An improved AS-SCLF decoding algorithm of polar codes based on the assigned set*

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An improved successive cancellation list bit-flip based on assigned set (AS-SCLF) decoding algorithm is proposed to solve the problems that the successive decoding of the successive cancellation (SC) decoder has error propagation and the path extension of the successive cancellation list (SCL) decoder has the decision errors in the traditional cyclic redundancy check aided successive cancellation list (CA-SCL) decoding algorithm. The proposed algorithm constructs the AS firstly. The construction criterion is to use the Gaussian approximation principle to estimate the reliabilities of the polar subchannel and the error probabilities of the bits under SC decoding, and the normalized beliefs of the bits in actual decoding are obtained through the path metric under CA-SCL decoding, thus the error bits containing the SC state are identified and sorted in ascending order of the reliability. Then the SCLF decoding is performed. When the CA-SCL decoding fails for the first time, the decision results on the path of the SC state in the AS are exchanged. The simulation results show that compared with the CA-SCL decoding algorithm, the SCLF decoding algorithm based on the critical set and the decision post-processing decoding algorithm, the improved AS-SCLF decoding algorithm can improve the gain of about 0.29 dB, 0.22 dB and 0.1 dB respectively at the block error rate (*BLER*) of 10^{-4} and reduce the number of decoding at the low signal-to-noise ratio (*SNR*), thus the computational complexity is also reduced.

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The polar code is a channel coding scheme that can achieve Shannon capacity[1,2]. In order to improve the decoding performance of polar codes, scholars have proposed an optimized decoding scheme called as the cyclic redundancy check aided successive cancellation list (CA-SCL) decoding algorithm[3-5]. The algorithm adds certain cyclic redundancy check (CRC) bits to the successive cancellation list (SCL) decoding algorithm[6] and outputs the codeword sequence which first passes the CRC check. Moreover, the successive cancellation list bit-flip (SCLF) decoding algorithm[7,8] is proposed to further improve the decoding performance of the CA-SCL algorithm. For the SCLF decoding, if no codeword sequence can pass the CRC, re-decoding will be executed to exchange the decision result of the path competition for a specific bit.

The SCLF based on critical set (CS-SCLF) decoding algorithm proposed in Ref.[7] quotes the critical set (CS) constructed by Ref.[9], and the CS contains the first bit of all the Rate-1 nodes in the codeword sequence. The CS-SCLF decoding algorithm reduces the error propagation of the decoding process by flipping a bit in the CS, thereby achieving an improvement in performance. However, the bits of the decision errors may not be identified quickly and accurately in the CS due to the destruction of dynamic noise. To predict the decision errors[10], Ref.[8] has proposed the decision post-processing (D-POST) decoding algorithm, which directly uses the path metric (*PM*) obtained from the CA-SCL decoding process to calculate the beliefs of the bits and preferentially flips the bit with the lower belief, thereby reducing the unnecessary re-decoding attempts. A new CS called as the assigned set (AS) is constructed in this paper. The AS can identify the bits of the decoding errors by judging the normalized beliefs of the bits, and the bits are arranged in ascending order according to their reliabilities, thus the error propagation of the SC decoder and the decision errors for the path competition of the SCL decoder are both reduced. An improved SCLF based on assigned set (AS-SCLF) decoding algorithm that can reduce the number of the decoding attempts and has better decoding performance is proposed in this paper.

Consider a binary (N, K) polar code where the input source sequence is denoted by $\boldsymbol{u}_1^N=(u_1,u_2,\ldots,u_N)$, and $\boldsymbol{u}_1^N$ is encoded by the generator matrix $\boldsymbol{G}_N$. The transmitted

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sequence $\boldsymbol{x}_1^N=(x_1,x_2,\ldots,x_N)$ constructed by $\boldsymbol{x}_1^N=\boldsymbol{u}_1^N\boldsymbol{G}_N$. $\boldsymbol{y}_1^N=(y_1,y_2,\ldots,y_N)$ represents the received sequence corresponding to $\boldsymbol{x}_1^N$.

Let the codeword be modulated by binary phase shift keying (BASK) and transmitted over an additive white Gaussian noise (AWGN) channel. The SC decoding calculates the transition probabilities according to $\boldsymbol{y}_1^N$ and $\hat{\boldsymbol{u}}_1^N$, where $\hat{\boldsymbol{u}}_1^N=(\hat{u}_1,\hat{u}_2,\ldots,\hat{u}_N)$ is the estimated sequence of $\boldsymbol{u}_1^N$. Subsequently, the estimated sequence is obtained by judging the transition probabilities in turn. The detailed decision process is as follows.

The logarithm likelihood ratio (*LLR*) for u_i is defined as[11]

$$L_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1})=\ln\frac{w_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1}\mid u_i=0)}{w_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1}\mid u_i=1)}, \tag{1}$$

where $i\in A^{\mathrm{C}}$, u_i is a frozen bit and determined as the predetermined value of the sender, and the value usually is 0. u_i is an information bit if $i\in A$. For $\forall\, i\in A$, if the *LLR* is used to obtain $\hat{\boldsymbol{u}}_1^N$, the decision criterion is as follows

$$\hat{u}_i=\begin{cases}0, & L_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1})\geq 0\\ 1, & L_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1})<0\end{cases}. \tag{2}$$

Consider the SCL decoding with the list size of *L*, $\forall 1<l<L$. The estimated sequence of $\boldsymbol{u}_1^i$ on the *l*-th path is denoted as $\hat{\boldsymbol{u}}_{1,l}^i=(\hat{u}_{1,l},\hat{u}_{2,l},\ldots,\hat{u}_{i,l})$. Let $PM_l^{(i)}$ denote the path metric of u_i on the *l*-th path, and $PM_l^{(i)}$ is initialized to 0. For $\forall i,\ l$, the *LLR* is defined as

$$L_{N,l}^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_{1,l}^{i-1})=\ln\frac{w_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_{1,l}^{i-1}\mid u_i=0)}{w_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_{1,l}^{i-1}\mid u_i=1)}. \tag{3}$$

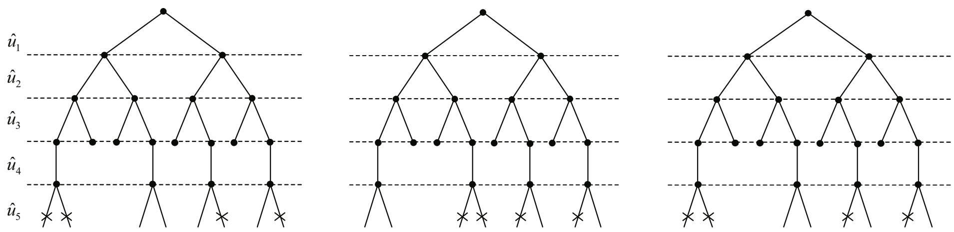
For u_i on the *l*-th path, *PM* is expressed as[12]

$$PM_l^{(i)}=\begin{cases}PM_l^{(i-1)}, \text{if } \hat{u}_{i,l}=\dfrac{1}{2}\left[1-\mathrm{sign}(L_{N,l}^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_{1,l}^{i-1}))\right]\\ PM_l^{(i-1)}+\left|L_{N,l}^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_{1,l}^{i-1})\right|, \quad \text{others}\end{cases}. \tag{4}$$

For $i\in A$, each decoding path for the SCL decoder will be extended to two paths where $\hat{u}_{i,l}=0$ or 1. If there are more than *L* decoding paths, the decoder will only retain the *L* decoding paths with the smallest *PM* value for subsequent decoding. For the CA-SCL decoder, the *L* decoding paths are checked by CRC in ascending order according to the *PM*, and then output the path that passes the check firstly.

While none of the *L* decoding paths for the CA-SCL decoder pass the check, the SCLF decoding algorithm will be adopted to flip the specific information bits, so as to achieve the purpose of error correction. For the SCL decoding and two bit-flip schemes, as revealed in Fig.1, suppose the list size *L*=4, u_1, u_2, u_3 and u_5 are information bits, and u_4 is a frozen bit. Moreover, assume that u_5 is chosen as the flip bit. The symbol "×"in Fig.1 means to delete the path. For the standard SCL decoding scheme, as illustrated in Fig.1(a), since the two extended paths of node A are deleted, this path is in deletion state. The two extended paths of node B are both preserved, so this path is in clone state. Nodes C and D just retain one extended path, which manifests as the SC decoding and means that this path is in the SC state.

Ref.[8] has shown that there are two bit-flipping schemes for the SCLF decoding algorithm as shown in Fig.1(b) and (c). Fig.1(b) illustrates the bit-flipping scheme presented in Ref.[8]. All the nodes in Fig.1(b) have to exchange the decision result of path competition, because the D-POST decoding algorithm in Ref.[8] just considers the belief related to the path metric to determine the flip bits. Fig.1(c) reveals the bit-flip scheme presented in Ref.[7]. Since the CS used for bit-flip in this scheme is constructed according to SC decoding, and the decoding paths corresponding to nodes C and D are in the SC state, the decision results of the path competition for nodes A and B are unchanged, and only the decision results of nodes C and D are exchanged. In this paper, because the bit error probability which is used to construct the AS is also related to the SC decoding, the bit-flip scheme in Fig.1(c) is adopted.



(a) Standard SCL decoding scheme (b) Bit-flipping scheme[8] (c) Bit-flipping scheme[7]

Fig.1 Decoding trees of the SCL and SCLF decoding schemes

Consider that a polar code with code length *N* uses BASK modulation under AWGN channel. The corresponding

LLR of the input sequence $\boldsymbol{u}_1^N$ is $\boldsymbol{L}_1^N=(L_1,L_2,\ldots,L_N)$. If $\boldsymbol{u}_1^N$ is an all-zero codeword, $\boldsymbol{L}_1^N$ follows $N(2/\sigma^2, 4/\sigma^2)$ distribution. It is clear that all *LLRs* are Gaussian random variables with variance twice the mean. Therefore, Eqs.(5)—(7) can be calculated according to the Gaussian approximation[13] as

$$\mathrm{E}[\boldsymbol{L}_N^{(2i-1)}]=\phi^{-1}(1-(1-\phi(\mathrm{E}[\boldsymbol{L}_{N/2}^{(i)}]))^2), \tag{5}$$

$$\mathrm{E}[\boldsymbol{L}_N^{(2i)}]=2\mathrm{E}[\boldsymbol{L}_{N/2}^{(i)}], \tag{6}$$

$$\mathrm{E}[\boldsymbol{L}_1^{(1)}]=2/\sigma^2, \tag{7}$$

where

$$\phi(x)=\begin{cases}1-\dfrac{1}{\sqrt{4\pi x}}\displaystyle\int_{-\infty}^{+\infty}\tanh\frac{\alpha}{2}\mathrm{e}^{-\frac{(\alpha-x)^2}{4x}}\mathrm{d}x, x>0\\ 1, \qquad\qquad\qquad\qquad\qquad x=0\end{cases}. \tag{8}$$

Through the recursive operation of Eqs.(5) and (6), the *LLR* expectation of each polar subchannel $\boldsymbol{W}_N^{(i)}$ is calculated finally. In addition, the greater the *LLR* expectation is, the higher the reliability of corresponding channel $\boldsymbol{W}_N^{(i)}$ becomes.

Ref.[14] has demonstrated that $\boldsymbol{L}_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1})$ is a Gaussian random variable if $\hat{\boldsymbol{u}}_1^{i-1}=\boldsymbol{u}_1^{i-1}=0$ for $\forall i=1,2,\ldots,N$. It indicates that $\boldsymbol{L}_N^{(i)}(\boldsymbol{y}_1^N,\hat{\boldsymbol{u}}_1^{i-1})$ is a Gaussian random variable if the previous bits are decoded correctly. Since the signal-to-noise ratio (*SNR*) of Gaussian random variables is $SNR=\mu^2/2\mu=\mu/2$, the error probability of the i-th bit can be calculated by

$$P_\mathrm{e}(i)=Q(\sqrt{\frac{1}{2}\mathrm{E}[\boldsymbol{L}_N^{(i)}]}), \tag{9}$$

where $Q(x)=\frac{1}{2}\mathrm{erfc}(x/\sqrt{2})$, and $\mathrm{erfc}(x)=(2/\sqrt{\pi})\int_x^{\infty}\mathrm{e}^{-\tau^2}\mathrm{d}\tau$ is the complementary error function.

Firstly, a novel AS is constructed in this paper. The detailed construction process is as follows.

For CA-SCL decoding with the list size of L, the least $\log_2 L$ information bits of A are denoted as $\boldsymbol{v}_1^{\log_2 L}=(v_1,v_2,\ldots,v_{\log_2 L})$. Since the information bits $v_1^{\log_2 L}$ are in clone state under SCL decoding and cannot perform the bit-flip, the AS must remove $v_1^{\log_2 L}$ from A. Then the initial set (IS) can be gotten and denoted as $IS=A/v_1^{\log_2 L}$.

For $i\in IS$, each decoding path $\hat{\boldsymbol{u}}_{1,l}^{i-1}$ will expand into $2L$ paths, and the *PM* values of these paths are denoted as $PM_1^{(i)},PM_2^{(i)},\ldots,PM_{2L}^{(i)}$. Without loss of generality, suppose $PM_1^{(i)},PM_2^{(i)},\ldots,PM_{2L}^{(i)}$ is arranged in ascending order. The probability that $\boldsymbol{u}_{1,l}^{i}$ have no decoding errors can be estimated by $\mathrm{e}^{-PM_l^{(i)}}$ [6]. Since SCL decoding always reserves the L paths with minimum *PM* value, the total probabilities of the L survival paths and the L removed paths are calculated by $\sum_{l=1}^{L}\mathrm{e}^{-PM_l^{(i)}}$ and $\sum_{l=1}^{L}\mathrm{e}^{-PM_{l+L}^{(i)}}$ respectively, then the normalized belief of the CA-SCL decoding can be determined via

$$C_{\mathrm{SCL}}(i)=\log\frac{\sum_{l=1}^{L}\mathrm{e}^{-PM_l^{(i)}}}{\sum_{l=1}^{L}\mathrm{e}^{-PM_{l+L}^{(i)}}}. \tag{10}$$

Similar to Eq.(10), the normalized belief estimated by Gaussian approximation can be expressed as[15]

$$C_{\mathrm{GA}}(i)=\log\frac{1-P_\mathrm{e}(i)}{P_\mathrm{e}(i)}. \tag{11}$$

In practical decoding, when the belief of u_i is lower than the theoretical value, there is a high probability to decode u_i incorrectly, so the flip bit must satisfy

$$C_{\mathrm{SCL}}(i)<C_{\mathrm{GA}}(i),\quad i\in IS. \tag{12}$$

Let the deletion set (DS) represent the set which consists of the bits containing the deletion state. Since $P_\mathrm{e}(i)$ is the error probability of u_i which is obtained by Gaussian approximation under the SC decoder, the DS should be removed from the *IS* to get the set *S* that only consists of the bits containing the SC state. Therefore, the set *S* is denoted as $S=IS\backslash DS$. In order to make the AS contain the bits of the decoding errors as much as possible, the AS can be deduced according to

$$AS=\left\{i\middle|C_{\mathrm{SCL}}(i)<C_{\mathrm{GA}}(i),\quad i\in S\right\}. \tag{13}$$

The D-POST part in Ref.[8] is flipped in ascending order of the belief. Different from the order of bit-flip in Ref.[8], in this paper, the constructed AS can be directly sorted in ascending order according to the *LLR* expectation obtained by the Gaussian approximation.

The AS is used in the improved AS-SCLF decoding algorithm based on the AS in this paper. The detail process of the algorithm is described in Tab.1, where t=1, 2,…, T represents the bit index, and T denotes the maximum allowed flip number, $T\leq|FS|$. The algorithm performs CA-SCL decoding firstly. If the first CA-SCL decoding is successful, the path that passes the CRC check is output directly. Once all the L paths of the first CA-SCL decoding failed in the CRC check, re-decoding will be performed to flip one bit in the AS in order. After the bit-flip, if the decoding result passes the CRC check, the bit-flip ends and the re-decoding result is output, otherwise, the next bit-flip is performed. If there is still no re-decoding result pass the CRC check until the flip number reaches the maximum value, the decoder will output the path with the smallest *PM* for the first CA-SCL decoding.

The simulation is under the AWGN channel and adopts BASK modulation. All simulated polar codes are constructed by Gaussian approximation at SNR=2.5 dB. Polar codes with code length N=1 024, code rate R=0.5, and list size L=8 include the 16-bit CRC with the generator polynomial $g_{16}(x)=x^{16}+x^{15}+x^2+1$. This section compares the accuracy of the AS in this paper with the CS in Ref.[7] firstly, and then analyzes the performances and computational complexities of different SCLF decoding algorithms. The number of simulation iterations for performance and computational complexity is 10^7. The maximum allowed flip number is T=50. The improved AS-SCLF decoding algorithm proposed in this paper is compared with the CA-SCL decoding algorithm in Ref.[6], the CS-SCLF decoding algorithm in Ref.[7] and the D-POST decoding algorithm in Ref.[8], and then the block error rate (*BLER*) and average number of decoding attempts are analyzed for the above algorithms.

Tab.1 Improved AS-SCLF decoding algorithm based on the AS

Input: received sequence y_1^N, assigned set AS;
Output: send sequence $\hat{u}_1^N$;
1: **if** the first CA-SCL decoding succeeds **then**
2: return the path that successfully passes the CRC check and has the smallest path metric
3: **else**
4: **for** $t=1,2,\cdots,T$ **do**
5: flip bit $AS(t)$
6: **if** bit-flip decoding succeeds **then**
7: return the path that successfully passes the CRC check and has the smallest path metric
8: **end if**
9: **end for**
10: **if** $t=T+1$ **then**
11: decoding failure, return the path with smallest *PM* of first CA-SCL decoding
12: **end if**
13: **end if**

The accuracy of the index set can be illustrated as the probability that the first error bit in the L paths is included in the index set after the first CA-SCL decoding fails. The accuracy of the AS and the CS is shown in Tab.2. The number of simulation iterations is 10^5. It can be concluded from Tab.2 that the accuracy of both the AS and CS increases with the increase of the *SNR*. And the accuracy reaches 1 at the *SNR* of 2.5 dB, which proves the validity of the AS and CS. Compared with the CS, the AS constructed in this paper has a higher probability of containing the first error bit under the same conditions, increased by about 3%, which illustrates that the AS is more suitable for the SCLF decoding.

Tab.2 Accuracy of AS and CS

Accuracy	E_b/N_0 (dB)			
	1	1.5	2	2.5
CS	0.89	0.93	0.98	1
AS	0.92	0.98	0.99	1

Fig.2 shows the *BLER* for different decoding algorithms. It can be seen from Fig.2 that the AS-SCLF decoding algorithm proposed in this paper achieves a best performance compared to other algorithms. Compared with the CA-SCL decoding algorithm, the CS-SCLF decoding algorithm and the D-POST decoding algorithm, this algorithm has gains of about 0.29 dB, 0.22 dB and 0.1 dB respectively at the *BLER* of 10^{-4}. Since the AS in this paper is more accurate than the CS in Ref.[7], the error propagation is further reduced to improve performance. Moreover, since the AS tries to contain the bits of decoding errors, the AS-SCLF decoding algorithm outperforms the D-POST decoding algorithm which only flip in ascending order according to the belief. Furthermore, with the increase of the *SNR*, the *BLER* of all the SCLF decoding algorithms gradually decrease according to their own curve trend in Fig.2.

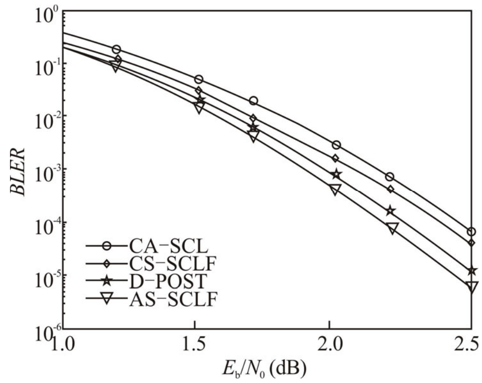


Fig.2 *BLER* for different decoding algorithms

The computational complexity can be described as the average number of decoding attempts. Obviously, the greater number of decoding attempts performs, the higher the computational complexity becomes. Fig.3 shows the average number of decoding attempts of different algorithms. From Fig.3, at the low *SNR* (that is, poor channel conditions), the AS-SCLF decoding algorithm, compared with other algorithms, has the smallest complexity. The average number of decoding attempts is about 26% and 12% lower than that of the CS-SCLF decoding algorithm and D-POST decoding algorithm respectively at the *SNR* of 1 dB. Due to the construction of the AS, the AS-SCLF decoding algorithm reduces the range of the flip index compared with the D-POST decoding algorithm and identifies error bit more accurately than the CS of the CS-SCLF decoding algorithm, thereby certain unnecessary re-decoding is reduced. With the increase of the *SNR*, all of the average number of decoding attempts for the SCLF decoding algorithms gradually decrease and are almost close to the CA-SCL decoding algorithm at the high *SNR*. The above results show that the proposed algorithm is more suitable for the SCLF decoding at the low *SNR*, and the complexity added to the SCL decoder is almost negligible at the high *SNR*.

To sum up, since the improved AS-SCLF decoding algorithm proposed in this paper can correct the error bits more accurately, it not only reduces the computational complexity, but also significantly improves the decoding performance compared with other SCLF decoding algorithms.

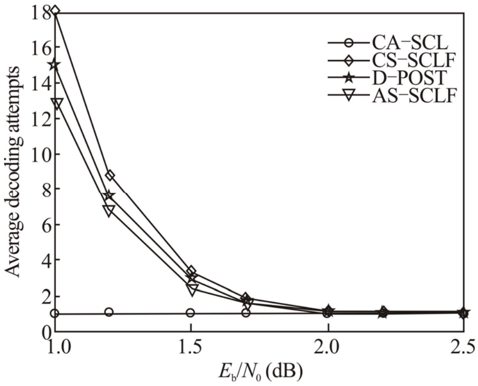


Fig.3 Average number of decoding attempts for different decoding algorithms

The three states of the path extension for the SCL decoder and the two schemes of the bit-flip for the SCLF decoder are analyzed in this paper. And then the Gaussian approximation principle is used to estimate the *LLR* expectation of each polar subchannel and the bit error probability of SC decoder. On this basis, the normalized beliefs of the bits containing the SC state are compared by using the decoding result of the CA-SCL decoder, thereby the bits with decoding errors are identified and sorted in ascending order of the *LLR* expectations to construct a more effective AS. When the first CA-SCL decoding fails, a bit-flip scheme that only exchanges the decision results on the path of the SC state is adopted to flip the bits from the AS in turns. Through the above methods, an improved AS-SCLF decoding algorithm is proposed. The simulation results show that compared with the CA-SCL decoding algorithm, the CS-SCLF decoding algorithm and the D-POST decoding algorithm, the proposed decoding algorithm not only improves the decoding performance but also reduces the computational complexity at the low *SNR*.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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