J. Ocean Univ. China (Oceanic and Coastal Sea Research)
DOI 10.1007/s11802-013-1894-5
ISSN 1672-5182, 2013 12 (1): 32-36
http://www.ouc.edu.cn/xbywb/
E-mail:xbywb@ouc.edu.cn

A Nonlinear Vortex Induced Vibration Model of Marine Risers

LIU Juan[1), 2)], and HUANG Weiping[1), *]

1) *Shandong Key Laboratory of Ocean Engineering*, *Ocean University of China*, *Qingdao* 266100, *P. R. China*
2) *Construction Engineering Institute*, *Agricultural University of Qingdao*, *Qingdao* 266009, *P. R. China*

(Received October 9, 2011; revised November 24, 2011; accepted August 28, 2012)

Abstract With the exploitation of oil and gas in deep water, the traditional vortex induced vibration (VIV) theory is challenged by the unprecedented flexibility of risers. A nonlinear time-dependent VIV model is developed in this paper based on a VIV lift force model and the Morison equation. Both the inline vibration induced by the flow due to vortex shedding and the fluid-structure interaction in the transverse direction are included in the model. One of the characteristics of the model is the response-dependent lift force with nonlinear damping, which is different from other VIV models. The calculations show that the model can well describe the VIV of deepwater risers with the results agreeing with those calculated by other models.

Key words vortex-induced vibration; fluid-structure interaction; lift force; nonlinear model

1 Introduction

In the development of the marine oil and gas industry, a large number of marine structures, particularly slender risers, have been built and put into use in the last thirty years (Huang *et al.*, 2009). The vortex-induced vibration (VIV) of risers has been studied for years and many analytic models from rigid to flexible cylinders have been developed (Gabbai and Benaroya, 2005; Sarpkaya, 2004; Lie and Kaasen, 2006), such as the Iwan-Blevins model, the Landl model, the Iwan model, and the Skop-Griffin model.

With the oil and gas exploitation in deep water, traditional VIV theory is challenged by that for deepwater risers because of their unprecedented flexibility. The major difference between the VIV of deepwater risers and traditional cylinders is that the former is of multi-mode vibration (Cai *et al.*, 2010; Zhang, 2009) and the flexibility and the fluid-structure interaction of deepwater risers can result in the system nonlinearity.

Previous studies (Huang *et al.*, 2009) mostly used the linear VIV lift force for the VIV analysis of deepwater risers, which has neglected the nonlinearity related to the fluid-structure interaction and the large displacement of deepwater risers. In this paper, a nonlinear VIV lift force model considering fluid-structure interaction is developed and applied to a deepwater riser.

2 Lift Force Model

VIV for a cylinder is induced by vortex shedding due to flow separation from the cylinder (Zhou *et al.*, 2010). The adverse pressure gradient imposed by the divergent geometry of the flow at the rear side of the cylinder causes the separation of the flow-cylinder boundary layer (Fig.1). As a result, a significant amount of vorticity is generated in the boundary layer and fed into the downstream shear layer leaving the separation point, and causes the shear layer to roll up into vortex. Relative to structures, different flows can lead to different separation points and vortex shedding models, which is fluid-structure interaction and considered in the VIV study of the wake oscillator model. Although the concept of the wake oscillator can well describe the interaction between fluid and cylinder, large displacement due to the interaction between fluid and deepwater riser cannot be modeled reasonably because of the disturbed flow field and the 'broken' wake oscillator. Large relative motion can also affect the flow separation and lead to difference in vortex shedding mode and frequency.

Based on the above analysis, the vortex shedding frequency

$$f_s = \frac{St \cdot U}{D} \tag{1}$$

and VIV lift force:

$$F_L = \frac{1}{2} C_L \rho D U^2 \cos 2\pi f_s t \tag{2}$$

can be respectively written as (Huang *et al.*, 2007a):

$$f_s' = \frac{St \cdot (U - \dot{x})}{D} \tag{3}$$

* Corresponding author. E-mail: wphuang@ouc.edu.cn

and

$$F_s^{'} = \frac{1}{2} C_L \rho D \left(U - \dot{x}\right)^2 \cos 2\pi f_s' t \tag{4}$$

where *St* is the Strouhal number, *U* is the flow velocity, *D* is the diameter of the cylinder, C_L is the lift force coefficient, ρ is the density of the fluid and t is time., $(U-\dot{x})$ is the relative fluid-structure velocity and $\dot{x}$ is the inline cylinder velocity.

The flow velocity *U* in Eqs. (1) and (2) are replaced by the relative fluid-structure velocity $(U-\dot{x})$ in Eqs. (3) and (4), which is based on the consideration of the fluid-structure interaction.

Using the measured $\dot{x}$ in Eqs. (3) and (4), the calculated results are plotted in Fig.2, which well agree with the measurements in Fig.3 (Zu and Huang, 2006).

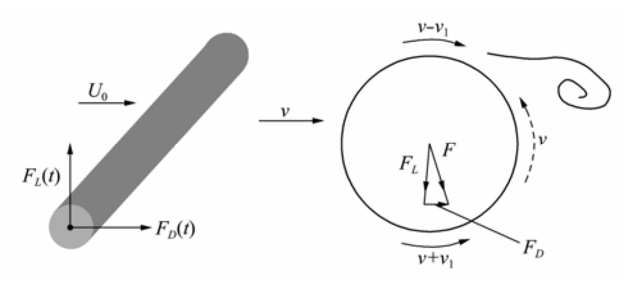

Fig.1 Flow regime around a circular cylinder.

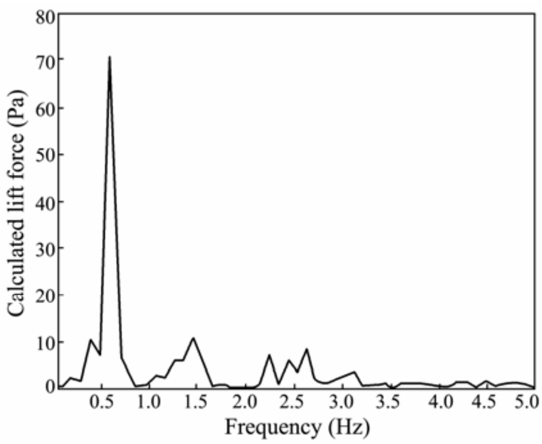

Fig.2 Calculated lift force spectrum.

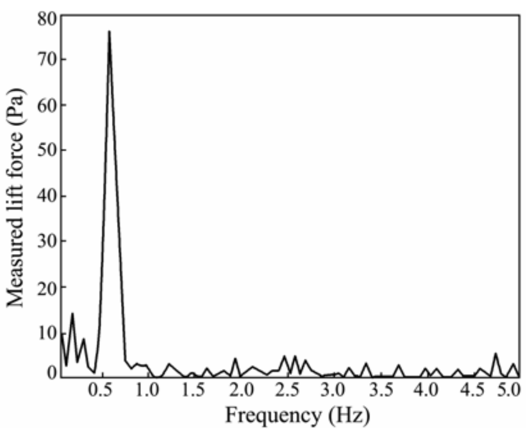

Fig.3 Measured lift force spectrum.

3 VIV Model

In the VIV analysis for cylinders, the fluctuation of the drag force caused by vortex shedding changes the inline vibration, and the relative fluid-cylinder velocity affects the vortex shedding frequency. As the mode and the frequency of both the lift force and the drag force change, the stochastic nature of the forces and the inline vibration or cross flow arise (Figs.4–7) (Huang *et al.*, 2007a).

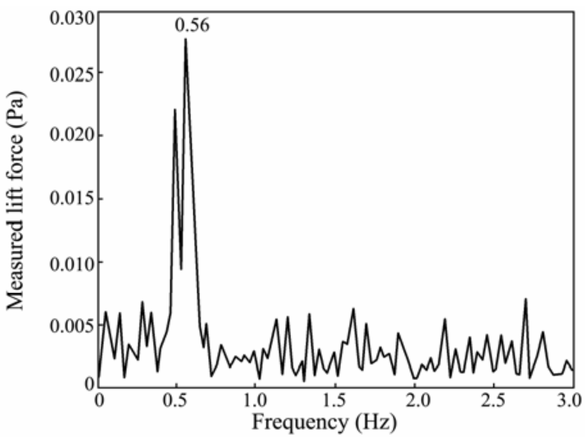

Fig.4 Measured lift force spectrum.

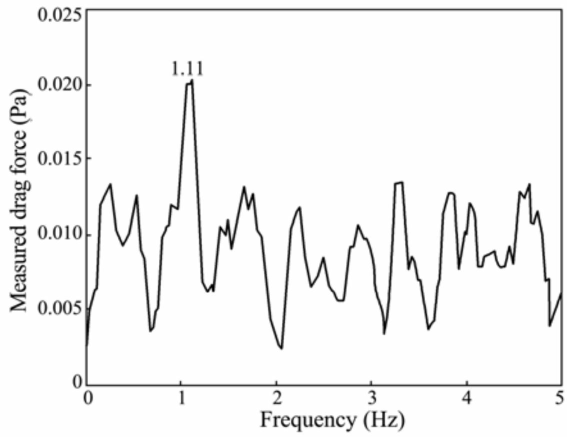

Fig.5 Measured drag force spectrum.

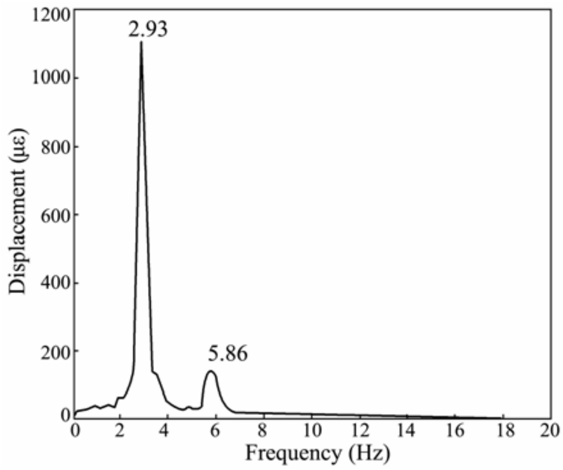

Fig.6 Spectrum of cross flow response.

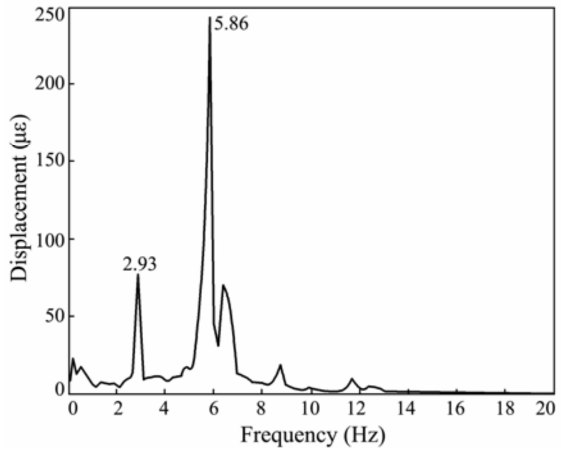

Fig.7 Spectrum of in-line response.

The vortex induced inline vibration has not drawn much attention in early VIV studies because it was considered much smaller than the cross flow (Huse *et al.*, 2002; Erik *et al.*, 2002; Martin 2003; Huang *et al.*, 2007b, Dahl *et al.*, 2006; Sanchis *et al.*, 2008). However, Figs.8 and 9 shows that the inline vibration of a flexible cylinder can generate a flow with the same order of displacment as that of cross flow.

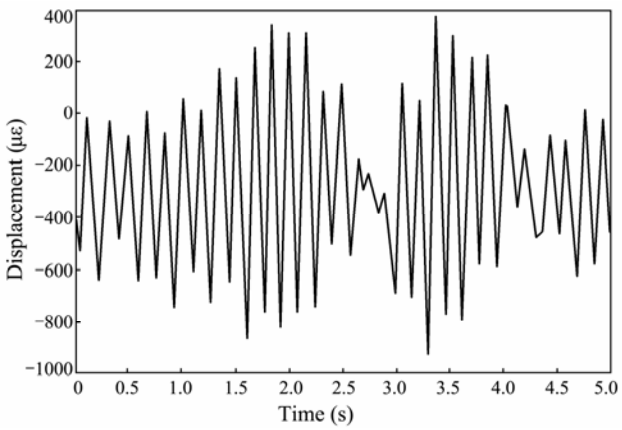

Fig.8 Time history of in-line response.

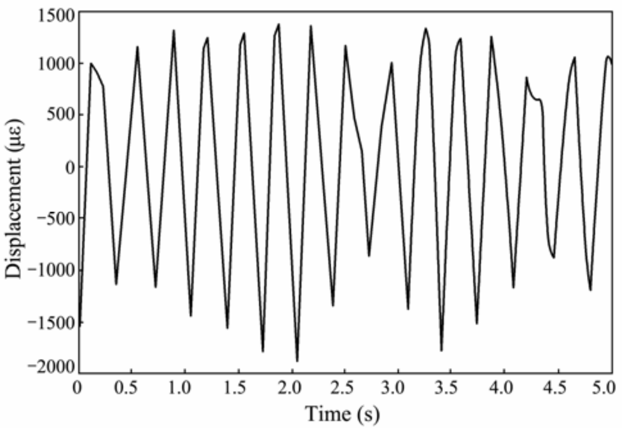

Fig.9 Time history of cross flow response.

Based on early study findings (Erik *et al.*, 2002; Martin, 2003) that the inline vibration frequency is twice as high as that of cross flow, this state-of-the-art nonlinear VIV model has included the inline vibration and fluid-structure interaction as shown in the following equations,

$$(m+m_a)\ddot{x}+c\dot{x}+kx=\frac{1}{2}C'_D\,\rho D(U-\dot{x})^2\cos 4\pi f'_s\,t+\frac{1}{2}C_D\rho D(U-\dot{x})\left|U-\dot{x}\right|, \tag{5}$$

$$\left(m+m_a\right)\ddot{y}+\left(c+c_a\right)\dot{y}+ky=\frac{1}{2}C_L\rho D\left(U-\dot{x}\right)^2\cos 2\pi f'_s t\ , \tag{6}$$

where m and c are the mass and damping of the cylinder, k is the stiffness of the cylinder, C'_D is the inline drag coefficient induced by VIV and ranges from 0.05 to 0.1, C_D is the drag coefficient, C_L is the lift force coefficient of cross flow vibration , and m_a and c_a are the added mass and added damping and can be calculated based on the relations in Morison equation:

$$m_a=\frac{\pi}{4}C_M\,\rho D^2\,, \tag{7}$$

$$c_a=\frac{1}{2}C_D\rho D\left|\dot{y}\right|, \tag{8}$$

where the added mass and added damping are due to the acceleration and the velocity of cylinder for steady flow, respectively.

4 Numerical Example

Based on the VIV model proposed above in Eqs. (5) and (6), an analytic program, RIWAV, which is written in the Matlab language, has been developed for top tensioned risers (TTRs).

The differential equations of tensioned beam are:

$$EI\frac{\mathrm{d}^4x}{\mathrm{d}z^4}-\frac{\mathrm{d}}{\mathrm{d}z}\left(T\frac{\mathrm{d}x}{\mathrm{d}\,z}\right)+\overline{m}\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\overline{c}\frac{\mathrm{d}x}{\mathrm{d}t}=f_x(z,t)\,, \tag{9}$$

$$EI\frac{\mathrm{d}^4y}{\mathrm{d}z^4}-\frac{\mathrm{d}}{\mathrm{d}z}\left(T\frac{\mathrm{d}y}{\mathrm{d}z}\right)+\overline{m}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+\overline{c}\frac{\mathrm{d}y}{\mathrm{d}t}=f_y(z,t)\,. \tag{10}$$

Eqs. (9) and (10) are solved for inline and cross flow vibration, where, EI is the bending stiffness of the cylinder, T is the tension in the cylinder, $\overline{m}$ and $\overline{c}$ is the mass and damping per unit length of cylinder, and the inline fluctuating drag force and cross flow lift force are represented by $f_x(z,\ t)$ and $f_y(z,\ t)$ in the equations, respectively.

A 1500 m TTR with double casing pipes is specified in the following calculations. The outer diameters of the casing pipes are 324 mm and 222 mm, respectively, and the outer diameter of the tube is 114 mm. The riser is simulated by a single pipe with the same outer diameter as the outer casing pipe and an inner diameter of 292 mm based on the equivalent bending stiffness of the pipe crosssection (Table 1). The current is uniform and has a speed of 0.21 m s^{-1}.

The RIWAV results are compared with those from a commercial software, Shear 7 for the TTR. Different boundary conditions have been specified for the two models, and the root-mean-square (RMS) values of the displacement and stress are calculated.

Figs.10–13 show the RMS displacement and RMS stress with different boundary specifications for RIWAV and Shear 7. The calculated RIWAV results agree well with the Shear 7 results. Therefore the VIV model developed here can describe the VIV process of deepwater risers as well as the vortex lift model.

Table 1 TTR parameters

Parameter	Value
Length of TTR	1500 m
Outer diameter	324 mm
Inner diameter	292 mm
Top tension coefficient	1.4
Modulus of elasticity	207 GPa
Density	7850 kg m^{-3}
Support conditions	Fixed-hinge and both hinges

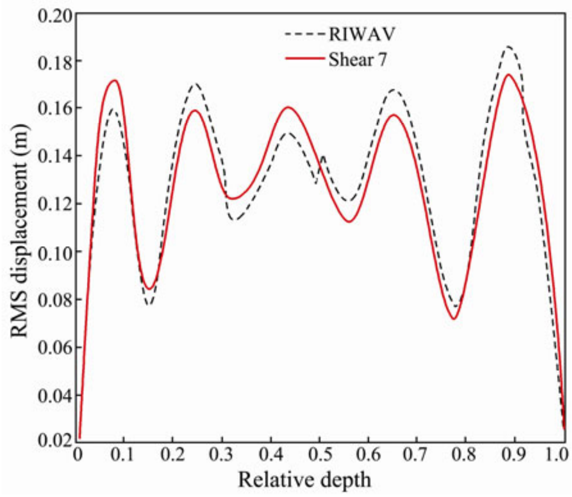

Fig.10 RMS displacement along TTR (Fixed-hinge).

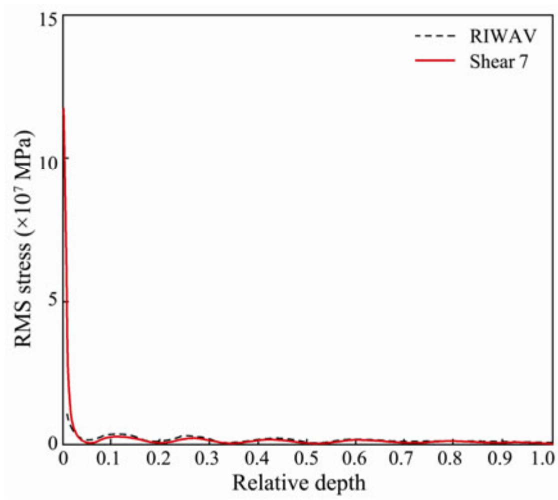

Fig.11 RMS Stress along TTR (Fixed-hinge).

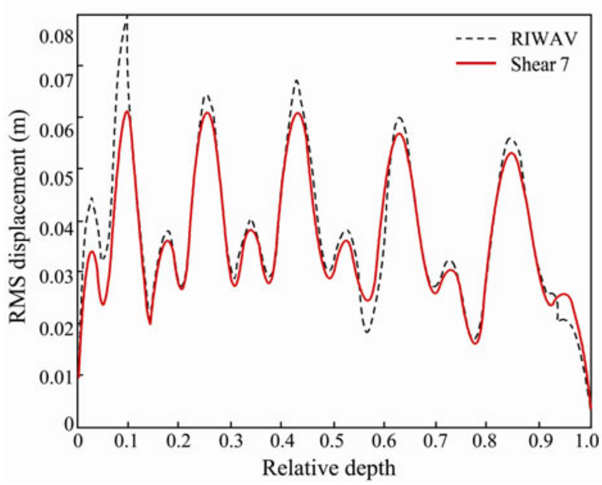

Fig.12 RMS Displacement along TTR (hinge-hinge).

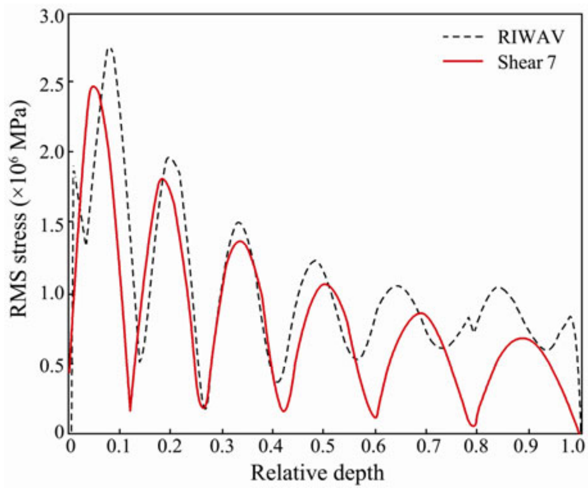

Fig.13 RMS Stress along TTR (hinge-hinge).

5 Conclusions

Traditional VIV models do not include the inline vibration and are only applicable to lock-in condition. But for deepwater risers, the lock-in has to be kept away by an elaborate design to avoid riser resonance. An updated VIV model is developed to simulate the cases under non-lock-in condition. The model is characterized by its nonlinear lift force and fluid-structure interaction. The fluid-structure interactions for both inline and cross flows are taken into account in the model.

The model well describes the VIV and fluid-structure interaction around a flexible cylinder, and the calculated RIWAV results show a good agreement with Shear 7. The study indicates that the fluid-structure interaction exists for a riser with large displacement. The greater the relative motion is, the stronger the interaction is. The fluid-structure interaction is successfully simulated by added mass and damping in the model.

Acknowledgements

The work is owned by National Natural Science funds of China (51079136, 51179179).

References

Cai, J., You, Y. X., Li, W., Shi, Z. M., and Qu, Y., 2010. The VIV characteristics of deep-sea risers with high aspect ratio in a uniform current profile. *Journal of Hydrodynamics*, **25** (1): 50-58 (in Chinese with English abstract).

Dahl, J. M., Hover, F. S., and Triantafyllou, M. S., 2006. Two-degree-of-freedom vortex-induced vibrations using a force assisted apparatus. *Journal of Fluids and Structures*, **22** (6-7): 807-818.

Erik, A. H., Mads, B., and Stefan, M., 2002. Interaction of in-line and cross-flow vortex induced vibrations in risers. In: *Proceedings of International Conference on Offshore Mechanics and Arctic Engineering*. ASME, Oslo, 551-557.

Gabbai, R. D., and Benaroya, H., 2005. An overview of modeling and experiments of vortex-induced vibration of circular cylinders. *Journal of Sound and Vibration*, **282** (3-5): 575-616.

Huang, W. P., Bai, X. L., and Li, H. J., 2009. State of the art of research and development of overseas deepwater steel catenary risers. *Periodical of Ocean University of China*, **39** (2): 290-294 (in Chinese with English abstract).

Huang, W. P., Wang, A. Q., and Li, H. J., 2007a. Experimental study on VIV of span of subsea pipeline and improved model of lift force. *Engineering Mechanics*, **24** (12): 153-157 (in Chinese with English abstract).

Huang, Z. Y., Pan, Z. Y., and Cui, W. C., 2007b. Numerical simulation of VIV of a circular cylinder with two degrees of freedom and low mass-ratio. *Journal of Ship Mechanics*, **11** (1): 1-9 (in Chinese with English abstract).

Huse, E.,Nielsen, F. G., and Reddy, J. N., 2002. Coupling between in-line and cross-flow VIV response. In: *Proceedings of International Conference on Offshore Mechanics and Arctic Engineering*. ASME, Oslo, 835-847.

Lie, H., and Kaasen, K. E., 2006. Modal analysis of measurements from a large-scale VIV model test of a riser in linearly sheared flow. *Journal of Fluids and Structures*, **22**: 557-575.

Martin, S., 2003. Experimental investigation of in-line and cross-flow VIV. In: *Proceedings of International Offshore and Polar Engineering Conference*. ISOPE, Honolulu, 585-592.

Sanchis, A., Sælevik, G., and Grue, J., 2008. Two-degree-of-freedom vortex-induced vibrations of a spring- mounted rigid cylinder with low mass ratio. *Journal of Fluids and Structures*, **24** (6): 907-919.

Sarpkaya, T., 2004. A critical review of the intrinsic nature of vortex-induced vibrations. *Journal of Fluids and Structures*, **18** (Suppl.): 389-447.

Zhang, J. Q., 2009. Experiment Investigation of Vortex-Induced Vibration of Long-Flexible Marine Risers. Master thesis. Dalian University of Technology, Dalian (in Chinese with English abstract).

Zhou, W. W., Cao, J., Zhou, L., and Zong, Z., 2010. Analysis of vortex-induced vibration induced fatigue damage for deep sea steel catenary riser. *Journal of Jiangsu University of Science and Technology* (*Natural Science Edition*), **24** (2): 112-115 (in Chinese with English abstract).

Zu, N., and Huang, W. P., 2006. Nonlinear dynamic analysis of submarine pipelines considering fluid-structure interaction. *Periodical of Ocean University of China*, **36** (1): 168-172 (in Chinese with English abstract).

(Edited by Xie Jun)