An integrated PHY-MAC analytical model for IEEE 802.15.7 VLC network with MPR capability[*]

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Considering that the collision caused by hidden terminal is particularly serious due to the narrow beams of optical devices, the multi-packet reception (MPR) is introduced to mitigate the collisions for IEEE 802.15.7 visible light communication (VLC) system. To explore the impact of MPR on system performance and investigate the interaction between physical (PHY) layer and media access control (MAC) layer, a three dimensional (3D) integrated PHY-MAC analytical model of carrier sense multiple access/collision avoidance (CSMA/CA) is established based on Markov chain theory for VLC system, in which MPR is implemented through the use of orthogonal code sequence. Throughput is derived to evaluate the performance of VLC system with MPR capability under imperfect optical channel. The results can be used for the performance optimization of a VLC system with MPR capability.

Visible light communication (VLC), as a favorable complementary technology for radio frequency (RF), has been a research hotspot in recent years. There is an increasing number of achievements in modulation[1,2] and channel modeling[3,4], while the results relevant to random access are rare. IEEE 802.15.7[5] is introduced to standardize the VLC system for connecting devices via visible light, in which a slotted carrier sense multiple access/collision avoidance (CSMA/CA) mechanism is given to coordinate channel access. The hidden terminal problem[6] is especially serious due to the narrow beams of optical devices, which will inevitably increase collision and decrease throughput in VLC system. It is necessary and urgent to optimize the performance of CSMA/CA in VLC network.

Nowadays, the research of CSMA/CA mostly focuses on the performance analysis and the optimization of 802.11 and 802.15.4 networks. Many researches have been carried out in the aspects of modeling, access control, backoff algorithms, parameter optimization, etc[7-10]. Based on multi-parameter differentiation, a mechanism to support priority MAC for IEEE 802.15.7 was discussed[11]. The performance of VLC system was analyzed by implementing a simulator with considering the physical (PHY) and media access control (MAC) layers[12]. However, all the studies mentioned above allow only one transmission, which is not applicable to the serious collisions in VLC system. The multi-packet reception

(MPR)[13] exactly provides a new perspective to solve this problem by allowing multiple packets transmitted in PHY layer simultaneously. So in this paper, a PHY-MAC federated model of CSMA/CA is built to analyze the impact of MPR capability on the performance of 802.15.7 VLC system which operates on imperfect optical channel. With the characteristics of both PHY layer and MAC layer taken into consideration, throughput is derived to investigate the role of MPR from a system point of view. This novel insight may reveal new potentials for VLC performance optimization.

We consider a VLC system operating in a one-hop star topology. To mitigate the collisions caused by hidden terminals, MPR is introduced in coordinator. The coordinator is capable of decoding the receiving packet which is coded with a selected orthogonal code sequence when it is transmitted. Suppose that there are N devices contending for one of the M available code sequences based on slotted CSMA/CA, where M is referred to the MPR capability. A packet is assumed to be transmitted successfully only when the selected code sequence is not used by other devices. We abstract the above process as a schematic diagram shown in Fig.1.

In slotted CSMA/CA of IEEE 802.15.7, when a device has packets to transmit, it first initializes three variables, which are the number of backoff stage (NB) as 0, the retransmission time (RT) as 0 and the backoff exponent (BE) as *macMinBE*. Then the device delays for a random

number of backoff periods selected in the range [0, $2^{BE}-1$]. At the expiration of delay, the clear channel assessment (CCA) is performed to check channel state (busy or idle). If the channel is idle, the device randomly selects a code sequence (from M available code sequences) to code the sending packet, and then starts packet transmission. Otherwise, NB and BE are increased by one up to *macMaxCSMABackoffs* and *macMaxBE*, respectively. If NB exceeds the maximum, packet is discarded due to the channel access failure. If the transmission encounters a collision or experiences error, the above trial is repeated until the transmission succeeds subject to a limit of *macMaxFrameRetries*. If RT exceeds its retry limit, packet is discarded.
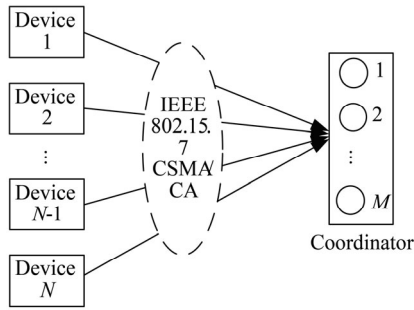
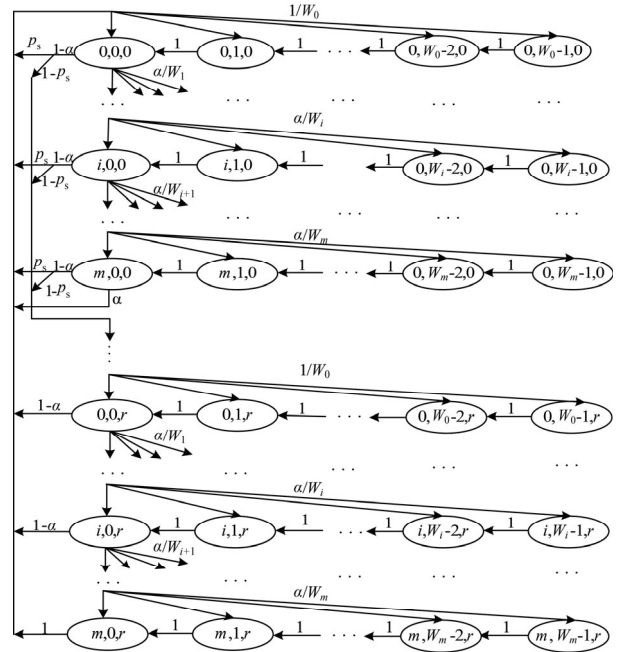**Fig.1 Schematic diagram for VLC system with MPR capability**

In order to analyze the performance of VLC system with MPR capability, an integrated PHY-MAC analytical model is established based on Markov chain theory, as shown in Fig.2.

A three dimensional (3D) Markov chain $(s(t), c(t), b(t))$ is adopted to describe the process of CSMA/CA, which takes

Fig.2 Markov chain model of IEEE 802.15.7 CSMA/CA in VLC system with MPR capability

MPR capability and transmission errors into consideration. Let $s(t)$, $c(t)$ and $b(t)$ be the stochastic processes representing the backoff stage, then the backoff counter value and the retransmission counter value experienced by a node at time t. The duration of the backoff window W_i is

$$W_i = \begin{cases} 2^i W_0, & i \le n \\ 2^n W_0, & i > n \end{cases}, \tag{1}$$

where $W_0 = 2^{macMinBE}$ and $n = macMaxBE - macMinBE + 1$. According to the procedure of CSMA/CA, we obtain transition probabilities of Markov chain as

$$\begin{cases} P\{i+1, j, k | i, 0, k\} = \alpha / W_{i+1}, & i \in [0, m-1], j \in [0, W_{i+1}-1], k \in [0, r] \\ P\{0, j, k+1 | i, 0, k\} = (1-\alpha)(1-P_s)/W_0, & i \in [0, m-1], j \in [0, W_0-1], k \in [0, r-1] \\ P\{0, j, 0 | i, 0, k\} = (1-\alpha)P_s / W_0, & i \in [0, m-1], j \in [0, W_0-1], k \in [0, r-1] \\ P\{i, j, k | i, j+1, k\} = 1, & i \in [0, m], j \in [0, W_i-2], k \in [0, r] \\ P\{0, j, 0 | i, 0, r\} = (1-\alpha)/W_0, & i \in [0, m-1], j \in [0, W_0-1] \\ P\{0, j, 0 | m, 0, k\} = \alpha / W_0, & j \in [0, W_0-1], k \in [0, r-1] \\ P\{0, j, 0 | m, 0, r\} = 1/W_0, & j \in [0, W_0-1] \end{cases} \tag{2}$$

Let $b_{i,j,k} = \lim_{t \to \infty} P\{s(t) = i, c(t) = j, b(t) = k\}$ denote the stationary distribution of the Markov chain in steady state, where $i \in (0, m), j \in (0, W_i-1), k \in (0, r)$. Owing to chain regularities and Eq.(2), we obtain

$$\begin{cases} b_{0,0,k} = \alpha^i b_{0,0,k} & 0 \le i \le m, 0 \le k \le r \\ b_{i,0,k} = \beta^k b_{i,0,0} & 0 \le i \le m, 0 \le k \le r \\ \quad \vdots \qquad\qquad \vdots \\ b_{i,j,k} = \dfrac{W_i - j}{W_i} b_{i,0,k} & 0 \le i \le m, 0 \le j \le W_i - 1 \end{cases}, \tag{3}$$

where α is the probability when channel is busy at CCA, and $\beta = (1-\alpha)(1-p_s)\sum_{i=0}^{m} \alpha^i = (1-p_s)(1-\alpha^{m+1})$. The channel is busy when at least one of the remaining $N-1$ nodes transmits simultaneously with the current transmitting node, thus α can be expressed as $\alpha = 1-(1-\tau)^{N-1}$, where τ is transmission probability.

Based on the normalization condition and transition probabilities of Markov chain, $b_{0,0,0}$ can be obtained as

$$b_{0,0,0} = \begin{cases} \dfrac{2(1-\beta)(1-2\alpha)(1-\alpha)}{\left(1-\beta^{r+1}\right)\left(W_0\left(1-(2\alpha)^{m+1}\right)(1-\alpha)+(1-2\alpha)\left(1-\alpha^{m+1}\right)\right)} & m \le n \\[4mm] \dfrac{2(1-\beta)(1-2\alpha)(1-\alpha)}{\left(1-\beta^{r+1}\right)\left(W_0\left(1-(2\alpha)^{n+1}\right)(1-\alpha)+(1-2\alpha)\left(1-\alpha^{m+1}\right)+2^n W_0 \alpha^{n+1}(1-2\alpha)\left(1-\alpha^{m-n}\right)\right)} & m > n \end{cases} . \tag{4}$$

Then we can express the probability that a device attempts CCA as

$$\omega = \sum_{i=0}^{m}\sum_{k=0}^{r} b_{i,0,k} = \frac{1-\alpha^{m+1}}{1-\alpha}\frac{1-\beta^{r+1}}{1-\beta} b_{0,0,0} . \tag{5}$$

Thus the probability that a device starts to transmit is given as

$$\tau = (1-\alpha)\sum_{i=0}^{m}\sum_{k=0}^{r} b_{i,0,k} = (1-\alpha^{m+1})\frac{1-\beta^{r+1}}{1-\beta} b_{0,0,0} . \tag{6}$$

For simplicity, $\Omega(b,N,a) = \binom{N}{b}(a)^b(1-a)^{N-b}$ is used to represent the binomial probability distribution function of random variable b with parameters N and a. A ready terminal is referred to be the one ready to transmit its packet in the next slot with probability of one, while the other terminals make transmissions with probability of τ. Transmission is successful only when there is neither collision nor transmission error. Then the success probability is

$$p_s = \begin{cases} (1-p_e)(1-\tau)^{N-1} & M=1 \\ (1-p_e)\displaystyle\sum_{k=0}^{N-1}\Omega(k,N-1,\tau)\left(1-\dfrac{1}{M}\right)^k & M \ge 2 \end{cases}, \tag{7}$$

where $p_e = 1-(1-BER)^{T_L}$ is packet error rate, and T_L is the packet payload size. The bit error rate (BER) of on-off keying (OOK) modulation scheme under line of sight (LOS) indoor wireless optical channel is given by[14]

$$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right), \tag{8}$$

where E_b is the average energy per bit, and N_0 is the power spectrum density of white Gaussian noise. $Q(\cdot)$ is the error function (erf), which is defined as

$$Q(x) = \frac{1}{\sqrt{2x}}\left[\frac{1}{2}-\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t^2}dt\right]. \tag{9}$$

We assume that the devices always have packets available for transmission, i.e., operate in saturation condition. Let q_{tr} be the probability that there is at least one transmission in the considered time slot, which can be expressed as

$$q_{tr} = \sum_{n=1}^{N}\Omega(n,N,\tau) = 1-(1-\tau)^N . \tag{10}$$

With the condition of the fact that there is at least one

transmission, Let q_{sk} be the probability that k concurrent packets are transmitted successfully, and it can be expressed as

$$q_{sk} = \left(\sum_{n=1}^{N}\Omega(n,N,\tau)\Omega(k,n,p_s)\right)/q_{tr} . \tag{11}$$

Thus the throughput S can be expressed as

$$S = \frac{\displaystyle\sum_{k=1}^{M} k q_{sk} q_{tr} E[T_L]}{(1-q_{tr})\sigma + q_{tr}q_s T_s + q_{tr}(1-q_s)T_c} , \tag{12}$$

where T_L, T_s and T_c are the time in unit of backoff slots to transmit packet payload, successful transmission and collision, respectively. σ is the backoff slot size, and $q_s = \displaystyle\sum_{k=1}^{M} q_{sk}$. In particular, when $M=1$, the throughput reduces to the one without MPR capability. T_s and T_c are expressed as

$$\begin{cases} T_s = T_{s,L} + T_{CCA} + T_{ex} \\ T_c = T_{c,L} + T_{CCA} + T_{ex} \end{cases}, \tag{13}$$

where T_{CCA} is the duration for performing a CCA. Assume $T_{s,L} = T_{c,L} = T_L$, where $T_{s,L}$, $T_{c,L}$ and T_{ex} are the durations corresponding to three states of transmitting packet successfully, transmitting packet unsuccessfully and keeping waiting for extra slots, respectively. Neglecting transmitting and waiting time of acknowledgement (ACK), T_s is the same as T_c. T_{ex} is assumed to be two backoff slots after a transmission regardless of success or failure.

Algorithm simulations are accomplished to validate our analysis. We analyze the system throughput when the number of code sequences M and the number of devices N change. We consider a VLC system with OOK, and the data rate is 11.67 kbit/s. E_b/N_0 is 8 dB, and data is 30 bit. If not specified, the system parameters are shown in Tab.1.

Tab.1 Simulation parameters

Parameter	Value
macMinBE	3
macMaxBE	5
macMaxFrameRetries	3
macMaxCSMABackoffs	4
Optical clock rate	200 kHz
Backoff period	20 optical clocks

Fig.3 shows the system throughput versus the number of devices N. From Fig.3, we can see the throughput increases firstly and then decreases as N increases. Moreover, the bigger the N is, the clearer the downward trend. A large number of devices leads to serious collisions due to the narrow beams of optical devices, which results in performance degradation. The use of MPR reduces the collision and increases the number of simultaneous transmissions in some degree. Thus in order to further analyze the impact of MPR capability on mitigating the hidden terminal in VLC system, we observe the throughput with different MPR capabilities, i.e., M ranges from 1 to 8. The six curves in Fig.4 have the same changing trend. The throughput increases approximately linearly with M increasing. A bigger M means more available code sequences, which contributes to solving the hidden terminal problem brought by narrow beams of optical devices. The simulation results have significant potential value on performance improvement in VLC system by configuring M appropriately.
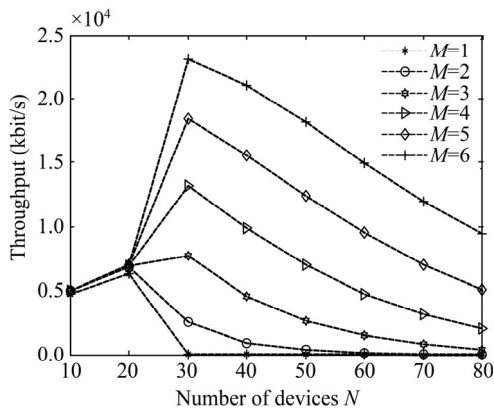

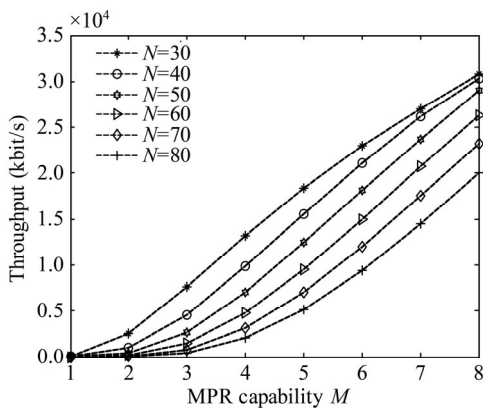**Fig.3 Throughput versus number of devices**

Fig.4 Throughput versus MPR capability

In summary, in this paper, the slotted CSMA/CA mechanism is analyzed by using MPR. Specially, a 3D Markov chain is used to model the access process of CSMA/CA in IEEE 802.15.7 VLC network, which takes both PHY layer and MAC layer into consideration. Then taking MPR capability and transmission errors into consideration, the performance of the VLC system is analyzed from a system perspective point. Furthermore, we discuss the impact of MPR capability in terms of throughput. Simulation results indicate that a reasonable set of MPR capability can effectively improve the system throughput. Our modeling and analysis make foundation for theoretical performance evaluation of 802.15.7 VLC network with MPR. Future work can focus on the optimization of VLC network with MPR capability through the cooperation between PHY layer and MAC layer.

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