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Metocean Design Parameter Estimation for Fixed Platform Based on Copula Functions

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Abstract Considering the dependent relationship among wave height, wind speed, and current velocity, we construct novel trivariate joint probability distributions via Archimedean copula functions. Total 30-year data of wave height, wind speed, and current velocity in the Bohai Sea are hindcast and sampled for case study. Four kinds of distributions, namely, Gumbel distribution, lognormal distribution, Weibull distribution, and Pearson Type III distribution, are candidate models for marginal distributions of wave height, wind speed, and current velocity. The Pearson Type III distribution is selected as the optimal model. Bivariate and trivariate probability distributions of these environmental conditions are established based on four bivariate and trivariate Archimedean copulas, namely, Clayton, Frank, Gumbel-Hougaard, and Ali-Mikhail-Haq copulas. These joint probability models can maximize marginal information and the dependence among the three variables. The design return values of these three variables can be obtained by three methods: univariate probability, conditional probability, and joint probability. The joint return periods of different load combinations are estimated by the proposed models. Platform responses (including base shear, overturning moment, and deck displacement) are further calculated. For the same return period, the design values of wave height, wind speed, and current velocity obtained by the conditional and joint probability models are much smaller than those by univariate probability. Considering the dependence among variables, the multivariate probability distributions provide close design parameters to actual sea state for ocean platform design.

Key words Archimedean copulas; univariate distribution; bivariate distribution; trivariate distribution; platform response

1 Introduction

Offshore platforms have been widely utilized for oil and gas production in the Bohai Sea (Yang and Zhang, 2013), which must be designed under low failure probabilities with complicated structures and high costs (Ewans and Jonathan, 2014). Fixed offshore platforms are subjected to various ocean environmental loads, such as waves, wind, current, storm surges, and earthquakes, during their lifetime. Designers need to estimate the return values of these environmental conditions, and extreme frequency analysis should be conducted (Zhang *et al.*, 2015). However, previous design criteria of ocean environmental parameters are calculated by univariate distribution (API, 2005), which regards ocean environmental factors (waves, wind, and current) as independent variables. This method may result in overestimation of the extreme loads and conservative designs. Considering the dependence among variables, multivariate joint probability distributions are more suitable to actual circumstances than univariate distributions (De Michele *et al.*, 2007).

Traditional multivariate statistical analyses of wave height and wind speed can be found in the literature. For example, Duan *et al.* (2002) established the joint probability distribution of wind speed and significant wave height based on multivariate extreme theory. Morton and Bowers (1996), Zachary *et al.* (1998), Nerzic and Prevosto (2000), Dong (2007), and Dong *et al.* (2008) accounted for the dependence between wave height and wind speed using the bivariate logistic model, bivariate extreme value model, and bivariate lognormal distribution. However, the prerequisite of the above models is the same type of marginal distribution for different variables.

Copula functions proposed by Sklar (1959) can combine the marginal distributions of different ocean environmental conditions with some correlations among them and eventually lead to a joint probability distribution (Nelsen, 2006). The margins can be chosen from different types of distributions, which may be useful in coping with limitations of traditional multivariate models. Copulas have an accurate and reliable statistical description of all the relevant margins, so they are efficient tools to construct multivariate joint distributions (Joe, 1997; Nelsen, 2006; Wist *et al.*, 2005; De Michele *et al.*, 2007; Muhaisen *et al.*, 2010; Dong *et al.*, 2012; Corbella and Stretch, 2013; Dong *et al.*, 2015). In recent years, copula functions have been extensively applied in coastal and offshore engineering. Qin *et al.* (2007) established the joint

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probability distribution of maximum effective wave height and maximum wind speed using copula functions. Chen (2011) used bivariate copula functions to analyze the joint probability distribution of extreme wave height and wind speed in the Shanwei sea area. Yang and Zhang (2013) utilized Gumbel-Hougaard and Clayton copulas to construct joint probability distributions of winds and waves. Zhai *et al*. (2014) built the bivariate distribution of annual maximum wave height and corresponding wind speed with Clayton copula, and they applied it in the calculation of load design standards for ocean platforms to decrease the load design standards. Dong and Li (2015) applied Plackett copula to construct the trivariate joint probability distribution of wave height, wind velocity, and current velocity. Dong *et al*. (2016) constructed multivariate distributions with the fully nested copulas to compute the reliability of vertical breakwater.

The bivariate copulas are widely used in coastal and ocean engineering, but the trivariate or higher dimensional copulas are rarely utilized. In this paper, the trivariate joint design criteria of wave height, wind speed, and current velocity are proposed in accordance with the 30-year data from the Bohai Sea. The content includes three aspects: the analysis of dependence among wave height, wind speed, and current velocity; the construction of their trivariate joint probability distributions; and the adoption of joint design criteria to calculate metocean parameters for platform design. Section 2 presents four kinds of bivariate and trivariate Archimedean copulas. Section 3 shows the model selection methods of univariate, bivariate, and trivariate probability models. In Section 4, a case study is conducted to verify the efficiency of joint distributions and design criteria. Some conclusions are summarized in Section 5.

2 Joint Probability Distributions Based on Copula Functions

2.1 Copula Theory

Copulas model dependence between random variables separately from a marginal distribution. Therefore, the use of copulas in environmental science is rapidly developing. If H is a multivariate distribution function with marginal cumulative function $u_i=F_X(x_i)$, then a n-dimensional copula C exists; for all $X\in R^n$, the link between a multivariate distribution H and the associated n-dimensional copula C is given by the functional identity stated by Sklar's Theorem (Sklar, 1959; Salvadori, 2004; Nelsen, 2006; Prokhorov, 2008):

$$H\left(x_1,x_2,\cdots,x_n\right)=C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,u_n\right)$$
$$=C_{F_1,F_2,\cdots,F_n}\left(F_1\left(x_1\right),F_2\left(x_2\right),\cdots,F_n\left(x_n\right)\right), \quad (1)$$

$$C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,u_n\right)>0,\ u_i\neq 0, \quad (2)$$

$$C_{U_1,U_2,\cdots,U_n}\left(0,u_2,\cdots,u_n\right)=C_{U_1,U_2,\cdots,U_n}\left(u_1,0,\cdots,u_n\right)$$
$$=C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,0\right)=0, \quad (3)$$

$$C_{U_1,U_2,\cdots,U_n}\left(1,u_2,\cdots,u_n\right)=C_{U_1,U_2,\cdots,U_n}\left(u_1,1,\cdots,u_n\right)$$
$$=C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,1\right)=1. \quad (4)$$

A multivariate copula $C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,u_n\right)$ is simply a joint distribution over $\boldsymbol{I}^n=[0, 1]^n$ with uniform marginal. If $u_i=F_{X_i}\left(x_i\right)$ is continuous, C is unique; otherwise, C is uniquely defined on $\mathrm{Ran}(u_1)\times\mathrm{Ran}(u_2)\times\cdots\times\mathrm{Ran}(u_n)$.

Let $F_{X_1,X_2,\cdots,X_n}\left(x_1,x_2,\cdots,x_n\right)$ represent the joint density distribution of $(X_1, X_2, \cdots, X_n)$. The density function $c[u_1, u_2, \cdots, u_n]$ of the copula together with a marginal probability density distribution $f_i(x_i)$ of variables x_i are then used to construct a joint probability density function $f(x_1, x_2,\cdots, x_n)$

$$h(x_1,x_2,\cdots,x_n)=\frac{\partial^N C_{U_1,U_2,\cdots,U_n}\left(u_1,u_2,\cdots,u_n\right)}{\partial u_1\partial u_2\cdots\partial u_n}\prod_{i=1}^{n}f_i(x_i)$$
$$=c\left(u_1,u_2,\cdots,u_n\right)\times\prod_{i=1}^{n}f_i(x_i), \quad (5)$$

where $c(u_1, u_2, \cdots, u_n)$ is the copula density, and $f_i(x_i)$ is the probability density function of the univariate variable x_i.

2.2 Archimedean Copulas

Sklar (1959) proposed the concept of copula to cope with the difficulty of constructing joint probability distribution for dependent random variables. Since then, various copulas have been proposed, such as Archimedean copula (Clayton copula, Frank copula, Gumbel-Hougaard copula, and Ali-Mikhail-Haq (AMH) copula), elliptic type of copula (Gaussian copula, and student t copula), Plackett copula, and extreme value copula (Nelson, 2006; Gudendorf and Segers, 2010). The Archimedean copulas, which are extensively used in various research fields, have a simple structure and less unknown parameters than the other copulas. Genest and MacKay (1986) define Archimedean copulas as follows (Genest and Rivest, 1993; Joe, 1997):

$$C\left(u_1,u_2,\cdots,u_n\right)$$
$$=\begin{cases}\varphi^{-1}\left(\varphi\left(u_1\right)+\varphi\left(u_2\right)+\cdots+\varphi\left(u_n\right)\right), & \sum_{i=1}^{n}\varphi\left(u_i\right)\leq\varphi(0)\\ 0, & \text{otherwise}\end{cases}, \quad (6)$$

where $\varphi(\cdot)$ is the generator of the copula, and it is decreasing for all $u\in[0, 1]$ and $\varphi(1)=0$; $\varphi^{-1}(\cdot)$ is the inverse function of $\varphi(\cdot)$, and $\varphi^{-1}(\cdot)$ satisfies

$$\varphi^{-1}(t)=\begin{cases}\varphi^{-1}(t), & 0\leq t\leq\varphi(0)\\ 0, & \varphi(0)\leq t\leq\infty\end{cases}. \quad (7)$$

Archimedean copulas play an important role because they present several desired properties through a simple symmetric structure with one parameter. Therefore, Clayton, Frank, Gumbel-Hougaard, and AMH copulas are adopted in this study to construct bivariate and multivariate joint distributions. These four kinds of common Archimedean copulas are as follows:

1) Bivariate and trivariate Clayton copula:

$$C(u_1,u_2;\theta)=\left(u_1^{-\theta}+u_2^{-\theta}\right)^{-\frac{1}{\theta}},\ \theta\in(0,\infty), \tag{8}$$

$$C(u_1,u_2,u_3;\theta)=\left(u_1^{-\theta}+u_2^{-\theta}+u_3^{-\theta}\right)^{-\frac{1}{\theta}},\ \theta\in(0,\infty). \tag{9}$$

2) Bivariate and trivariate Frank copula:

$$C(u_1,u_2;\theta)=-\frac{1}{\theta}\ln\left\{1+\frac{\left[\exp(-\theta u_1)-1\right]\left[\exp(-\theta u_2)-1\right]}{\left[\exp(-\theta)-1\right]}\right\}$$

$$\theta\in R, \tag{10}$$

$$C(u_1,u_2,u_3;\theta)=$$

$$-\frac{1}{\theta}\ln\left\{1+\frac{[\exp(-\theta u_1)-1][\exp(-\theta u_2)-1][\exp(-\theta u_3)-1]}{[\exp(-\theta)-1]^2}\right\}$$

$$\theta\in R. \tag{11}$$

3) Bivariate and trivariate Gumbel-Hougaard copula:

$$C(u_1,u_2;\theta)=\exp\left\{-\left[(-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right]^{\frac{1}{\theta}}\right\},\ \theta\in[1,\infty) \tag{12}$$

$$C(u_1,u_2,u_3;\theta)=$$

$$\exp\left\{-\left[(-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}+(-\ln u_3)^{\theta}\right]^{\frac{1}{\theta}}\right\},$$

$$\theta\in[1,\infty). \tag{13}$$

4) Bivariate and trivariate AMH copula:

$$C(u_1,u_2;\theta)=\frac{u_1u_2}{\left[1-\theta(1-u_1)(1-u_2)\right]},\ \theta\in[-1,1), \tag{14}$$

$$C(u_1,u_2,u_3;\theta)=\frac{u_1u_2u_3}{\left[1-\theta(1-u_1)(1-u_2)(1-u_3)\right]},\ \theta\in[-1,1), \tag{15}$$

2.3 Parameter Estimation

The common parameter estimation methods of copula functions are Maximum Likelihood Method (MLM), Correlation Index Method, Inference Function for Margins Method, and Moment Method. Among these methods, the Kendall rank correlation coefficient is usually applied on bivariate copulas. However, it is unsuitable for trivariate or much higher dimensional copulas. The MLM maximizes sample data and copula functions, so it is adopted to estimate parameters for bivariate and trivariate copulas. The basic idea of the MLM for copulas is expressed as follows.

Assuming that the sample is $(u_1, u_2, \cdots, u_n)\in[0, 1]$, the expression of the likelihood function in the case of copulas (Favre *et al.*, 2004; Hou *et al.*, 2010) is as follows:

$$L(\theta)=\prod_{i=1}^{n}c(u_1,u_2,\cdots,u_n;\theta)=\prod_{i=1}^{n}\frac{\partial^n C(u_1,u_2,\cdots,u_n;\theta)}{\partial u_1\partial u_2\cdots\partial u_n}. \tag{16}$$

The logarithm likelihood function $\ln L(\theta)$ (Eq. (17)) is associated with the likelihood function $L(\theta)$, and they reach the maximum simultaneously.

$$\ln L(\theta)=\prod_{i=1}^{n}c(u_1,u_2,\cdots,u_n;\theta)=\sum_{i=1}^{n}\ln c(u_1,u_2,\cdots,u_n;\theta). \tag{17}$$

The maximum likelihood estimator $\hat{\theta}_{ML}$ satisfies

$$\ln L\left(\hat{\theta}_{ML}\right)\geq\ln L(\theta),\ \forall\theta\in\Theta, \tag{18}$$

where Θ is the parameter space.

Taking $\ln L$, a partial derivative with respect to θ (Eq. (19)), the parameter θ of the copulas can be obtained by solving the likelihood equations.

$$\frac{\partial\ln L(\theta)}{\partial\theta}=0. \tag{19}$$

3 Model Selection

Before using multivariate copulas to construct the joint probability distribution model, tests of goodness must be conducted to ensure that these models fit the sample well. Three tests are conducted, namely, 1) Kolmogorov-Smirnov (K-S) test for univariate margins, 2) Pearson χ^2 test for bivariate copulas, and 3) root mean square error (RMSE) method and Akaike information criterion (AIC) for trivariate distributions.

3.1 K-S Test for Univariate Distributions

The K-S test is a goodness-of-fit statistics to assess if a random variable X can have the hypothesized, continuous, and cumulative distribution function. Assume that $F(x)$ denotes the actual distribution for the sample data and $F_0(x)$ is the theoretical distribution (Zhai *et al.*, 2014). Choose the statistics $D_n=\sup_{-\infty<x+\infty}\left|F_n(x)-F_0(x)\right|$, and

$$\begin{cases} d_k^{(1)}=\left|F_n(x_k)-F_0(x_k)\right| \\ d_k^{(2)}=\left|F_n(x_{k+1})-F_0(x_k)\right| \end{cases} (k=1,2,\cdots,n). \tag{20}$$

The observation of D_n can be defined as

$$\hat{D}_n=\max_{1\leq k\leq n}\left\{d_k^{(1)},d_k^{(2)}\right\}. \tag{21}$$

If the significance level is α, the K-S critical value is $D_n(\alpha)$ for different sample size n. If $\hat{D}_n<D_n(\alpha)$, the hypothetical theoretical distribution is accepted to fit the sample data; otherwise, we refuse the hypothetical distribution.

3.2 Pearson χ^2 Test for Bivariate Copulas

Hu (2002) introduced a statistics of M, which follows χ^2 distribution and can be utilized to estimate the fitting quality between the bivariate copula and observations. Statistical results will test whether this model can describe the dependence between these two variables (Tao *et al.*, 2013).

Suppose that the observation data of X and Y are $\{x_t\}$ and $\{y_t\}$ and the marginal distributions of X and Y are $F_X(x)$ and $F_Y(y)$. Let $u_t=F_X(x_t)$, $v_t=F_Y(y_t)$, $t=1, 2, \cdots, n$. Build a square matrix with $k \times k$ dimension. The unit in the ith row and jth column is denoted by $R(i, j)$, $i, j=1, 2, \cdots, k$. For each $\{u_t, v_t\}$, if both $(i-1)/k \le u_t \le i/k$ and $(j-1)/k \le v_t \le j/k$ exist, then denote $\{u_t, v_t\} \in R(i, j)$. Let A_{ij} be the number of actual observation points that fall into the unit $R(i, j)$, and B_{ij} be the number of predicted points produced by the copula model that fall into the unit $R(i, j)$. Then,

$$M=\sum_{i=1}^{k}\sum_{j=1}^{k}\frac{(A_{ij}-B_{ij})^2}{B_{ij}}\sim\chi^2\left((k-1)^2\right). \quad (22)$$

If the significance level is α, the rejection region is $\{M > \chi_{\alpha}^{2}((k-1)^2)\}$, in which $\chi_{\alpha}^{2}((k-1)^2$ is the downside $1-\alpha$ quantile of the χ^2 distribution with $(k-1)^2$ free degrees. If $M > \chi_{\alpha}^{2}((k-1)^2$, the copula is refused; otherwise, the copula is accepted to construct bivariate models.

3.3 RMSE

The RMSE is used to measure the goodness of fit of the univariate or multivariate distribution. RMSE can be expressed as

$$\mathrm{RMSE}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[F_c(i)-P_0(i)\right]^2}, \quad (23)$$

where n is the sample size, F_c is the theoretical probability obtained from a model, and P_0 is the empirical probability: $P_0=m_i/(n+1)$, where m_i is the number of $x \le x_i$, $x \le x_i$; $y \le y_i$, or $x \le x_i$, $y \le y_i$; and $z \le z_i$ for different dimensional distributions.

3.4 AIC

The AIC is proposed to identify the appropriate probability distribution (Akaike, 1974). The AIC includes two parts (Zhang and Singh, 2007): lack of fit of the model, which can be obtained either by maximizing the likelihood function of the distribution or the MSE of the model, and the unreliability of the model due to the number of model parameters. The AIC is expressed as

$$\mathrm{AIC}=-2\ln(L)+2m, \quad (24)$$

$$\mathrm{AIC}=n\ln(\mathrm{MSE})+2m, \quad (25)$$

$$\mathrm{MSE}=E\left(F_c-P_0\right)^2=\frac{1}{n-m}\sum_{i=1}^{n}\left[F_c(i)-P_0(i)\right]^2, \quad (26)$$

where L is the likelihood function of the model, and m is the number of model parameters. In this paper, Eq. (25) is applied to calculate AIC. A small AIC implies that the model has a good fit.

4 Case Study

4.1 Univariate Data Analysis

The data for a period between 1970 and 1999 at the Bohai Sea are hindcast and sampled in this study. The data set includes observations of simultaneously occurred wave height (H), wind speed (W), and current velocity (C) (Fig.1). Annual extreme environmental element series can be selected for frequency analysis.

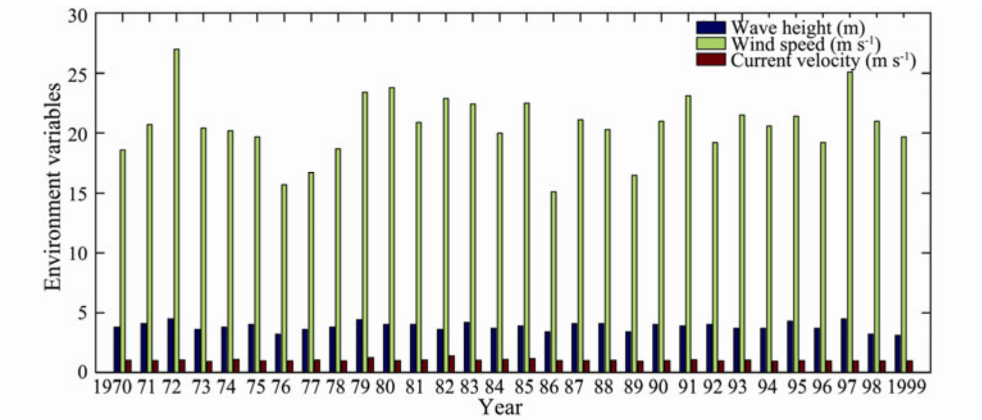

Fig.1 Observations of ocean environmental elements (H, W, and C).

In coastal and offshore engineering, the Gumbel, Log-normal, Weibull, and Pearson Type III distributions are commonly used for frequency analysis (Muir and El-Shaarawi, 1986; Dong *et al.*, 2003; Tao *et al.*, 2013). Table 1 shows these four probability distributions adopted to fit H, W, and C.

The MLM is applied to fit the parameters of all the above distribution curves. The K-S test results are listed in Table 2. The actual statistic values $\hat{D}_n$ of all distributions for H, W, and C are smaller than the test statistic $D_{30}(0.05)=0.2417$; thus, these distributions pass the statistical test. The fitting curves of H, W, and C are shown in Figs.2–4, respectively. A low RMSE or AIC implies good fitting result of the theoretical distribution. For H, W, and C, the RMSE and AIC values of Pearson Type III are all smaller than those of the other distribution types. Therefore, Pearson Type III is the preferred marginal distribution for H, W, and C to construct a joint probability model.

4.2 Bivariate Data Analysis

4.2.1 Dependence among variables

Copula functions are used to describe the interdependency of random variables. In this paper, Kendall's rank correlation coefficient (Eq. (27)) (Li *et al.*, 2013; van Doorn *et al.*, 2016) and Spearman's rank correlation coefficient (Eq. (29)) (Hou *et al.*, 2010) are utilized to evaluate the dependence among H, W, and C (Table 3).

$$\tau=\frac{2}{n(n-1)}\sum_{1\le i<j\le n}\operatorname{sign}\left[(x_i-x_j)(y_i-y_j)\right], \tag{27}$$

where τ is the Kendall rank correlation coefficient; n is the sample size; (x_i, y_i) are the pairs of H and W, W and C,

Table 1 Four univariate probability distributions

Distribution type	Probability distribution function $F(x)$	Parameter
Gumbel	$\exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]$	μ-location parameter; σ-scale parameter.
Lognormal	$\frac{1}{\sqrt{2\pi}\sigma}\int_0^x \frac{1}{t}\exp\left[-\frac{(\ln t-\mu)^2}{2\sigma^2}\right]\mathrm{d}t$	μ_y-mean value of ln x; σ_y-standard deviation of ln x.
Weibull	$1-\left[1-\exp\left[-\left(\frac{x-\mu}{\alpha}\right)^{\gamma}\right]\right]$	α-scale parameter; γ-shape parameter; μ-location parameter.
Pearson Type III	$\int_0^x \frac{\beta^{\alpha}}{\Gamma(\alpha)}(t-a_0)^{\alpha-1}\mathrm{e}^{-\beta(t-a_0)}\mathrm{d}t$	a_0-location parameter; α-shape parameter; β-scale parameter.

Table 2 Parameter estimation and goodness-of-fit test results for different distributions of H, W, and C

Distribution		PA	PB	PC	$\hat{D}_n$	$\hat{D}_n(0.05)$	RMSE	AIC
H	Gumbel	3.6788	0.2850	–	0.1882	0.2417	0.0433	−182.36
	Lognormal	1.3419	0.0967	–	0.1470		0.0318	−200.75
	Weibull	2.4253	1.5560	4.3923	0.1636		0.0418	−181.36
	Pearson Type III	1.9348	13.2096	25.2102	0.1437		0.0311	−199.09
W	Gumbel	19.4318	2.0469	–	0.1389	0.2417	0.0633	−159.56
	Lognormal	3.0179	0.1300	–	0.1016		0.0457	−179.11
	Weibull	12.2288	9.3129	3.5416	0.1157		0.0425	−180.34
	Pearson Type III	4.4×10^{-5}	3.8748	79.8724	0.1237		0.0384	−186.44
C	Gumbel	0.9864	0.0773	–	0.1434	0.2417	0.0670	−156.12
	Lognormal	0.0266	0.0881	–	0.1961		0.0969	−133.99
	Weibull	0.9395	0.0882	0.9242	0.1987		0.0397	−183.92
	Pearson Type III	0.9300	11.3354	1.1450	0.0899		0.0312	−198.84

Note: PA, PB, and PC denote the location parameter, scale parameter, and shape parameter, respectively.

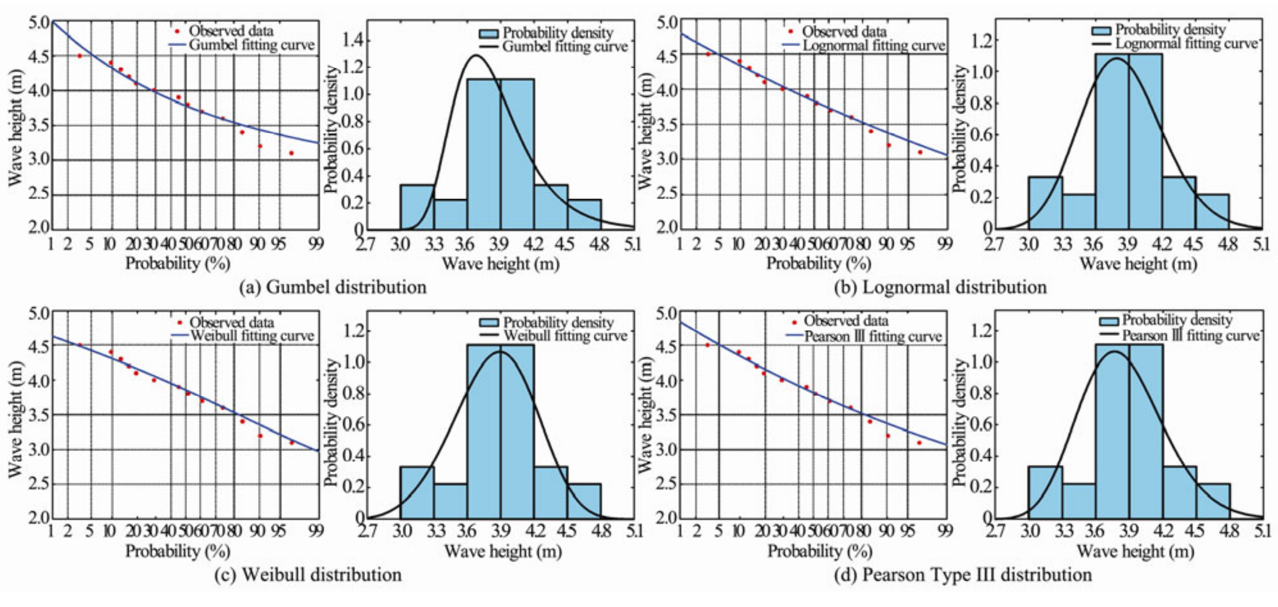

Fig.2 Univariate fitting curves of H.

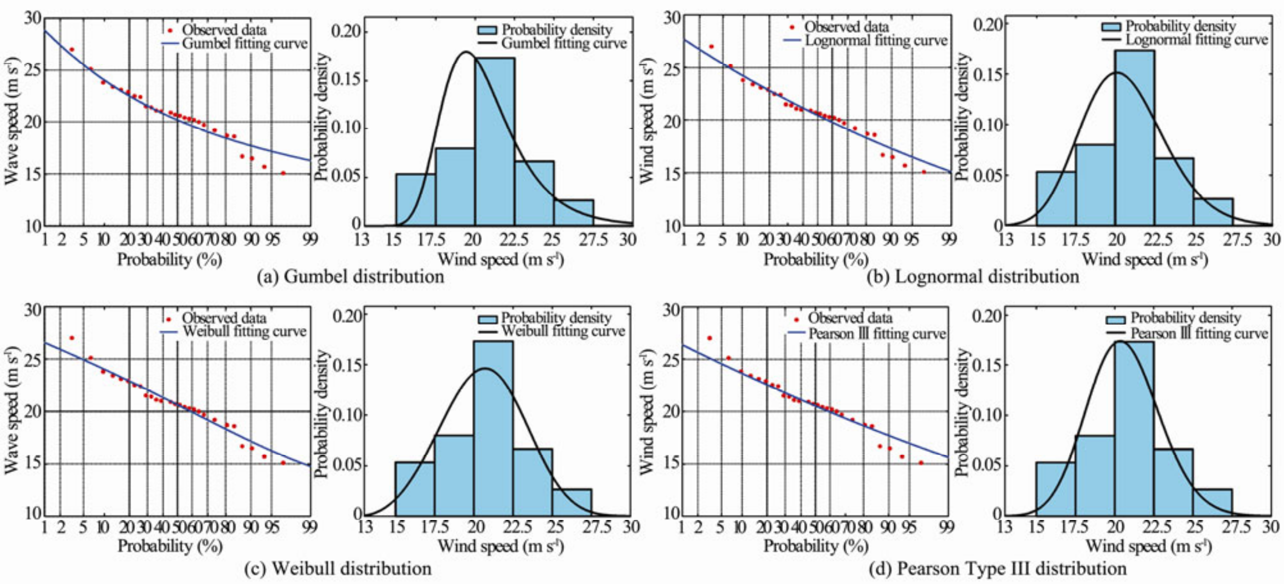

Fig.3 Univariate fitting curves of *W*.

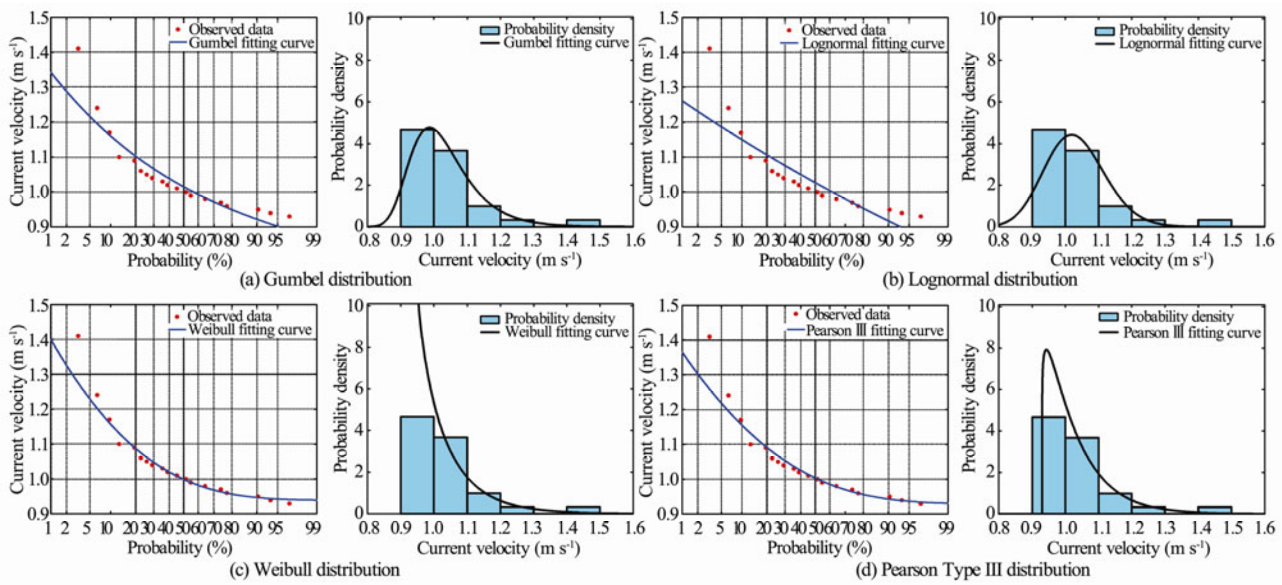

Fig.4 Univariate fitting curves of *C*.

or *H* and *C*; and $sign(\cdot)$ is the sign function, and it satisfies

$$sign(\cdot)=\begin{cases}1, & (x_i-x_j)(y_i-y_j)>0\\ -1, & (x_i-x_j)(y_i-y_j)<0\\ 0, & (x_i-x_j)(y_i-y_j)=0\end{cases}, \tag{28}$$

$$\rho_n=\frac{\sum_{i=1}^{n}(R_i-\bar{R})(S_i-\bar{S})}{\sqrt{\sum_{i=1}^{n}(R_i-\bar{R})^2\sum_{I=1}^{N}(S_i-\bar{S})^2}}, \tag{29}$$

$$\bar{R}=\frac{1}{n}\sum_{i=1}^{n}R_i=\frac{n+1}{2}=\frac{1}{n}\sum_{i=1}^{n}S_i=\bar{S}, \tag{30}$$

where ρ_n is Spearman's rank correlation coefficient, and R_i and S_i are ranks of random variables *X* and *Y*, respectively.

According to the Table 3, the Kendall's rank correlation coefficients τ and the Spearman's rank correlation coefficients ρ_n of *H* and *W*, *W* and *C*, and H and *C* are more than zero. That is to say, there exists a positive correlation between *H* and *W*, *W* and *C*, and *H* and *C*, respectively. In addition, for *H* and *W*, and *W* and *C*, their p values of τ and ρ_n are smaller than the significant level α=0.05, then the correlation is significantly different from zero, which shows significant correlation.

4.2.2 Parameter estimation of bivariate copulas

Table 3 shows that the dependence among variables is strong. Four types of bivariate Archimedean copulas (Clayton, Frank, Gumbel-Hougaard, and AMH copulas) and margins (Pearson Type III distribution) are applied to construct the joint distribution models of *H* and *W*, *W* and

C, and *H* and *C*. The estimated correlated parameters θ for the above four models are shown in Table 4.

4.2.3 Goodness-of-fit test of bivariate copulas

The comparisons of empirical and theoretical frequentcies and P-P plot of the observed combinations of *H* and *W*, *W* and *C*, and *H* and *C* are depicted in Figs.5–7, respectively. The empirical and theoretical frequencies exhibit a robust fit. In addition, P-P plots show that these points are around the 45˚ oblique line. Intuitive graphical analysis methods demonstrate that the joint distribution models based on the above four copula functions are rational and robust to fit the observed combinations of *H* and *W*, *W* and *C*, and *H* and *C*.

Simultaneously, Pearson χ^2 test is used to evaluate the goodness of fit for bivariate copulas. To confirm sufficient unit cells for model evaluation and guarantee sufficient sample points of each unit cell, the *k* value is taken from 4, 5, 6, and 7 (corresponding to 16, 25, 36, and 49 equal-sized unit cells). Table 5 shows the Pearson's χ^2 statistics *M* of different copulas for the combinations *H*

Table 3 Dependence between variables

Dependence index	Kendall τ	*P*-value	Spearman ρ_n	*P*-value
H and *W*	0.4704	0.0004	0.6037	0.0004
W and *C*	0.2750	0.0378	0.3946	0.0309
H and *C*	0.1818	0.1826	0.2579	0.1688

Table 4 Parameter estimations θ of bivariate copulas

Copula function	*H* and *W*	*W* and *C*	*H* and *C*
Clayton	0.5745	0.0539	0.1132
Frank	4.8576	2.9494	1.9158
Gumbel–Hougaard	1.7315	1.2057	1.1629
AMH	0.8918	0.6966	0.7458

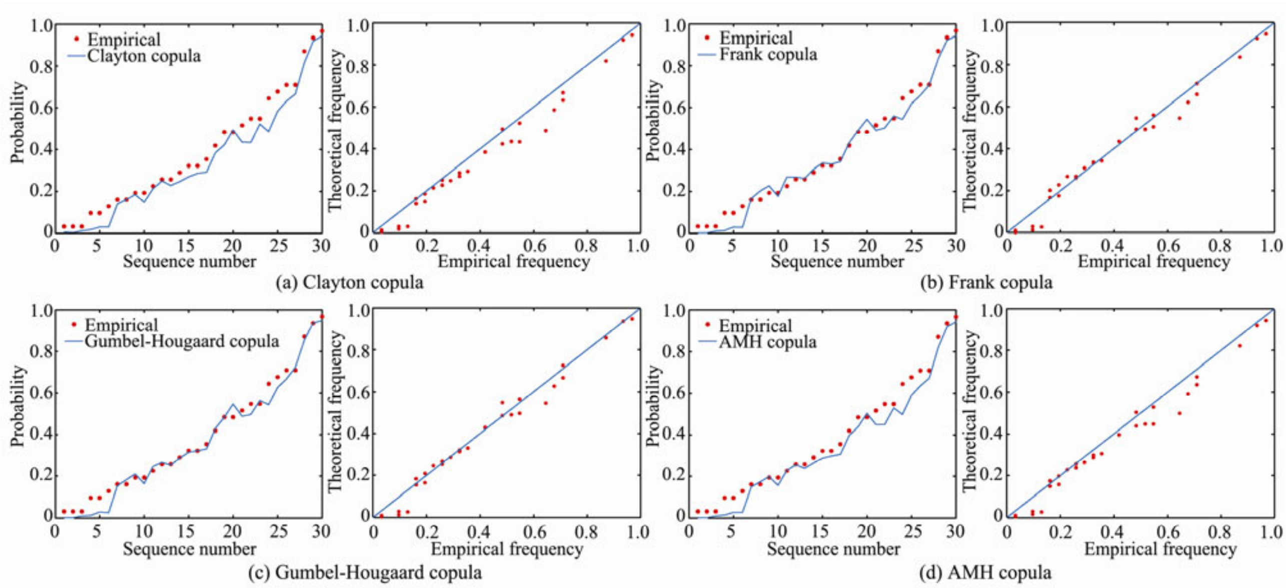

Fig.5 Comparison of empirical and theoretical frequencies and P-P plot of the observed combination of *H* and *W*.

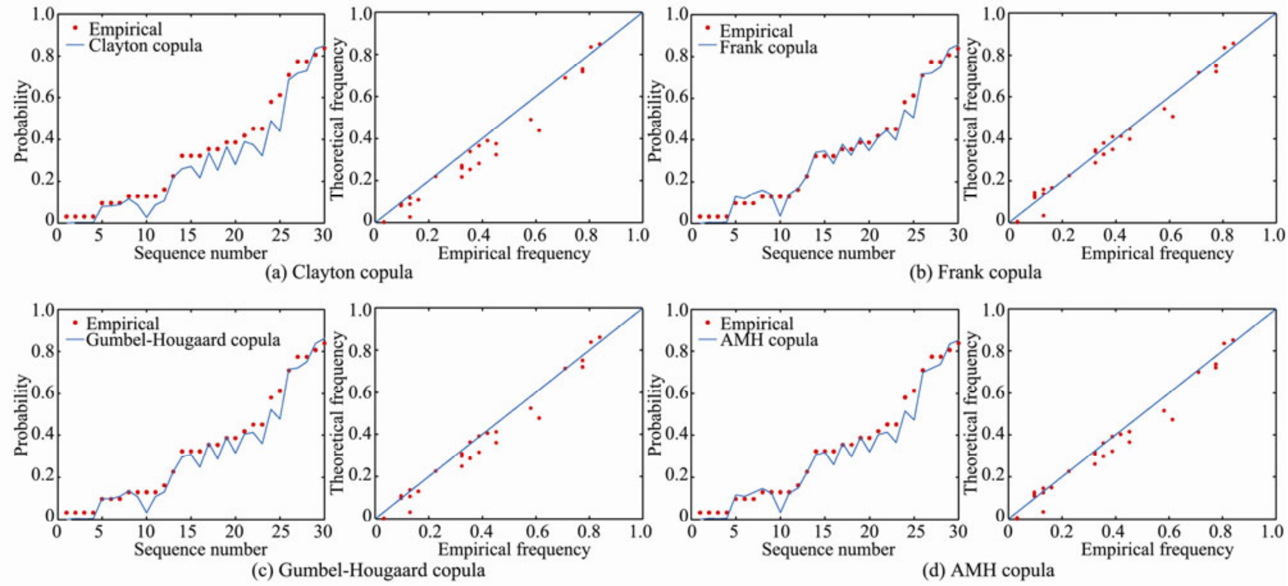

Fig.6 Comparison of empirical and theoretical frequencies and P-P plot of the observed combination of *W* and *C*.

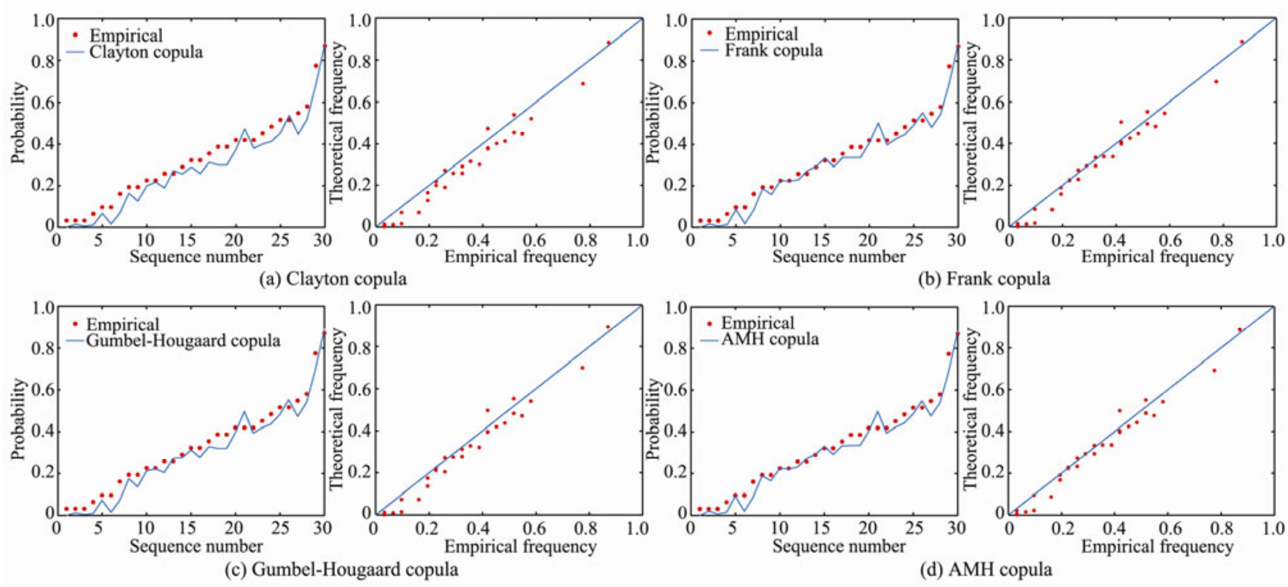

Fig.7 Comparison of empirical and theoretical frequencies and P-P plot of the observed combination of *H* and *C*.

Table 5 Pearson's χ^2 test for copulas

Copula function	*H* and *W*				*W* and *C*				*H* and *C*			
	4×4	5×5	6×6	7×7	4×4	5×5	6×6	7×7	4×4	5×5	6×6	7×7
Clayton	8.2705	21.7326	36.1300	33.0379	12.2765	31.2313	38.9959	50.8658	8.2804	25.5546	40.5358	36.8410
Frank	4.3215	21.0239	34.7175	33.5098	10.9651	24.3043	34.7454	46.4509	8.6412	25.5608	41.5038	36.9851
Gumbel-Hougaard	5.9200	19.8065	33.4622	31.0905	9.3226	27.1242	34.7292	46.5444	9.5535	26.0660	40.9920	36.7794
AMH	6.8230	20.8487	34.6043	32.0040	10.9029	27.3808	36.1211	47.0161	7.3995	24.2203	39.5780	35.6161

Notes: For Pearson's χ^2 test, the χ^2 test standard can be expressed as: $k=4$, $\chi^2_{0.05}(k-1)^2=16.9190$; $k=5$, $\chi^2_{0.05}(k-1)^2=26.2962$; $k=6$, $\chi^2_{0.05}(k-1)^2=37.6525$; $k=7$, $\chi^2_{0.05}(k-1)^2=50.9985$.

and *W*, *W* and *C*, and *H* and *C*. For the joint distribution of *H* and *W*, Clayton, Frank, Gumbel-Hougaard, and AMH copulas all pass the χ^2 tests when *k* takes any value of 4–7. For the joint distributions of *W* and *C*, only Frank copula can pass the χ^2 tests when *k* takes any value of 4–7. For the joint distributions of *H* and *C*, these four copulas can pass the χ^2 tests when *k* takes any value of 4, 5, and 7. According to Table 5, different selections of *k* can lead to varying orders of *M*. Thus, the best model cannot be selected based only on the χ^2 test.

The RMSE and AIC criteria are calculated to obtain the optimal copula to construct the joint distribution model (Table 6). A low RMSE or AIC value indicates a well-fitting model. Based on Table 6, Gumbel-Hougaard, Frank, and AMH copulas are separated to select the optimal copula to construct the joint distributions of *H* and *W*, *W* and *C*, and *H* and *C*. The joint probability density contours and joint return periods of *H* and *W*, *W* and *C*, and *H* and *C* are shown as Figs.8–10, respectively.

Table 6 Goodness-of-fit test of bivariate copulas

Copula	*H* and *W*		*W* and *C*		*H* and *C*	
	RMSE	AIC	RMSE	AIC	RMSE	AIC
Clayton	0.0596	−167.19	0.0654	−161.61	0.0564	−170.54
Frank	0.0428	−187.06	0.0379	−194.34	0.0405	−190.32
Gumbel-Hougaard	0.0409	−189.77	0.0476	−180.69	0.0466	−181.91
AMH	0.0543	−172.79	0.0464	−182.18	0.0403	−190.62

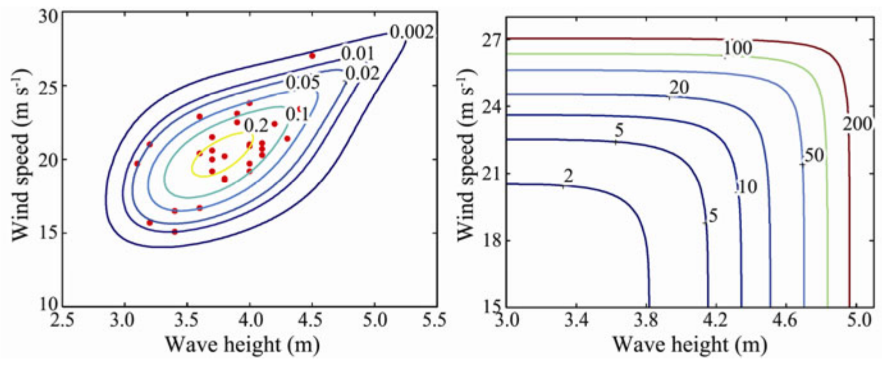

Fig.8 Joint probability density and return period contours of the observed combination of *H* and *W* (Gumbel-Hougaard copula).

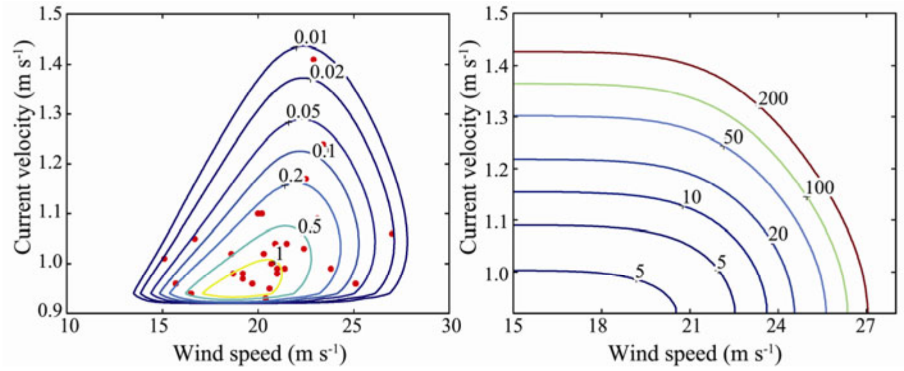

Fig.9 Joint probability density and return period contours of the observed combination of *W* and *C* (Frank copula).

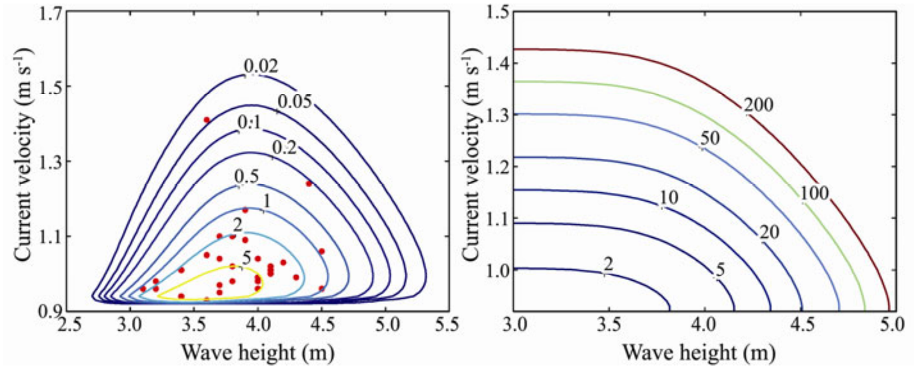

Fig.10 Joint probability density and return period contours of the observed combination of *H* and *C* (AMH copula).

4.3 Multivariate Models and Analysis

4.3.1 Trivariate joint model

Four kinds of trivariate Archimedean copulas (Clayton, Frank, Gumbel-Hougaard, and AMH copulas) are used to construct the joint probability distribution model of *H*, *W*, and *C*. The estimated values of correlated parameters θ for the above four models are shown in Table 7. The fitting curves of these four trivariate copulas are presented as Fig.11.

As Fig.11 illustrates, Frank and Gumbel-Hougaard copulas are better than Clayton and AMH copulas to construct the joint probability distribution model of *H*, *W*, and *C*. To obtain the best-fitting model, we calculate the RMSE and AIC values (Table 8). Based on the results in Table 8 and Fig.11, Frank copula is selected as the optimal fitting model to construct the trivariate joint probability distribution model of *H*, *W*, and *C*.

Table 7 Parameter estimations θ of trivariate copulas

Parameter	Clayton	Frank	Gumbel-Hougaard	AMH
θ	0.1650	2.8516	1.2916	0.4114

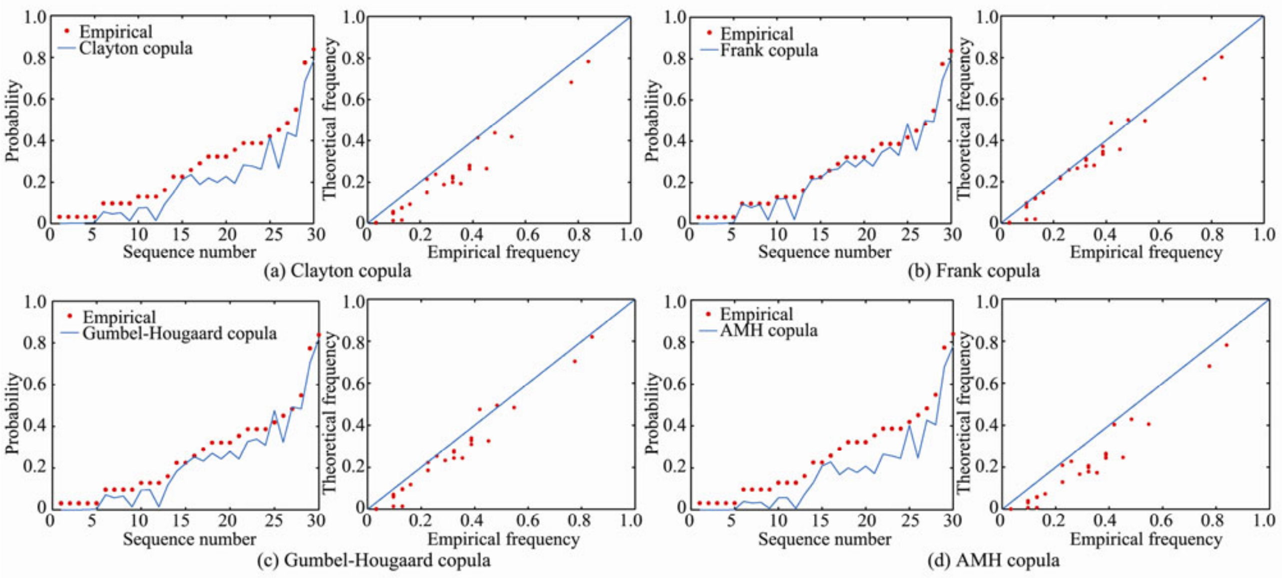

Fig.11 Comparison of empirical and theoretical frequencies and P-P plot of the combinations of *H*, *W*, and *C*.

Table 8 Goodness-of-fit test of trivariate copulas

Copula	Clayton	Frank	Gumbel-Hougaard	AMH
RMSE	0.0862	0.0453	0.0577	0.0989
AIC	−145.05	−183.70	−169.13	−136.81

4.3.2 Design value under the multivariate joint distribution model

1) Conditional probability method

For many offshore structures, the dominating load is the one associated with the wave load, wind load, or current load. When wave height, wind speed, or current velocity is given, the trivariate joint distribution is transformed as a bivariate joint distribution. We use Eq. (31) to calculate the conditional probability density of the other two variables. When the conditional probability density reaches its maximum with a given variable, the other two variables are most likely to occur (modes). Therefore, three pairs of data, namely, the maximum wave height extremes with concomitant wind speed and current velocity (H_{max}, W, and C), the maximum wind speed extremes with concomitant wave height and current velocity (W_{max}, H, and C), and the maximum current velocity extremes with concomitant wave height and wind speed (C_{max}, H, and W), are considered for trivariate analysis.

$$f(x,y|z)=\frac{f(x,y,z)}{f(z)}=c(x,y,z)\cdot f(x)\cdot f(y)\cdot f(z). \tag{31}$$

The conditional probability density contours are shown in Figs.12–14 for maximum wave height, wind speed, or current velocity in the return periods of 100 year, 50 year, 20 year, and 10 year. Design values and joint probabilities of wave height, wind speed, and current velocity calculated by univariate and joint probability methods are listed in Table 9.

Table 9 shows that the design wave heights, wind speeds, and current velocities calculated by the conditional probability method are all smaller than those by the univariate probability method. Taking T (H, W, or C) = 100 years as an example, H, W, and C by the univariate probability method are 4.84 m, 26.36 m s^{-1}, and 1.36 m s^{-1}, respectively. The combination of H_{max}, W, and C by the conditional probability method is 4.84 m, 21.86 m s^{-1}, and 1.04 m s^{-1}, respectively. The values of W and C decrease by

Table 9 Design values of H, W, and C for the conditional probability method

T (year)	Univariate probability method				H_{max} with concominant W and C				W_{max} with concominant H and C				C_{max} with concominant H and W			
	H (m)	W (m s^{-1})	C (m s^{-1})	P	H (m)	W (m s^{-1})	C (m s^{-1})	P	H (m)	W (m s^{-1})	C (m s^{-1})	P	H (m)	W (m s^{-1})	C (m s^{-1})	P
200	4.96	27.04	1.43	2.13×10^{-6}	4.96	21.87	1.04	2.34×10^{-3}	4.03	27.04	1.04	2.39×10^{-3}	4.12	22.36	1.43	1.61×10^{-3}
100	4.84	26.36	1.36	1.63×10^{-5}	4.84	21.86	1.04	4.67×10^{-3}	4.03	26.36	1.04	4.75×10^{-3}	4.11	22.32	1.36	3.30×10^{-3}
50	4.70	25.62	1.30	1.21×10^{-4}	4.70	21.84	1.04	9.32×10^{-3}	4.03	25.62	1.04	9.42×10^{-3}	4.11	22.30	1.30	6.57×10^{-3}
25	4.56	24.82	1.24	8.31×10^{-4}	4.56	21.82	1.04	1.84×10^{-2}	4.00	24.82	1.03	2.03×10^{-2}	4.09	22.20	1.24	1.38×10^{-2}
20	4.51	24.55	1.22	1.51×10^{-3}	4.51	21.65	1.03	2.50×10^{-2}	3.99	24.55	1.03	2.56×10^{-2}	4.08	22.15	1.22	1.76×10^{-2}
10	4.34	23.62	1.15	8.86×10^{-3}	4.34	21.42	1.02	5.33×10^{-2}	3.96	23.62	1.02	5.38×10^{-2}	4.05	21.96	1.15	3.72×10^{-2}

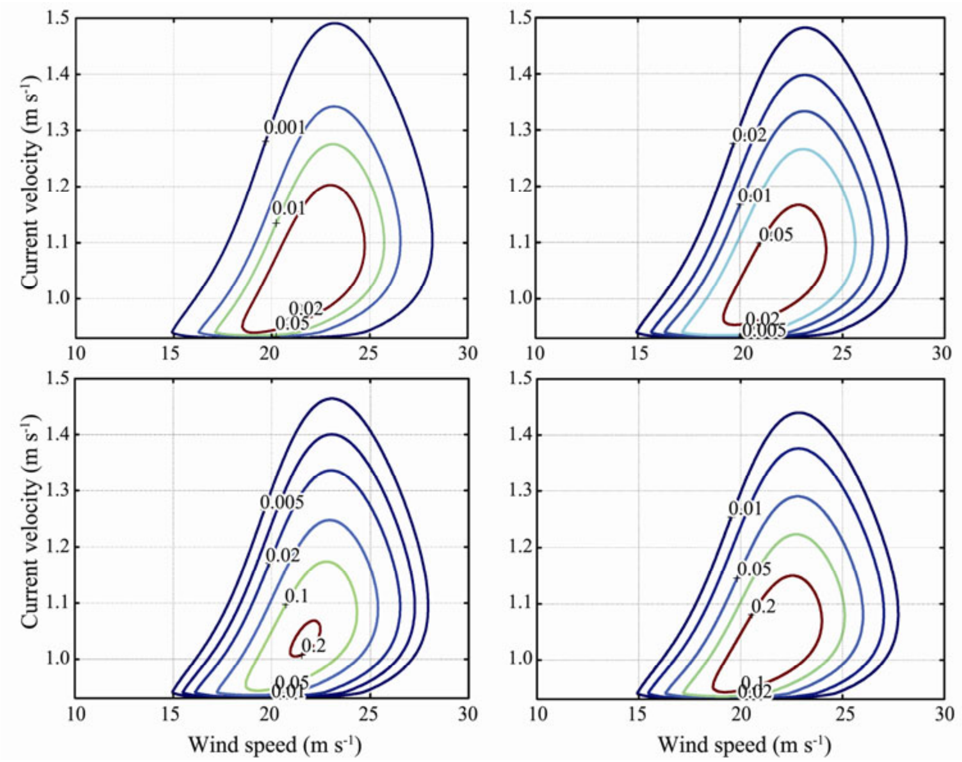

Fig.12 Conditional probability density contours of H_{max} with concominant W and C.

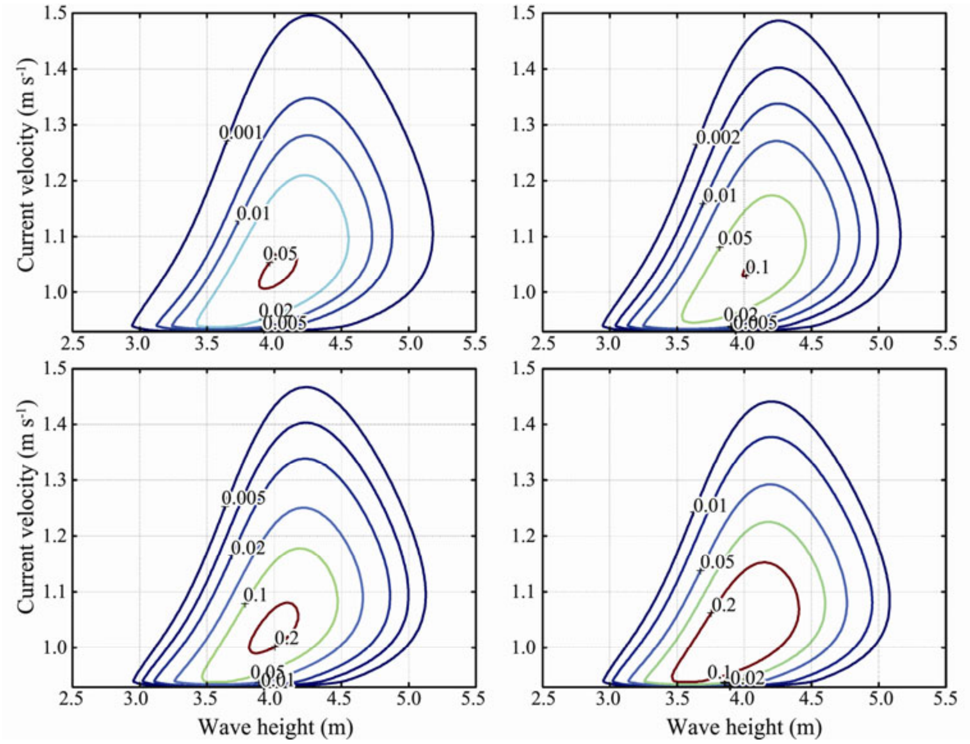

Fig.13 Conditional probability density contours of W_{max} with concominant *H* and *C*.

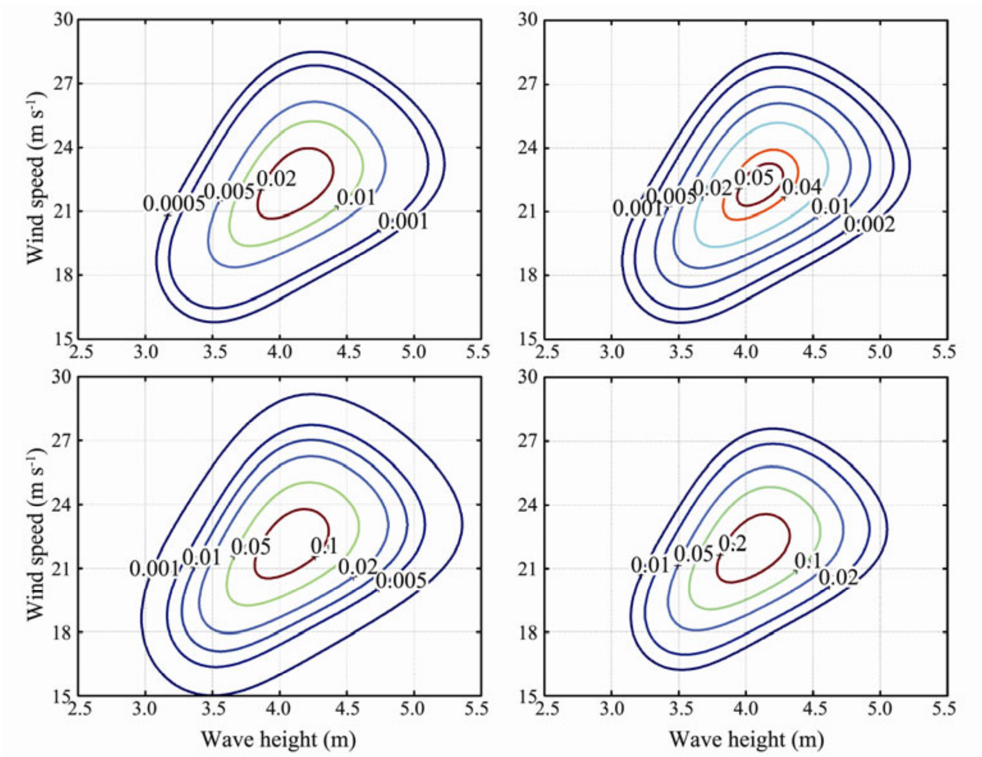

Fig. 14 Conditional probability density contours of C_{max} with concominant *H* and *W*.

17.07% and 23.53%, respectively. For the case of W_{max}, *H*, and *C*, the values of *H* and *C* are reduced by 16.74% and 23.53%, respectively. For C_{max}, *H*, and *W*, the values of *H* and *W* fall by 15.08% and 15.33%, respectively. Evidently, the joint probability of H_{max}, W_{max}, and C_{max} is larger than the arithmetic product of the probability of the three independent variables.

2) Joint probability method

Clayton copula is applied to construct the joint probability model of *H*, *W*, and *C*. Fig.15 shows the joint return period isosurface and side views under *T* (*H*, *W*, and *C*) = 100 year. Moreover, different joint return period isosurfaces of (*H*, *W*, *C*) are shown as Fig.16. According to Figs.15 and 16, many pairs of (*H*, *W*, *C*) exist for one certain joint return period. The maximum joint probability density (f_{max}) and maximum platform responses (*e.g.*, maximum base shear (Q_{max}), maximum overturning moment (M_{max}), or maximum deck displacement (D_{max})) are considered constraints to calculate the return values of (H, W, and C). The design values of H, W, and C calculated by the trivariate joint probability method for different return periods are listed in Table 10.

Table 10 Design values of H, W, and C under different return periods T

T (year)	f_{max}			Q_{max}			M_{max}			D_{max}		
	H (m)	W (m s^{-1})	C (m s^{-1})	H (m)	W (m s^{-1})	C (m s^{-1})	H (m)	W (m s^{-1})	C (m s^{-1})	H (m)	W (m s^{-1})	C (m s^{-1})
200	4.43	24.27	1.14	4.27	25.92	0.93	4.27	25.92	0.93	3.30	27.03	0.93
100	4.36	23.82	1.12	4.21	25.34	0.93	4.21	25.34	0.93	3.29	26.34	0.93
50	4.28	23.35	1.10	4.15	24.69	0.93	4.15	24.69	0.93	3.28	25.61	0.93
25	4.19	22.80	1.07	4.08	23.98	0.93	4.08	23.98	0.93	3.26	24.81	0.93
20	4.16	22.60	1.06	4.06	23.72	0.93	4.06	23.72	0.93	3.26	24.54	0.93
10	4.04	22.00	1.03	3.96	22.90	0.93	3.96	22.90	0.93	3.22	23.61	0.93

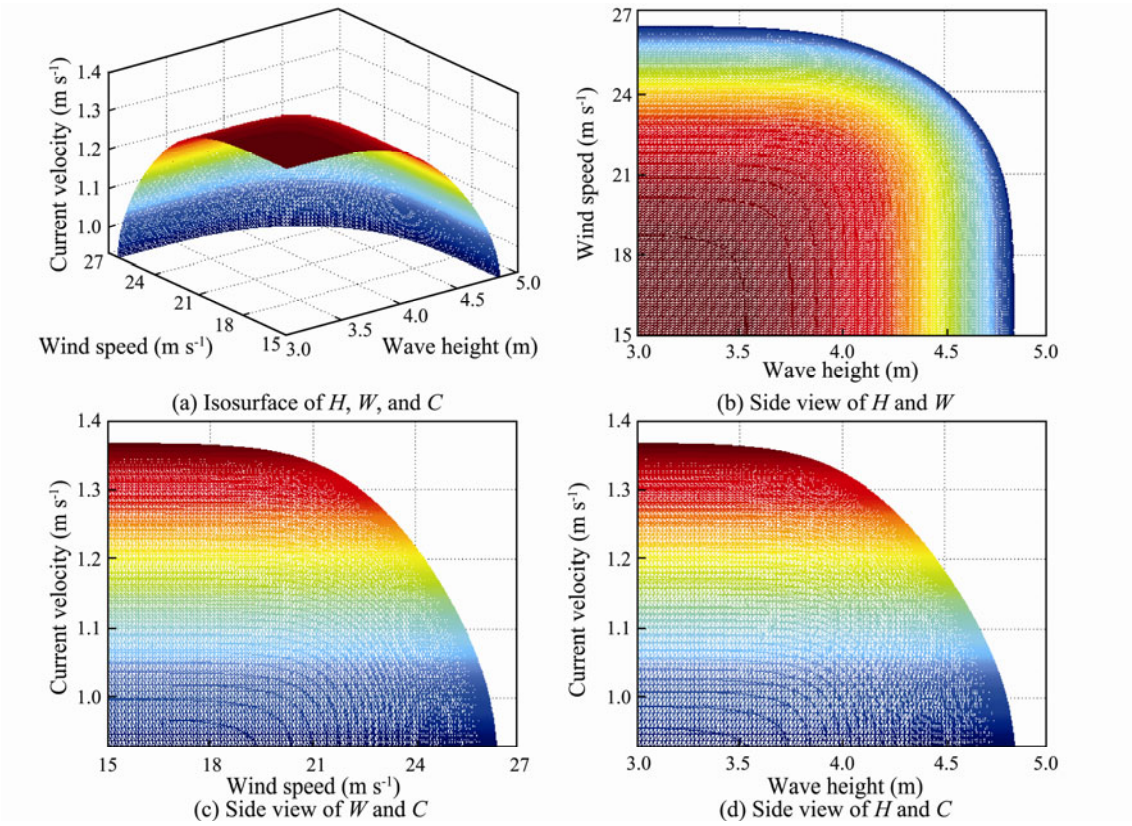

Fig.15 Joint probability model of H, W, and C under 100 year return period.

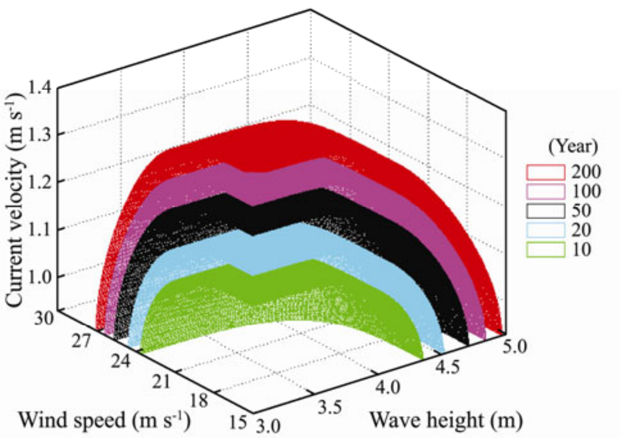

Fig.16 Joint return period isosurfaces of H, W, and C.

4.4 Application of Joint Distribution in Fixed Platform

The empirical functions of base shear, overturning moment, and deck displacement of H, W, and C for a fixed platform in the Bohai Sea are expressed as Eqs. 31–33, respectively (Xu *et al.*, 2013).

$$Q=20.3797H^{1.9414}+31.9567C^{-0.2970}+79.7442H^{0.7220}C^{1.3861}+0.1153W^2, \quad (31)$$

$$M=234.2491H^{1.3465}+286.0988C^{1.8859}+38.5857H^{1.9339}C^{1.144}+0.2908W^2, \quad (32)$$

$$D=0.0001\cdot(H-1.0868)^2+0.0001\cdot(C-0.4412)^2+1.3879\times10^{-6}\cdot(W+8.9107)^2+0.0087, \quad (33)$$

where Q (kN) is base shear, M (kN·m) is overturning moment, and D (m) is deck displacement.

According to Section 4.3.2, eight kinds of design values calculated by the univariate probability, conditional probability (H_{max}, W, and C; W_{max}, H, and C; C_{max}, H, and W), and joint probability methods (f_{max}, Q_{max}, M_{max}, and D_{max}) are substituted in Eqs. (31)–(33), respectively. Subsequently, the platform responses are obtained (see Table 11). Table 11 shows that the structural response by the joint probability model is smaller than that by the univariate probability method. Namely, the joint distribution can result in low design values of ocean environmental factors.

Table 11 Design values of response under given return periods

T (year)	Univariate method			Conditional method -H_{max}, W, and C			Conditional method -W_{max}, H, and C			Conditional method -C_{max}, H, and W		
	Q	M	D	Q	M	D	Q	M	D	Q	M	D
200	985	4083	0.01209	812	3367	0.01155	651	2648	0.01140	768	3178	0.01107
100	927	3834	0.01192	784	3254	0.01146	647	2638	0.01133	744	3078	0.01105
50	868	3587	0.01174	756	3136	0.01136	643	2627	0.01126	723	2989	0.01104
25	809	3340	0.01155	726	3011	0.01125	629	2579	0.01116	697	2884	0.01101
20	790	3260	0.01148	711	2953	0.01120	626	2567	0.01113	689	2848	0.01100
10	729	3009	0.01128	674	2799	0.01107	612	2519	0.01103	661	2736	0.01095
T (year)	Joint method -f_{max}			Joint method -Q_{max}			Joint method -M_{max}			Joint method -D_{max}		
	Q	M	D	Q	M	D	Q	M	D	Q	M	D
200	745	3073	0.01139	789	3286	0.01133	788	3289	0.01129	742	3059	0.01151
100	722	2978	0.01130	759	3160	0.01124	758	3163	0.01120	714	2946	0.01140
50	696	2872	0.01121	727	3029	0.01115	727	3031	0.01111	688	2840	0.01129
25	667	2757	0.01110	694	2890	0.01104	693	2892	0.01100	658	2720	0.01117
20	658	2719	0.01106	682	2843	0.01100	682	2844	0.01097	649	2683	0.01113
10	623	2577	0.01093	644	2687	0.01088	644	2689	0.01084	615	2548	0.01099

5 Conclusions

Based on the 30-year hindcast data of wave height, wind speed, and current velocity, bivariate and trivariate joint probability distributions are constructed by Archimedean copulas. Univariate distribution and conditional and joint probability methods are applied to estimate design wave heights, wind speeds, and current velocities with different return periods in the Bohai Sea. The combinations of joint design ocean environmental factors, joint return periods, and design platform response are obtained. The following conclusions are drawn from this study.

1) Compared with Gumbel, lognormal, and Weibull distributions, Pearson Type III distribution is the optimal statistical model to fit sampled data series of wave height, wind speed, and current velocity in this study.

2) The bivariate or trivariate Clayton, Frank, Gumbel-Hougaard, and AMH copulas are evaluated for goodness of fit. Gumbel-Hougaard, Frank, and AMH copulas are optimal for constructing the bivariate distributions of wave height and wind speed, wind speed and current velocity, and wave height and current velocity, respectively. Trivariate Clayton copula is optimal to establish the joint distribution of wave height, wind speed, and current velocity.

3) Compared with univariate probability, conditional and joint probability provide a smaller combination of design environmental conditions. With the constraints of structural responses (maximum base shear, maximum overturning moment, and maximum deck displacement), the design parameters of environmental conditions by joint probability are also smaller than those by univariate probability.

4) The multivariate joint probability model considers the dependence among variables; therefore, the design values calculated by the multivariate model are more suitable to actual circumstances and more reasonable than those by univariate probability. Joint distribution can result in low investment cost for fixed platform construction, thereby indicating its important role in marginal oil field exploitation.

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