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Stress Analysis of Top Tensioned Riser Under Random Waves and Vessel Motions

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Abstract The bending stresses of top tensioned riser (TTR) under combined excitations of currents, random waves and vessel motions are presented in this paper, and the effect of the internal flowing fluid on the riser stresses is also considered. The computation programs which are used to solve the differential equations in the time domain are compiled and the principal factors of concern including the angular movements at the upper and lower ends of the riser, lateral displacements and bending stresses are presented. Then the effects of current velocity, random wave, top tension, vessel mean offset, low frequency motion and internal flow velocity on the bending stresses of the riser are analyzed in detail.

Key words top tensioned riser (TTR); internal flow; vessel motion; dynamic response

1 Introduction

Offshore oil-gas exploitation and production in deep waters present many challenges, one of them being the design of technically feasible and cost effective riser systems. As one of the main configurations of the riser, the top tensioned riser (TTR) serves in the offshore structural system as the link between the platform and the well head at the seabed and can be used for drilling, injection, well completion and production. Generally, the riser encounters severe current and wave forces and it is excited by the vessel motion at the top end. Besides these loads and excitations, the riser may experience additional excitation arising from the internal flowing fluid. The natural frequencies of the riser usually fall within the range of these excitation frequencies. Consequently, these excitations can produce large displacement and significant dynamic cyclic stresses in the riser which may accelerate the fatigue damage of the riser.

In recent years, marine risers have attracted considerable attention in offshore engineering and academic circles due to their importance and complexity. The Galerkin method was used by Kirk (1985) in the solution of the marine riser differential equation and compared the dynamic bending stresses in a tension-leg-platform riser calculated by the linearised single wave and linearised spectral analysis methods. Ahmad and Datta (1989) investigated the dynamic response of marine risers under both regular and random waves in the time domain using a time marching numerical integration scheme to solve the motion equation. Ormberg *et al.* (1997) analyzed the coupling dynamics of the vessel motions and mooring and riser system. Atadan *et al.* (1997) investigated the forced dynamics of riser system which was connected to floating platform under the action of ocean waves and ocean currents. Li (1999) studied the force dynamics of the riser system, connected to a floating platform and conveying fluid, in the presence of ocean waves and currents. Kaewunruen *et al*. (2005) analyzed the influence of marine riser parameters such as flexural rigidity, top tension, internal flow velocity and static offset on the nonlinear free vibrational behaviors by reformulating the governing equation to an eigenvalue problem. Chatjigeorgiou and Mavrakos (2002) dealt with the nonlinear dynamic response in the transverse direction of vertical marine risers or tensioned cable legs subjected to parametric excitation at the top of the structure. Special attention was given to the effect of the hydrodynamic drag for the parametric excitation frequencies that guide the dynamic system to lie within a region of coupled instability. Kuiper *et al.* (2008) studied the parametric excitation stability of a deepwater riser using a numerical time-domain technique. Chang *et al.* (2008) studied the dynamic response of deepwater drilling risers subjected to random waves and vessel motions and concluded that the vessel motion, rather than wave force, produced the principal dynamic loading in the nonlinear dynamic response of the risers. Li *et al.* (2010) analyzed the fatigue life of top tensioned risers under vortex-induced vibrations (VIVs) and the influences of the riser's parameters such as flexural

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as flexural rigidity, top tension and internal flow velocity on the fatigue life of the riser are analyzed in detail. However, more intensive study on the nonlinear dynamic response of top tensioned riser under combined excitations of ocean currents, random waves and vessel motion is necessary; besides the effect of internal flowing fluid on the riser bending stresses has not been analyzed in most of the literature mentioned above.

In the present paper, the bending stresses of top tensioned riser under combined excitations of currents, random waves and vessel motion with consideration of the effect of the internal flowing fluid on the riser stresses are presented. The corresponding computation programs which are used to solve the governing differential equations in the time domain are compiled and the principal factors of concern including the angular movements at the upper and lower ends of riser, lateral displacements and bending stresses of the riser are proposed. Then the effects of the current velocity, random waves, top tension, vessel mean offset, low frequency motion and internal flow velocity on the bending stresses on the riser are analyzed in detail.

2 Mathematical Models

2.1 Governing Equation for Marine Riser

The coordinate system is defined as follows: the z axis, positive upward, coincides with the vertical axis of the riser in its undeflected configuration with the origin at the bottom of the riser, the x axis is parallel to the flow velocity, and the y axis is perpendicular to both. Only the zx plane dynamics of the riser is considered in the following analysis while the in-line and cross-flow vortex-induced vibration is ignored. The riser governing equation can be expressed as

$$(m_r+m_i)\frac{\partial^2 x}{\partial t^2}+c\frac{\partial x}{\partial t}+2m_iV\frac{\partial^2 x}{\partial z\partial t}+ EI\frac{\partial^4 x}{\partial z^4}-(T_e-m_iV^2)\frac{\partial^2 x}{\partial z^2}=F_x\,, \tag{1}$$

where m_r is the mass of unit length of riser, m_i the internal fluid mass of unit length of riser, c the structural damping ratio of riser system, V the internal flowing fluid velocity, EI the bending stiffness of the riser, T_e the effective tension in the riser, and F_x the hydrodynamic force per unit length on the riser. F_x is calculated by Morison's Equation,

$$F_x=\frac{\rho_{\text{sea}}\pi D_o^2}{4}\frac{\partial u}{\partial t}+C_A\frac{\rho_{\text{sea}}\pi D_o^2}{4}\left(\frac{\partial u}{\partial t}-\frac{\partial^2 x}{\partial t^2}\right)+ C_D\frac{\rho_{\text{sea}}D_o}{2}\left|u+U_c-\frac{\partial x}{\partial t}\right|\left(u+U_c-\frac{\partial x}{\partial t}\right), \tag{2}$$

where u is the wave particle velocity, U_c the current velocity, ρ_{sea} the seawater density, D_o the external diameter of the riser, C_D the drag coefficient, and C_A the added mass coefficient.

Substituting Eq. (2) into Eq. (1), moving the hydrodynamic added mass to the left side of the equation, then the following governing equation can be obtained,

$$(m_r+m_i+m_a)\frac{\partial^2 x}{\partial t^2}+(c+c')\frac{\partial x}{\partial t}+ 2m_iV\frac{\partial^2 x}{\partial z\partial t}+EI\frac{\partial^4 x}{\partial z^4}-(T_e-m_iV^2)\frac{\partial^2 x}{\partial z^2}=C_M\frac{\rho_{\text{sea}}\pi D_o^2}{4}\dot{u}+ C_D\frac{\rho_{\text{sea}}D_o}{2}\left|u+U_c-\frac{\partial x}{\partial t}\right|(u+U_c)\,, \tag{3}$$

where C_M=C_A+1 is the inertia coefficient and

$$c'=C_D\frac{\rho_{\text{sea}}D_o}{2}\left|u+U_c-\frac{\partial x}{\partial t}\right|$$

is the fluid damping.

2.2 Boundary Conditions

The lower end of the top tensioned riser is connected to wellhead on the seabed through a universal joint and the bending stiffness of the universal joint is assumed to be zero. Then the lower boundary conditions can be expressed as

$$x\big|_{z=0}=0,\ \ EI\frac{\partial^2 x}{\partial z^2}\bigg|_{z=0}=0\,. \tag{4}$$

The upper end of the riser is connected to floating platform through another universal joint. Neglecting the heaving movement of the platform, the upper boundary conditions can be expressed as

$$x\big|_{z=L}=S(t),\ \ EI\frac{\partial^2 x}{\partial z^2}\bigg|_{z=L}=0\,, \tag{5}$$

where $S(t)$ represents the vessel horizontal motion and L represents the total length of the riser.

2.3 Vessel Motions

Vessel motion and station keeping performance have a significant effect on the riser design and operation. Certain operations such as riser running or pulling, drilling, workover and through-bore may be restricted or require shutdown, depending upon vessel motions and environmental limits. It is a common practice to estimate the rig offsets from a set of Response Amplitude Operators (RAO) which relate the rig or vessel response to the significant wave height h. This offset is then introduced as a moving boundary condition in the differential equations describing the riser motion. The vessel motion model was given by Sexton and Agbezuge (1976) as follows

$$S(t)=S_0+S_L\sin(\frac{2\pi t}{T_L}-\alpha_L)+ \sum_{i=1}^{N}S_n\cos(k_nS(t)-\omega_nt+\phi_n+\alpha_n)\,, \tag{6}$$

where S_0 represents the mean offset,

$$S_L\sin(\frac{2\pi t}{T_L}-\alpha_L)$$

the low frequency motion,

$$\sum_{i=1}^{N}S_n\cos(k_nS(t)-\omega_nt+\phi_n+\alpha_n)$$

the resultant motion contributed by all wave frequencies involved. S_L is the single amplitude of vessel drift, T_L the period of vessel drift, α_L a phase angle between drift motion and wave (usually taken as zero), S_n the amplitude response of the vessel to a wave of period T_n (T_n=2π/ω_n) and amplitude A_n, k_n the wave number, ϕ_n the wave phase angle, α_n the phase angle between the vessel response and the wave of period T_n.

2.4 Random Waves

It is common to assume that the sea state is stationary for a duration of 20 min to 3–6 h. A sea state can be characterized by a set of environmental parameters such as the significant wave height H_s and the peak period T_p. The Pierson-Moskowitz spectrum presented by DNV-RP-C205 (2007) is as follows

$$S_{\eta\eta}(\omega)=\frac{5}{16}H_s^2\omega_p^4\omega^{-5}\exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right), \tag{7}$$

where $S_{\eta\eta}(\omega)$ is the PM (single-sided) sea surface elevation spectrum and $\omega_p=2\pi/T_p$ is the angular spectral peak frequency.

The random waves and corresponding water particle kinematics are simulated by wave superposition techniques from their respective spectra.

3 Dynamic Response Analyses of Riser

The parameters and their values employed in the calculations are taken as follows (Leklong *et al.*, 2008): water depth d=300 m, riser length L=300 m, top tension $T_{\rm top}$=476 198 N, riser pipe outside diameter D_o=0.26 m, inside diameter D_i=0.2 m, specific weight of the sea water $\rho_{\rm sea}$=1025 kg m^{-3}, specific weight of the fluid in the riser bore ρ_i=998 kg m^{-3}, specific weight of the riser wall material ρ_r=7850 kg m^{-3}, modulus of elasticity E=2.07×10^{11} N m^{-4}, drag coefficient C_D=0.7, and the inertia coefficient C_M=2.

The basic parameters of environmental load condition in the analysis are as follows: current velocity U_c=0.5 m s^{-1}, significant wave height H_s=6.5 m, the peak period T_p=12.82 s, vessel mean offset S_0=5 m, the single amplitude of vessel drift S_L=5 m, the period of vessel drift T_L=200 s, and the internal flowing fluid velocity V=0 m s^{-1}.

The fundamental frequency of the riser at different internal flow velocities in comparison with the result of Leklong *et al.* (2008) is shown in Table 1. It can be found that they are in very good agreement. Time history of the wave elevation and vessel motion are presented in Figs.1 and 2, respectively.

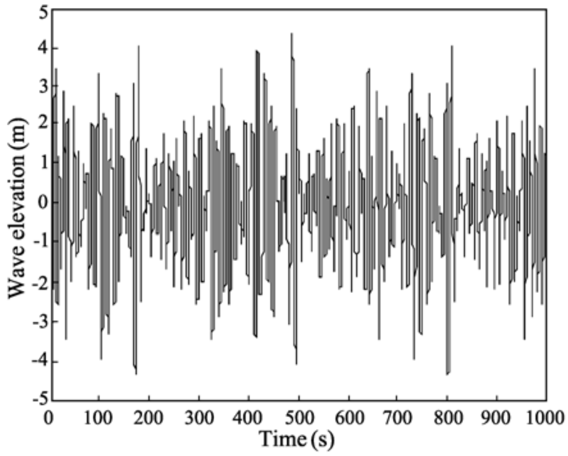

Fig.1 Time history of the wave elevation.

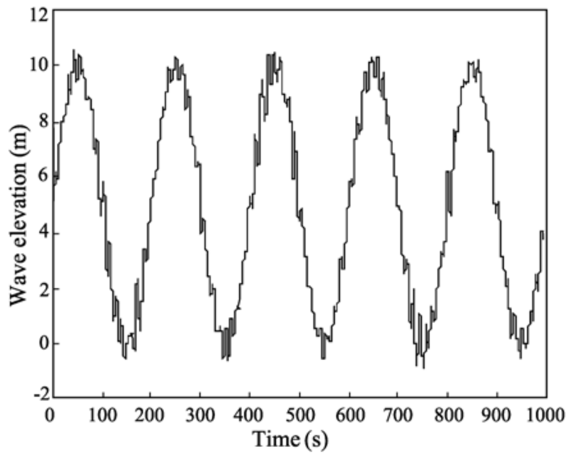

Fig.2 Time history of the vessel motion.

Table 1 Fundamental angular frequency of the riser at different internal flow velocities

Internal flow speed (m s^{-1})	Numerical solution [A]	Leklong's result [B]	This study	% Difference from [A] and [B] [A]	% Difference from [A] and [B] [B]
0	0.2987	0.3005	0.2989	0.0670	−0.5324
5	0.2979	0.2996	0.2982	0.1007	−0.4673
10	0.2956	0.2990	0.2961	0.1691	−0.9699
15	0.2916	0.2956	0.2924	0.2743	−1.0825
20	0.2858	0.2870	0.2871	0.4549	0.0348
25	0.2782	0.2798	0.2801	0.6830	0.1072
30	0.2683	0.2685	0.2710	1.0063	0.9311
35	0.2558	0.2590	0.2594	1.4073	0.1544

3.1 Dynamic Responses of the Riser Under Random Waves, Currents and Vessel Motion

The dynamic responses of the riser under random waves, currents and vessel motion are presented in Figs.3, 4, 5 and 6. The root mean square (RMS) bending stresses of the riser are shown in Fig.3. It can be seen that the riser has two extreme stress points which are located at 30 m and 285 m from the bottom of the riser. The corresponding time histories of the displacement and bending stresses of the two points are presented in Figs.5 and 6, respectively. It can be noted that the riser vibrates both at the wave frequencies and vessel low frequency. The bending stress history at location 30 m is also vibrating at both the wave frequencies and vessel low frequency while it is not distinctive at location 285 m. The lower part of riser vibrates at relatively high stress levels and the stress amplitude of the riser is very high, which may accelerate the fatigue failure of the riser. The American Petroleum Institute (API, 1998) and Det Norske Veritas(DNV, 2001) also recommend that the fatigue analyses performed on top tensioned risers should include the effects of wave cycles and slow drift cycles. Fig.4 shows the angular movements at the upper and lower ends of riser. It can be observed that the angle of the lower end is relatively high and sensitive to the vessel low frequency motion. The bending angles for the upper and lower ends of the joints are also important considerations in the design and production. The American Petroleum Institute has issued the corresponding prescripts of the allowable maximum angles. Due to the vessel motion and the dynamic vibrations of the riser, the upper and lower angular movements might exceed the angle limit, which may result in damages to the riser end joints.

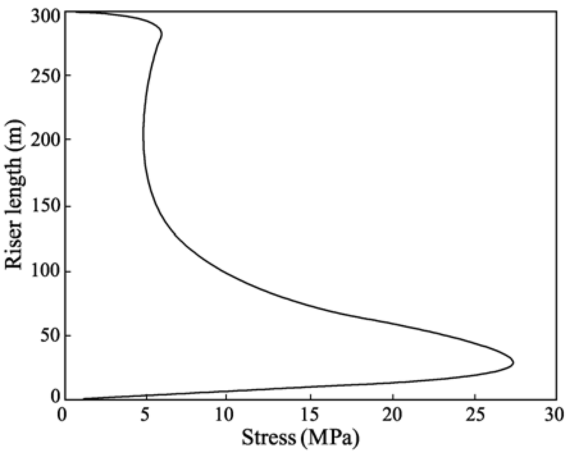

Fig.3 RMS stresses of the riser.

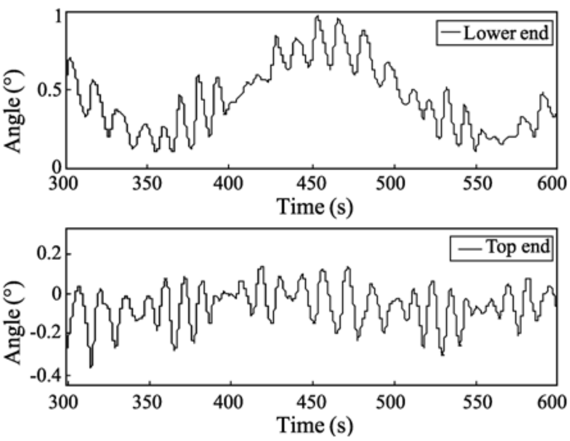

Fig.4 Time history of the angular movements.

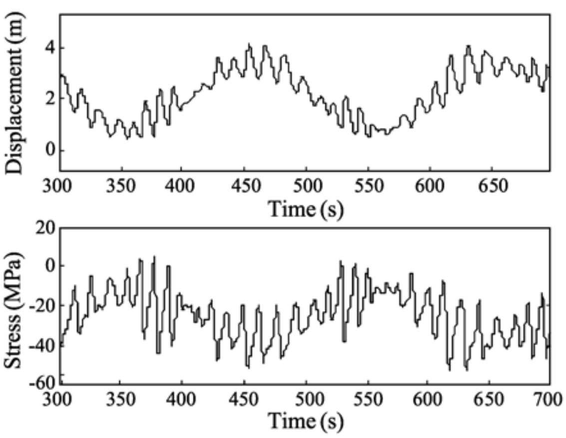

Fig.5 Time history of the riser displacement and stresses at location 30 m.

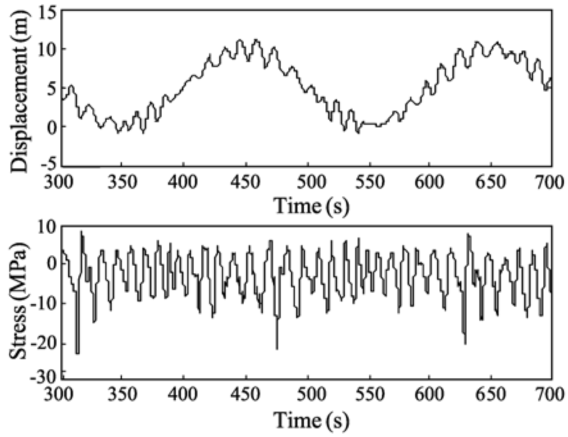

Fig.6 Time history of the riser displacement and stresses at location 285 m.

3.2 Effect of Current and Random Waves on the Bending Stresses of the Riser

Fig.7 presents RMS bending stresses of the riser at three different current velocities, *i.e.* 0.3, 0.5 and 0.7 m s^{-1}, and the other environmental parameters are assumed unchanged. It can be noted that with the increase of the current velocity, the stresses at the lower part increase greatly while the stresses at the upper part decrease.

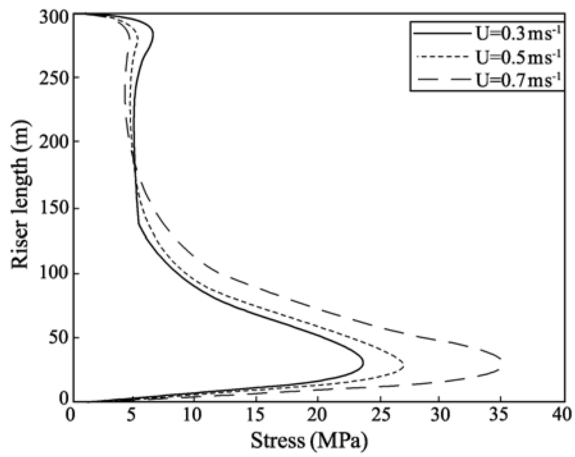

Fig.7 RMS stresses of the riser at different current velocities.

Fig.8 presents RMS bending stresses of the riser at three different significant wave heights, *i.e.* 5, 6.5 and 8 m, and the corresponding peak periods, *i.e.* 11.22, 12.82 and 14.28 s. It is clear that the bending stresses of the riser increase with the increase of the significant wave height.

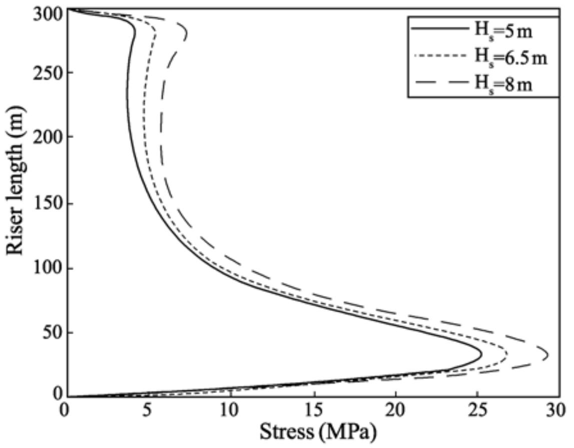

Fig.8 RMS stresses of the riser at different wave heights.

3.3 Effect of Top Tension on the Bending Stresses of the Riser

The effect of the top tension on the riser bending stress is shown in Fig.9. Three top tensions 476198, 600000 and 700000N are selected. It can be observed that with the simple increase of the top tension, the bending stresses of the riser decrease rapidly, which may augment the riser fatigue life greatly, especially for the lower part of the riser. Therefore it is recommended that the top tension can be as high as possible within the allowable range.

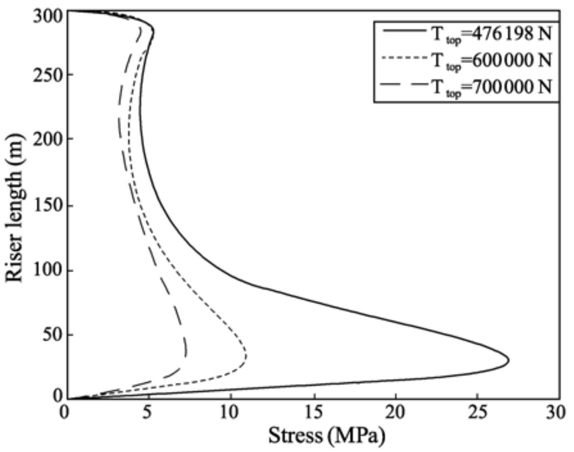

Fig.9 RMS stresses of the riser at different top tensions.

3.4 Effect of Vessel Mean Offset on the Bending Stresses of the Riser

Fig.10 presents RMS bending stresses of the riser at three different vessel mean offset, *i.e.* 3, 5 and 7m. It can be observed that with the increase of the vessel mean offset, the bending stresses increase greatly, especially for the lower part of the riser.

3.5 Effect of Low Frequency Motion on the Bending Stresses of the Riser

The effect of the low frequency motion of the vessel on the riser bending stress is shown in Figs.11 and 12. The three different low frequency motion amplitudes 3, 5 and 7m and periods 120, 160 and 200s are selected. It can be noted that the low frequency motion amplitude mainly affects the bending stresses of the lower part of the riser. With the low frequency motion amplitude increasing, the RMS stresses of the riser increase gradually. However, the low frequency motion period mainly affects the bending stresses of the upper part of the riser. With the low frequency motion period increasing, the RMS stresses of the riser decrease gradually.

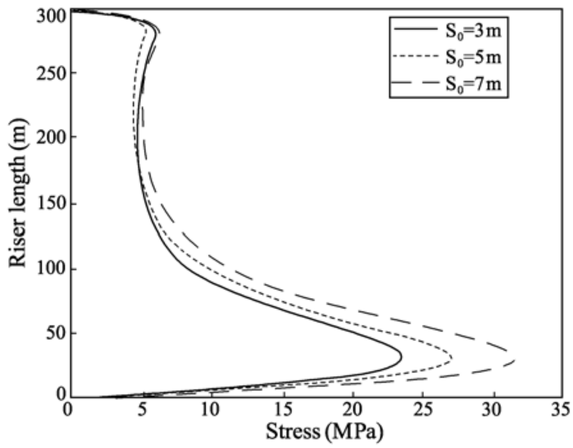

Fig.10 RMS stresses of the riser at different vessel mean offsets.

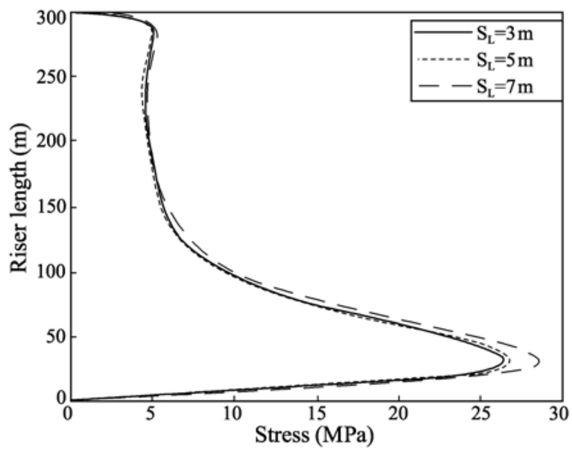

Fig.11 RMS stresses of the riser at different low frequency motion amplitudes.

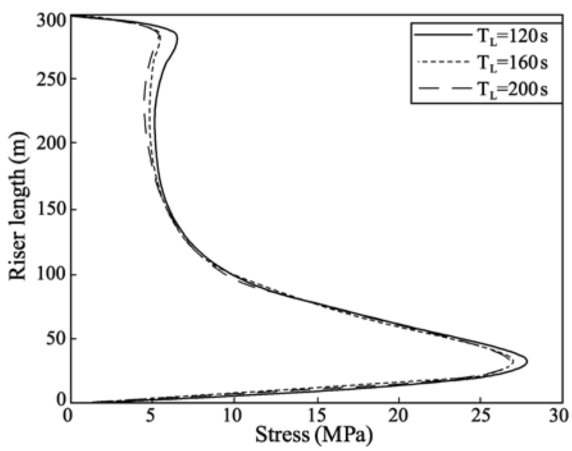

Fig.12 RMS stresses of the riser at different low frequency motion periods.

3.6 Effect of Internal Flow Velocity on the Bending Stresses of the Riser

The effect of the internal flow velocity on the riser bending stress is shown in Fig.13. Three different velocities of 0, 10 and 20 $m s^{-1}$ are chosen. It can be observed that the internal flow velocity has great effect on the riser bending stresses, especially for the lower part of the riser. Generally, with the internal flow velocity increasing, the RMS stresses of the riser increase; for high internal flow velocities this trend is more remarkable.

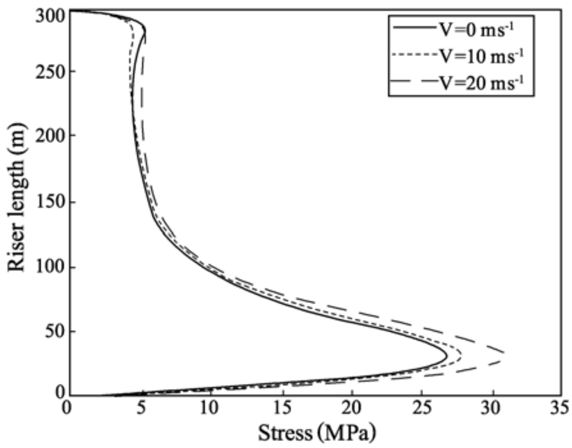

Fig.13 RMS stresses of the riser at different internal velocities.

4 Conclusions

In this paper, the bending stresses of top tensioned riser under combined excitations of currents, random waves and vessel motion with consideration of the effect of the internal flowing fluid on the riser are presented. The angular movements at the upper and lower ends of the riser, lateral displacements and bending stresses of the riser are presented and the effects of the current velocity, random waves, top tension, vessel mean offset, low frequency motion and internal flow velocity on the bending stresses of the riser are analyzed in detail. The following conclusions can be drawn:

1) The riser mainly has two points of extreme bending stress values under the excitations of random waves, currents and vessel motion which are located at the lower and upper parts of the riser, respectively. These two parts are prone to over-bending and hence ancillary devices should be incorporated into the design to increase the riser bending stiffness for protecting the riser.

2) With the increase of the current velocity, the stresses at the lower part increase greatly while the stresses at the upper part decrease. The bending stresses of the riser increase with the increase of the significant wave height.

3) The bending stresses of the riser decrease rapidly with the increase of the top tension while they increase with the increase of the vessel mean offset.

4) The low frequency motion amplitude mainly affects the bending stresses of the lower part of the riser while the low frequency motion period mainly affects the bending stresses of the upper part of the riser.

5) Internal flow velocity has great effect on the bending stresses of the riser and it should be given enough attention during riser analysis and design.

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References

Ahmad, S., and Datta, T. K., 1989. Dynamic response of marine risers. *Eng. Struct.*, **11** (3): 179-188.

API, 1998. Recommended practice API-RP-2RD: Design of risers for Floating Production Systems (FPSs) and Tension-Leg Platforms (TLPs). American Petroleum Institute.

Atadan, A. S., Calisal, S. M., Modi, V. J., and Guo, Y., 1997. Analytical and numerical analysis of the dynamics of a marine riser connected to a floating platform. *Ocean Eng.*, **24** (2): 111-131.

Chang, Y. J., Chen, G. M., Sun, Y. Y., Xu, L. B., and Peng, P., 2008. Nonlinear dynamic analysis of deepwater drilling risers subjected to random loads. *Chin. Ocean Eng.*, **22** (4): 683-691.

Chatjigeorgiou, I. K., and Mavrakos, S. A., 2002. Bounded and unbounded coupled transverse response of parametrically excited vertical marine risers and tensioned cable legs for marine applications. *Appl. Ocean Res.*, **24** (6): 341-354.

DNV, 2001. Offshore Standard DNV-OS-F201: Dynamic Riser. Det Norske Veritas.,91pp.

DNV, 2007. Recommended Practice DNV-RP-C205: Environmental Conditions and Environmental Loads. Det Norske Veritas.,123pp.

Kaewunruen, S., Chiravatchradej, J., and Chucheepsakul, S., 2005. Nonlinear free vibration of marine risers/pipes transporing fluid. *Ocean Eng.*, **32** (3-4): 417-440.

Kirk, C. L., 1985. Dynamic response of marine risers by single wave and spectral analysis methods. *Appl. Ocean Res.*, **7** (1): 12-13.

Kuiper, G. L., Brugmans, J., and Metrikine, A. V., 2008. Destabilization of deep-water risers by a heaving platform. *J. Sound Vib.*, **310** (3): 541-557.

Leklong, J., Chucheepsakul, S., and Kaewunruen, S., 2008. Dynamic responses of marine risers/pipes transporting fluid subject to top end excitations. *Proceedings of the Eighth ISOPE Pacific/Asia Offshore Mechanics Symposium*, Bangkok, Thailand, 105-112.

Li, Y. S., 1999. Dynamic response of marine risers to wave-crest loads. Offshore Technology Conference, Houston, Texas, USA, 1-6, paper 10930.

Li, X. M., Guo, H. Y., and Meng, F. S., 2010. Fatigue life assessment of top tensioned riser under vortex-induced Vibrations. *J. Ocean Univ. Chin.*, **9** (1): 43-47. doi: 10.1007/s11802-010-0043-7.

Ormberg, H., Fylling, I. J., Larsen, K., and Sodal, N., 1997. Coupled analysis of vessel motions and mooring and riser system dynamics. In: *Proceedings of the 16th International Conference on Offshore Mechanics and Arctic Engineering*, New York, 91–100.

Sexton, R. M., and Agbezuge, L. K., 1976. Random wave and vessel motion effects on drilling riser dynamics. Offshore Technology Conference, Dallas, Texas, USA, 391-404, paper 2650. doi: 10.4043/2650-MS.

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