows:

$$u_2=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\frac{a_1^2H_1+a_1^2H_2-b_1^2H_3+c_1^2H_4}{f}-\frac{(a_1H_1-a_1H_2-b_1H_{3'}+c_1H_{4'})^2}{f^2}\right], \tag{19}$$

$$\begin{cases} H_1=\dfrac{c_1^4k_3^2}{4a_1^4}e^{a_1x-\frac{\left(a_1^2+3c_1^2\right)a_1}{4\lambda_1}y+\frac{9}{16}\delta_1\left(a_1^4+10a_1^2c_1^2+5c_1^4\right)t+a_4} \\ H_2=e^{-a_1x+\frac{\left(a_1^2+3c_1^2\right)a_1}{4\lambda_1}y-\frac{9}{16}\delta_1\left(a_1^4+10a_1^2c_1^2+5c_1^4\right)t-a_4} \\ H_4=k_3\cosh\left(c_1c-\dfrac{\left(c_1^2+3a_1^2\right)a_1}{4\lambda_1}y+\dfrac{9}{16}c_1\delta_1\left(c_1^4+10a_1^2c_1^2+5a_1^4\right)t+c_4\right) \end{cases},$$

where a_1, a_4, c_1, c_4, k_1, k_3, and k_4 are the arbitrary values.

Moreover, we obtained twelve sets of solutions as mentioned above, we neglect to bring those categories of solutions (see Fig.4).

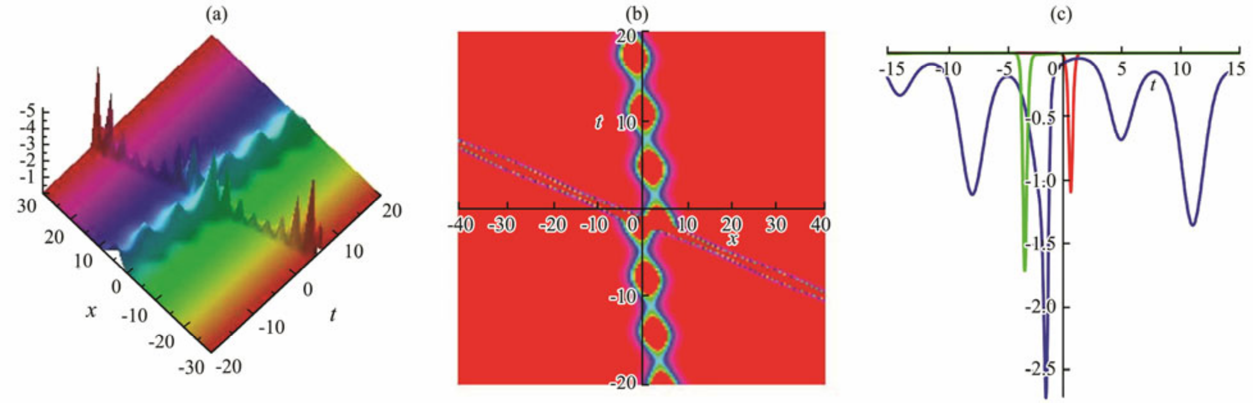

Fig.4 Diagram of periodic wave (19) using values $a_1=0.5$, $a_4=1$, $c_1=1.5$, $c_4=1$, $\delta_1=0.5$, $\lambda_0=-1$, $\lambda_1=1$, $b_9(t)=\cos(t)$, $k_1=1$, $k_3=1.5$, $k_4=2$, $y=-4$, and (a) 3D plot, (b) density plot, and (c) 2D plot with (red $x=-10$, blue $x=0$, and green $x=10$).

6 Novel Cross-Kink Wave Solutions of the VC CDGKS Equation

To get for the cross-kink wave solutions of the VC CDGKS equation, we would like to commence from a function as below form

$$\begin{cases} f=H_1+H_2+H_3+H_4+k_4 \\ H_1=k_1 e^{a_1x+a_2y+a_3t+a_4} \\ H_2=e^{-a_1x-a_2y-a_3t-a_4} \\ H_3=k_2\sin(b_1x+b_2y+b_3t+b_4) \\ H_4=k_3\sinh(c_1x+c_2y+c_3t+c_4) \end{cases},$$

$$u=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\frac{\partial^2}{\partial x^2}\ln(f)=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\frac{a_1^2H_1+a_1^2H_2-b_1^2H_3+c_1^2H_4}{f}-\frac{(a_1H_1-a_1H_2+b_1H_{3'}+c_1H_{4'})^2}{f^2}\right], \tag{20}$$

where a_i, b_i, c_i, k_j ($i=1, \cdots, 4$; $j=1, 2, 3$) are the free parameters in which are to find later. Plugging Eq. (20) into the Eq. (7) and then collecting the coefficients, we get to system of the nonlinear algebraic equations.

Solving the obtained equations we achieve to obtained cases:

$$\begin{cases} a_1=a_1,\ a_2=-\dfrac{a_1^3}{\lambda_1},\ a_3=9a_1^5\delta_1,\ a_4=a_4,\ b_2=\dfrac{b_1^3}{\lambda_1},\ b_3=9b_1^5\delta_1,\ b_4=b_4 \\ c_2=-\dfrac{c_1^3}{\lambda_1},\ c_3=9\delta_1c_1^5,\ c_4=c_4,\ k_1=k_1,\ k_2=k_2,\ k_3=k_3,\ k_4=k_4 \end{cases}. \tag{21}$$

Plugging Eq. (21) into relations (20), we get a cross-kink wave solution of the Eq. (2) as follows:

$$\begin{cases} u_4=2\lambda_0 e^{\int b_9(t)\mathrm{d}t}\left[\dfrac{a_1^2H_1+a_1^2H_2-b_1^2H_3+c_1^2H_4}{f}-\dfrac{(a_1H_1-a_1H_2+b_1H_{3'}+c_1H_{4'})^2}{f^2}\right], \\ f=H_1+H_2+H_3+H_4+k_4 \end{cases} \tag{22}$$

$$\begin{cases} H_1=k_1 e^{a_1x-\frac{a_1^3}{\lambda_1}y+9a_1^5\delta_1t+a_4},\ H_2=e^{-a_1x+\frac{a_1^3}{\lambda_1}y-9a_1^5\delta_1t-a_4} \\ H_3=k_2\sin\left(b_1x+\dfrac{b_1^3}{\lambda_1}y+9b_1^5\delta_1t+b_4\right) \\ H_4=k_3\sinh\left(c_1x-\dfrac{c_1^3}{\lambda_1}y+9\delta_1c_1^5t+c_4\right) \end{cases},$$

where a_1, a_4, b_1, b_4, c_1, c_4, k_1, k_2, k_3 and k_4 are the arbitrary values.

7 Novel Solitary Wave Solutions of the VC CDGKS Equation

To get for the new solitary wave solutions of the VC CDGKS equation, we would like to commence from a function as below form

$$\begin{cases} f = H_1 + H_2 + H_3 + H_4 \\ H_1 = k_1 \mathrm{e}^{a_1x+a_2y+a_3t+a_4} \\ H_2 = \mathrm{e}^{-a_1x-a_2y-a_3t-a_4} e \\ H_3 = k_2 \tan(b_1x+b_2y+b_3t+b_4) \\ H_4 = k_3 \tanh(c_1x+c_2y+c_3t+c_4) \end{cases},$$

$$u = 2\lambda_0 \mathrm{e}^{\int b_9(t)\mathrm{d}t} \frac{\partial^2}{\partial x^2}\ln(f)$$
$$= 2\lambda_0 \mathrm{e}^{\int b_9(t)\mathrm{d}t} \left[\frac{a_1^2H_1 + a_1^2H_2 + 2b_1^2H_3\left(1+\frac{H_3^2}{k_2^2}\right) - 2c_1^2H_4\left(1+\frac{H_4^2}{k_3^2}\right)}{f} - \frac{\left(a_1H_1 - a_1H_2 + k_2b_1\left(1+\frac{H_3^2}{k_2^2}\right) + k_3c_1\left(1+\frac{H_4^2}{k_3^2}\right)\right)^2}{f^2} \right], \quad (23)$$

where a_i, b_i, c_i, k_j ($i=1, \cdots, 4$; $j=1, 2, 3$) are the free parameters in which are to find later. Plugging Eq. (23) into the Eq. (7) and then collecting the coefficients, we get to the following results case:

$$\begin{cases} a_1 = a_1,\ a_2 = a_2,\ a_3 = a_3,\ a_4 = a_4 \\ b_1 = 0, b_2 = 0,\ b_3 = b_3,\ b_4 = b_4 \\ c_1 = 0,\ c_2 = 0,\ c_3 = c_3,\ c_4 = c_4 \\ k_1 = 0,\ k_2 = k_2,\ k_3 = k_3 \end{cases}. \quad (24)$$

Appending Eq. (24) into relations (23), we get a cross-kink wave solution of the Eq. (2) as follows:

$$\begin{cases} u_1 = 2\lambda_0 \mathrm{e}^{\int b_9(t)\mathrm{d}t} \left[\frac{a_1^2H_2}{f} - \frac{a_1^2H_2^2}{f^2} \right], \\ f = H_2 + H_3 + H_4 \end{cases} \quad (25)$$

$$\begin{cases} H_2 = \mathrm{e}^{-a_1x-a_2y-a_3t-a_4} \\ H_3 = k_2 \tan\left(b_3t+b_4\right) \\ H_4 = k_3 \tanh\left(c_3t+c_4\right) \end{cases},$$

where a_1, a_2, a_3, a_4, b_3, b_4, c_3, c_4, k_2 and k_3 are the arbitrary values.

We obtained twelve sets of solutions as mentioned above, we neglect to bring those categories of solutions. The three-dimensional dynamic graphs of the wave and corresponding density plots, contour plots, and two-dimensional plots were successfully depicted in Figs.1–4 with the help of the Maple. We can see that the exponential function, the sine function, and the hyperbolic sine function react with each other and move forward. Due to analyzing the dynamics properties briefly, we would like to discuss the evolution characteristic.

8 Conclusions

Through the symbolic calculation and employing the Hirota bilinear operator, we have discovered some novel analytic solutions for the VC CDGKS equation. As a consequence, some new solutions, which include the new multi-wave, breathers, periodic, cross-kink wave solutions were catched. Through of Maple, the evolution phenomenon of these waves is seen in Figs.1–4, respectively. The obtained solutions for solving the VC CDGKS equation shown some localized waves such as soliton, periodic and cross-kink solutions in which have been investigated by the approach of the bilinear method. Mainly, by choosing specific parameter constraints in all cases the two-dimension, and three-dimension in solitons can be captured from the multi-wave, breathers, periodic, cross-kink wave solutions. The obtained solutions are extended with numerical simulation to analyze graphically, which results in multi-wave, breather wave, periodic, cross-kink wave solutions. The attained solutions are in broad-ranging form and the definite values of the included parameters of the attained solutions yield the soliton solutions and help to analyze the quantum mechanics, the signal processing waves, the meteorology, and biomedical engineering, *etc.* That will be extensively used to report many attractive physical phenomena in the fields of acoustics, heat transfer, fluid dynamics, classical mechanics, and so on. Moreover, the established results have shown that the Hirota bilinear method is further general, straightforward, and more powerful and helped to examine traveling wave solutions of NLPDEs.

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