

# Simulation of false-alarm area of laser guidance based on Mie scattering model\*

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In order to study the influence of backscattering of indicating laser in laser guidance process and laser guidance countermeasure test, the scattering function and volume extinction coefficient of typical aerosol distribution are calculated, and the backscattering detection model of 1.06  $\mu\text{m}$  horizontally transmitted laser is established, based on Mie scattering theory and scattering function optimization algorithm; the model is used to study the change of backscattering energy detected by the detector at different positions and different detection angles, and the false-alarm area of laser guidance along the indicating laser path under different detection thresholds is obtained. The results can help to deepen the understanding of the influence of atmospheric scattering on the laser guidance process, and provide theoretical reference for the scheme design of the laser guidance countermeasure test.

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Laser guidance is an effective way of precision guidance and has been widely used and developed, the typical guidance processes are: the laser beam is directed to irradiate the target, and the laser seeker keeps track of the reflected laser signal of the target, so as to finally hit the target accurately<sup>[1,2]</sup>. In order to improve the guidance accuracy of the guidance system and its adaptability to different environments, for the whole process of laser guidance, researches on the laser reflection characteristics of different target surfaces, the transmission and scattering characteristics of the indicating laser under the atmospheric conditions of raindrops, haze, solid particles, etc., the construction of laser guidance simulation system, the anti-jamming and performance test of the laser seeker have been carried out at home and abroad<sup>[3-7]</sup>.

The above work mainly focuses on the transmission and detection of the indicating laser itself, and the backscattering of the indicating laser has also been studied. The White Sands Missile Range of the United States has established the corresponding test safety rules<sup>[8]</sup>, which are used to avoid the influence of backscattering on the laser guided weapon test; In China, some beneficial theoretical and experimental researches on the backscattering problem in the static erection test and simulation test of laser seeker are also conducted, and the theoretical and experimental results of backscattering detection at a single position have been obtained<sup>[9-12]</sup>. In this paper, aiming at the typical laser guidance process and atmos-

pheric environment, based on the Mie scattering model, the backscattered light detected by laser seeker in different positions in the area near the indicating laser path is simulated and calculated, and the backscattering distribution received by the detection equipment in the area near the indicating beam is obtained. Due to the diversity and complexity of the actual conditions, this work focuses on theoretical research, but the results can provide references for the subsequent experimental studies of backscattering under different conditions, the design of test scheme, the layout of test positions, etc.

The suspended solid and liquid particles in the atmosphere are called atmospheric aerosols. The number density of aerosol particles changes with the altitude and aerosol scale. In this paper, the aerosol distribution model proposed by McClatchey et al is used<sup>[13,14]</sup>. As shown in Eq.(1) and Tab.1, the model gives the aerosol size distribution function with normalization parameters  $c$  and the vertical distribution of the particle density  $N$  under different visibility conditions. After  $N$  is determined, the normalized parameters  $c$  can be calculated by  $N = \int f(r)dr$ , and the distribution functions of aerosol particles spectrum at different altitudes can be obtained as

$$f(r) = \begin{cases} c \times r^{-4} (0.1 \mu\text{m} < r \leq 10 \mu\text{m}) \\ c \times 10^4 (0.02 \mu\text{m} \leq r \leq 0.1 \mu\text{m}) \\ 0 (r < 0.02 \mu\text{m}, r > 10 \mu\text{m}) \end{cases} \quad (1)$$

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**Tab.1 The vertical distribution of the particle density**

Altitude (km)	Particle density $N$ (particles/cm <sup>3</sup> )	
	23 km visibility	5 km visibility
0	2 828	13 780
1	1 224	5 030
2	537.1	1 884
3	225.6	673.1
4	119.2	245.3

For example, when the atmospheric visibility is 5 km, the particle density at the ground is  $N=13\ 780$ , and the normalized results is  $c=12.158\ 8$ . The aerosol size distribution function at the ground can be obtained by Eq.(1).

Under the assumption that the aerosol particles are homogeneous and spherical, the Mie scattering light field of a single particle can be accurately solved by solving Maxwell equation. If the electric field intensity of incident plane wave is  $\mathbf{E}$  and the magnetic field intensity is  $\mathbf{H}$ , then the radiation flux density or average energy flux density of incident light field in the air medium is

$$\mathbf{S}_{av} = \frac{1}{2} \text{Re}[\mathbf{E} \times \mathbf{H}] = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} |\mathbf{E}_0|^2 \mathbf{e}_z, \quad (2)$$

where  $\mathbf{e}_z$  is the unit vector determined by the right-hand spiral rule, that is, the incident light intensity is

$$I_0 = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} |\mathbf{E}_0|^2, \text{ and then the scattered light intensity is}$$

$$I = \frac{I_0 \lambda^2}{4\pi l^2} F(\beta, \varphi) (\theta \in [0, \pi], \varphi \in [0, 2\pi]), \quad (3)$$

where  $l$  is the distance from the scattering center to the observation point,  $\lambda$  is the incident light wavelength,  $\beta$  is the scattering angle,  $\varphi$  is the angle between the incident plane and the scattering plane,  $F(\beta, \varphi)$  is called Mie scattering function. When the incident light is un-polarized, the scattering function can be expressed as<sup>[10,15]</sup>:  $F(\beta, \varphi) = [ |S_1(\beta)|^2 + |S_2(\beta)|^2 ] / 2$ , where  $S_1(\beta)$  and  $S_2(\beta)$  are the scattering amplitude functions, which are independent of  $\varphi$  and can be calculated as

$$S_1(\beta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n), \quad (4)$$

$$S_2(\beta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n), \quad (5)$$

where  $\pi_n$  and  $\tau_n$  are scattering angle functions, and the calculation equations are as follow:

$$\begin{cases} \pi_n = \frac{2n-1}{n-1} \cos \beta \times \pi_{n-1} - \frac{n}{n-1} \pi_{n-2} \\ \tau_n = n \cos \beta \times \pi_n - (n+1) \pi_{n-1} \\ \pi_0 = 0 \\ \pi_1 = 1 \end{cases}, \quad (6)$$

$a_n$  and  $b_n$  are Mie scattering coefficients, and the calculation equations are:

$$a_n = \frac{\phi_n(\alpha) \phi_n'(m\alpha) - m \phi_n'(\alpha) \phi_n(m\alpha)}{\xi_n(\alpha) \phi_n'(m\alpha) - m \xi_n'(\alpha) \phi_n(m\alpha)}, \quad (7)$$

$$b_n = \frac{m \phi_n(\alpha) \phi_n'(m\alpha) - \phi_n'(\alpha) \phi_n(m\alpha)}{m \xi_n(\alpha) \phi_n'(m\alpha) - \xi_n'(\alpha) \phi_n(m\alpha)}, \quad (8)$$

where  $m$  is the refractive index of aerosol particles and  $\alpha=2\pi r/\lambda$  is the dimensionless parameter related to the size of aerosol particles.  $\phi_n$  and  $\xi_n$  are Riccati-Bessel functions, and the solutions require to calculate the first and second kind of semi-odd order Bessel functions directly<sup>[16,17]</sup>. When  $\alpha$  or the imaginary part of  $m$  are relatively large, the high order term of Bessel function will lead to slow calculation or overflow of calculation results. Shen Jianqi decomposes  $a_n$  and  $b_n$  into new recursive terms by deforming the above formula, as shown in Eq.(9), which can effectively avoid the phenomenon that the data caused by the calculation of Bessel function alone is too large and then overflows<sup>[18]</sup>.

$$\begin{cases} a_n = A_n(\alpha) \times T_{a_n}(m, \alpha) \\ b_n = A_n(\alpha) \times T_{b_n}(m, \alpha) \end{cases}, \quad (9)$$

where

$$\begin{cases} T_{a_n}(m, \alpha) = \frac{D_n(m\alpha) / m - D_n(\alpha)}{D_n(m\alpha) / m - B_n(\alpha)} \\ T_{b_n}(m, \alpha) = \frac{m D_n(m\alpha) - D_n(\alpha)}{m D_n(m\alpha) - B_n(\alpha)} \\ A_n(\alpha) = \phi_n(\alpha) / \xi_n(\alpha) \\ B_n(\alpha) = \xi_n'(\alpha) / \xi_n(\alpha) \\ D_n(\alpha) = \phi_n'(\alpha) / \phi_n(\alpha) \end{cases}. \quad (10)$$

The recurrence relations are

$$\begin{cases} A_n(\alpha) = A_{n-1}(\alpha) \frac{B_n(\alpha) + n / \alpha}{D_n(\alpha) + n / \alpha} \\ B_n(\alpha) = -n / \alpha + \frac{1}{n / \alpha - B_{n-1}(\alpha)} \\ D_{n-1}(\alpha) = n / \alpha - \frac{1}{n / \alpha + D_n(\alpha)} \end{cases}. \quad (11)$$

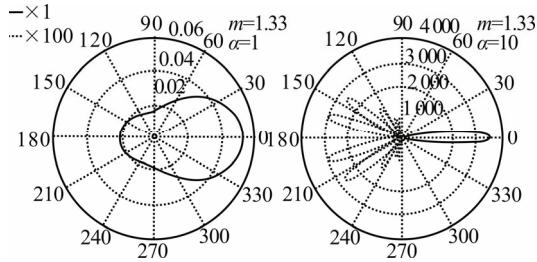
The initial conditions are

$$\begin{cases} A_0(\alpha) = \frac{1}{1 + i \frac{\cos \alpha}{\sin \alpha}} \\ B_0(\alpha) = -i \end{cases}. \quad (12)$$

In order to improve the calculation accuracy, according to Eq.(10), the back recurrence method is used to calculate the value of each  $D_n$ , but the first recurrence term  $D_N$  is calculated by Lentz continued fraction<sup>[19,20]</sup>. The number of cut-off terms  $N$  is twice of the calculation result of empirical formula  $\alpha+7.5\alpha^{1/3}+2$ <sup>[18]</sup>, and the maximum number of different cut-off terms corresponding to different  $\alpha$  is taken to ensure the accuracy of calculation.

When  $m=1.33$ , the angle distribution diagram of Mie scattering function under different scale parameter  $\alpha$  is drawn. It can be seen from Fig.1 that when  $\alpha$  is small, the

forward scattering and backward scattering are almost the same. With the increase of  $\alpha$ , the scattered light gradually concentrates in the front. By observing the image of scattering intensity magnified by 100 times as shown in the dotted line within the range of  $90^\circ$ — $270^\circ$  in the right figure of Fig.1, it can be seen that when  $\alpha$  is large, the back scattering intensity fluctuates greatly with the scattering angle.



**Fig.1 Angle distribution of Mie scattering function under different  $\alpha$**

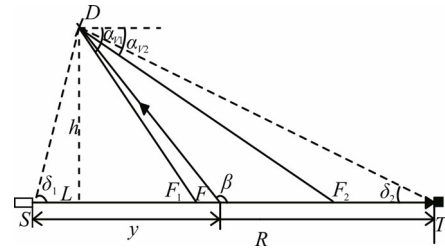
The calculation accuracy of Mie scattering function can be verified by calculating extinction efficiency factor  $Q_{ext}$  and scattering efficiency factor  $Q_{sca}$ <sup>[15]</sup>. The calculation results of the particles with different scale parameters and refractive indices are shown in Tab.2, which are consistent with those calculated by MIECPP method<sup>[21]</sup> in different refractive index and large scale range. The accuracy of the results is mainly related to recurrence method, calculation accuracy setting, selection of cut-off terms, etc.

**Tab.2 Results of  $Q_{ext}$  and  $Q_{sca}$ <sup>[21]</sup>**

$m$	$\alpha$	$Q_{ext}$	$Q_{sca}$
0.75	0.099	$7.41786 \times 10^{-6}$	$7.41786 \times 10^{-6}$
$1.33 \times 10^{-5}i$	10 000	2.004 09	1.723 86
1.5-i	0.055	0.101 491	$1.13169 \times 10^{-5}$
1.5-i	100	2.097 50	1.283 70
10-10i	10 000	2.005 91	1.795 39

In order to establish the detection model of the backscattering of the indicating laser, it is advisable to set the indicating laser to transmit horizontally at the ground level. The detector is located near the indicating beam and can receive the backscattering of the indicating beam. The spatial position relationship of the detection model is shown in Fig.2, indicating laser is emitted from the laser source  $S$  and transmitted along the horizontal light path, the diameter of the beam at the exit is  $D_0$ , the energy of the single pulse is  $Q_0$ , the pulse width is  $t_0$ , and the beam divergence angle is  $\theta$ ; the horizontal distance between the target  $T$  and  $S$  is  $R$ , the off-axis distance of the detector  $D$  is  $h$ , the horizontal distance of the detector  $D$  is  $L$ , and the angles between  $DS$ ,  $DT$  and optical path are  $\delta_1$  and  $\delta_2$ , respectively; the field angle of the detector is  $V_\phi$ ,

the angle between the lower edge of the field of view and the horizontal direction is  $\alpha_{V1}$  and it intersects with the optical axis at  $F_1$ , the angle between the upper edge of the field of view and the horizontal direction is  $\alpha_{V2}$  and it intersects with the optical axis at  $F_2$ ; the distance between  $S$  and the scattering point  $F$  in the field of view is  $y$  and the scattering angle is  $\beta$ , the angle between the scattered light and the optical axis of the detection field of view is  $\gamma$  (not shown in Fig.2).



**Fig.2 Sketch map of backscattering detection of indicating laser**

According to Eq.(3) and the attenuation law of laser propagation in the atmosphere<sup>[11,13]</sup>, the backscattered laser energy density of aerosol particles in volume element  $S_y dy$  at position  $F$  detected by detector  $D$  is as follows:

$$dW_s(y) = \frac{Q_0}{(D_0 + \theta y)^2 / 4} \exp(-\tau(y + \sqrt{h^2 + (y-L)^2})) \times \int \frac{F(\beta, \alpha) \cos \gamma f(r) dr}{k^2 (h^2 + (y-L)^2)} \pi \frac{(D_0 + \theta y)^2}{4} dy \quad (13)$$

After simplification, we get:

$$dW_s(y) = \frac{Q_0 \exp(-\tau(y + \sqrt{h^2 + (y-L)^2}))}{\pi (h^2 + (y-L)^2)^2} \int \frac{F(\beta, \alpha) \cos \gamma}{k^2} f(r) dr dy \quad (14)$$

where  $k=2\pi/\lambda$ ,  $\tau$  is the extinction coefficient. When only considering the extinction caused by Mie scattering,  $\tau$  is equal to Mie scattering extinction coefficient  $\tau_{ext}$ . According to  $\tau_{ext} = \pi \int_{r_1}^{r_2} r^2 f(r) Q_{ext} dr$ ,  $\tau_{ext}$ <sup>[15,20]</sup> can be obtained, and the unit of  $\tau_{ext}$  can be converted into  $\text{km}^{-1}$ . For  $1.06 \mu\text{m}$  laser, under the condition of  $m=1.33$  and  $V=5 \text{ km}$ ,  $\tau_{ext}=0.281 \text{ km}^{-1}$  can be calculated. At this time, according to formula  $I/I_0=\exp(-\tau_{ext}L)$ , the laser energy at the position 5km away from  $S$  decreases by 75.5%.

Without considering the influence of the diffuse reflection light, by integrating the scattering volume elements in the field of view along the optical path  $ST$ , the total energy density of scattered light detected by the detector is  $W_s = \int_{y_1}^{y_2} dW_s(y)$ ,  $y_1$  and  $y_2$  are the distances

from  $S$  to  $F_1$  and  $F_2$  respectively. Polar coordinate system is established in  $DST$  plane, with detector  $D$  as the pole, horizontal direction as the polar axis and clockwise direction as positive. Suppose that the deflection angle

between the optical axis direction of the detector field of view and the horizontal direction is  $\phi_0$ , then the deflection angle between the lower edge  $DF_1$  of the field of view and the horizontal direction is  $\alpha_{V1}=\phi_0+V_\phi/2$ , the deflection angle between the upper edge  $DF_2$  of the field of view and the horizontal direction is  $\alpha_{V2}=\phi_0-V_\phi/2$ , the deflection angle between the laser source  $S$  and the detector  $D$  is  $\delta_S=\pi-\delta_1=\pi-\text{actg}(h/L)$ , and the deflection angle between the target  $T$  and the detector  $D$  is:

$$\delta_T = \delta_2 = \text{arctg}\left(\frac{h}{R-L}\right). \quad (15)$$

For different situations,  $y_1$  and  $y_2$  have different values. When  $\delta_S > \alpha_{V1} > \delta_T > \alpha_{V2}$ , there are:

$$\begin{cases} y_1 = \text{ctg}\alpha_{V1} \times h + L \\ y_2 = R \end{cases}. \quad (16)$$

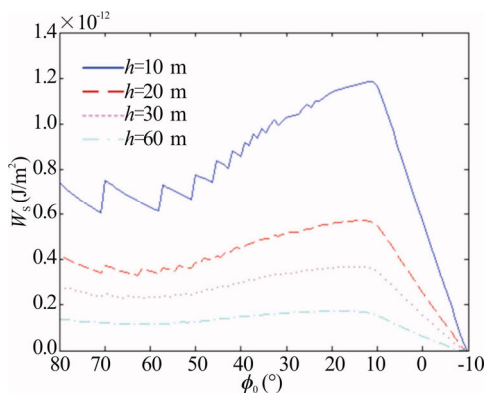
When  $\delta_S > \alpha_{V1} > \alpha_{V2} > \delta_T$ , there are:

$$\begin{cases} y_1 = \text{ctg}\alpha_{V1} \times h + L \\ y_2 = \text{ctg}\alpha_{V2} \times h + L \end{cases}. \quad (17)$$

In the above cases, the scattered light from  $y_1$  to  $y_2$  enters the detector's field of view. When the scattering signal strength  $W_S$  detected by the detector exceeds the detector's detection threshold, the scattering signal can cause false alarm interference to the detector.

According to the calculation process of Mie scattering model and backscattering detection model, the specific parameters are set as follow: laser wavelength  $\lambda=1.06\mu\text{m}$ , spot diameter at the light outlet  $D_0=1\text{ cm}$ , beam divergence angle  $\theta=1\text{ mrad}$ , visibility  $V=5\text{ km}$ , target distance  $R=9\text{ km}$ , detector field of view  $V_\phi=20^\circ$ , aerosol particle refractive index  $m=1.33$ , and the aerosol particle size distribution at the ground is represented by Eq.(1).

By changing the deflection angle  $\phi_0$ , when  $L=0\text{ km}$ , at different  $h$ , the curves of scattering intensity  $W_S$  with  $\phi_0$  show a similar trend. As shown in Fig.3,  $W_S$  first fluctuates and increases, then decreases rapidly, and there is an approximate peak position on each curve.

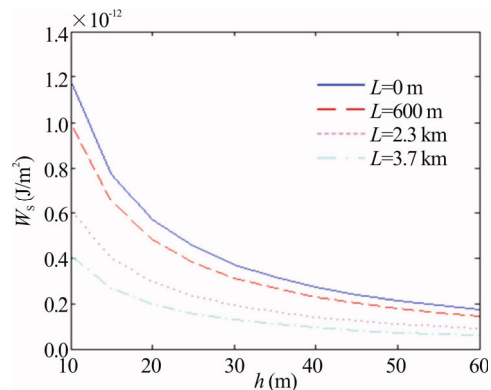


**Fig.3 Energy density detected as a function of detection angle ( $L=0\text{ km}$ )**

The factors leading to the change of  $W_S$  are the change of scattering angle, the change of the length of the scattering beam in the detection field of view and the attenuation of laser energy: when the  $\phi_0$  decreases from

$80^\circ$ , the length of the scattering beam in the field of view increases, the laser energy decays, and the scattering coefficient fluctuates with the scattering angle, the path increment plays an important role in the change of  $W_S$ , and the detection energy curve shows a fluctuating and rising trend; when the  $\phi_0$  is reduced to  $10^\circ$ , that is, the target  $T$  is in the field of view, with the decrease of  $\phi_0$ , the length of the scattering beam in the field of view is shortened, at the same time, the laser energy attenuation increases, and the detected scattering energy drops rapidly, affected by these factors, the detection energy curve reaches the peak value near  $10^\circ$  after the fluctuation and rise; when the detection angle is in the range of  $30^\circ-80^\circ$ , the detection energy curve will fluctuate, and when the detection angle is less than  $10^\circ$ , the detection energy curve will decline smoothly, this change trend of detection energy curve is mainly affected by the change of backscattering with angle, that is, the total backscattering of particle ensemble has different change trends in different angle ranges, and when the scattering angle is close to the incident optical axis, the total backscattering keeps a smooth trend with the change of scattering angle.

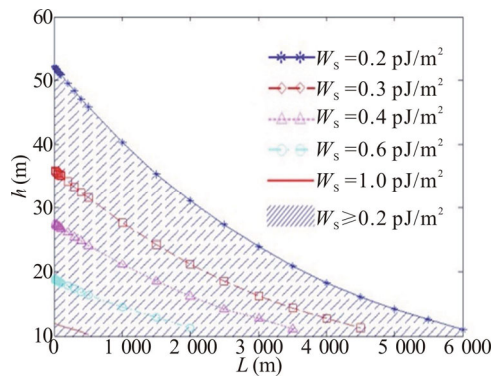
It can be seen from Fig.3 that the detected scattering energy fluctuates greatly with the change of detection angle  $\phi_0$ , and when  $\phi_0$  is different there is no uniform trend of the detected scattering energy with the change of  $h$ . However, when observing the change of maximum detection energy with  $h$ , a series of curves of the maximum detected energy density varying with  $h$  are obtained as shown in Fig.4. It can be found that the detection energy peak decreases smoothly with the increase of  $h$ , and when  $L$  is different, the different curves are approximately distributed. With the increase of  $L$ , the maximum detection energy density corresponding to  $h$  decreases, and the curve also tends to be flat.



**Fig.4 Energy density detected as a function of detection height**

By observing the change of maximum detection energy with  $h$  and  $L$ , the spatial distribution curves of detection energy density in the area near the indicating laser path are obtained as shown in Fig.5.





**Fig.5 The spatial distribution curves of detection energy density and section of the false-alarm area**

The spatial distribution curves of detection energy density show similar changes, and the detection energy decreases smoothly with the increase of off-axis distance  $h$  and horizontal distance  $L$ ; taking the spatial distribution curves as the generatrices, the spatial distribution surfaces of the detection energy density can be formed, and the surfaces can form approximate cone areas, when the threshold value of detector is lower than the detection energy corresponding to a certain surface, the detector in the corresponding cone area can be interfered by the back scattering light of the indicating beam, the area is the false alarm area of detector; when the threshold value of the detector decreases, the boundary of the false alarm area expands outwards, that is, the false alarm area gradually expands with the decrease of the threshold value of the detector.

The above results are calculated under the condition that each model parameter is fixed. Because the laser guidance process or laser countermeasure test is affected by aerosol distribution, target direction, target distance, indicating laser parameter, detector field of view, detector sensitivity and other factors, the next step, different conditions will be set to study the influence of the above factors on the laser guidance false alarm area, and experimental verification will also be carried out.

The Mie scattering function and the atmospheric extinction coefficient of single Mie scattering are calculated based on the Mie scattering model of laser atmospheric transmission. On this basis, a detection model of backscattering of indicating laser is established, the variation of detection energy with detection direction and detector position is analyzed and studied, and the spatial distribution of the false alarm area near the light path is obtained. The laser seeker and other detection equipment in the area are vulnerable to the interference of backscattering, the area distribution is affected by the aerosol distribution, target distance, indicating laser parameters, detector field of view and detector sensitivity. The research results are helpful to deepen the understanding of the influence of atmospheric scattering on the laser guidance process, and provide a theoretical reference for

the scheme design of the laser guidance countermeasure test.

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