Modulation instability in negative refractive materials with saturable nonlinearity*

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Starting directly from the nonlinear propagation equation including saturable nonlinearity, the first- and the second-order nonlinear dispersions, the dispersion relation, instable condition, gain spectra, and the dimensionless cut-off frequency and gain spectra of modulation instability (MI) in the negative refractive material are deduced by adopting the linear stability analysis and Drude electromagnetic model. And the variations of the dimensionless gain spectra with the normalized angular frequency and normalized incident power are calculated and discussed for different sign relations between the linear dispersion and the third-order nonlinear coefficients. The results show that in the negative refractive index region, MI can occur irrespective of the sign relation between the linear dispersion and the third-order nonlinear coefficients. And depending on different dimensionless angular frequencies and different sign relations, the variations of the dimensionless gain spectra with incident power take on several different forms. Namely, the peak gain and the cut-off frequency of MI may increase then decrease with the increase of the incident power, or decrease monotonously. Moreover, MI may even have a threshold incident power for some cases.
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Modulation instability (MI) is an important content of nonlinear physics, and can be observed in many fields, such as plasmas, fluid mechanics, Bose-Einstein condensation and optics, etc. So far, MI has been extensively investigated theoretically, experimentally and numerically in conventional optical materials. The involved effects have developed from the low-order nonlinear and dispersion effects[1] to high-order dispersion[2,3] and various high-order nonlinear effects, such as saturable nonlinearity[2,4], quintic nonlinearity[3,5] and self-steepening effect, etc. And the optical materials also have developed from the conventional fiber to fiber grating[6], fiber lasers[7], and so on.

The occurrence of MI in new circumstances means new methods to generate and control optical solitons. Thus, the studies on MI have been extended to artificial synthetic materials, such as photonic crystals and negative-refractive materials in recent years. At present, the researches on negative refractive materials have developed from linear to nonlinear characteristics, including the propagation properties of ultra-short optical pulses[8-10], harmonic generation, MI[11-14], etc. For example, Kourakis et al[11] investigated the nonlinear instability in negative refractive materials, and obtained the modulation stability picture of the coupled plane wave solution. Wen et al[12-14] investigated effects on MI of self-steepening effect, second-order nonlinear dispersion and fake quintic nonlinearity. Afterwards, Kourakis et al[15] investigated the MI in saturable nonlinear negative refractive materials. Recently, starting from the normalized propagation equation, Wen et al deduced the expressions for the dispersion relation, gain spectra and the peak gain of MI in negative refractive materials and calculated the effects of the saturable nonlinearity, the first-and the second-order nonlinear dispersions on MI[16]. In this paper, starting directly from the nonlinear propagation equation including saturable nonlinearity, the first- and the second-order nonlinear dispersions, dispersion relation, instable condition, gain spectra, and the dimensionless cut-off frequency and gain spectra of MI in the negative refractive material are all deduced by adopting the linear stability analysis and Drude electromagnetic model. Moreover, regarding the characteristic that the first- and the second-order nonlinear dispersion coefficients are related to the normalized angular frequency, the varia-

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tions of the dimensionless gain spectra with the normalized angular frequency and incident optical power are calculated and discussed in detail for different sign relations between the linear dispersion and the third-order nonlinear coefficients.

Taking the saturable nonlinearity, the first- and the second-order nonlinear dispersion effects into account, the nonlinear propagation equation of the optical pulse in the negative refractive material can be expressed as[16]

$$\frac{\partial A}{\partial z}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 A}{\partial T^2}=\mathrm{i}\gamma_0\left\{\frac{|A|^2}{1+\Gamma|A|^2}A+\mathrm{i}S_1\frac{\partial}{\partial T}\left[\frac{|A|^2}{1+\Gamma|A|^2}A\right]-S_2\frac{\partial^2}{\partial T^2}\left[\frac{|A|^2}{1+\Gamma|A|^2}A\right]\right\}, \quad (1)$$

where β_2, γ_0, S_1, S_2, $\Gamma=1/P_{\mathrm{sat}}$, P_{sat}, A, T and z are the second-order group velocity dispersion coefficient, the third-order nonlinear coefficient, the first-order nonlinear dispersion coefficient, the second-order nonlinear dispersion coefficient, the saturable parameter, the saturable optical power, the slow varying amplitude of the optical field, the time in pulse coordinate and the propagation distance, respectively. The following lossless Drude electromagnetic model[17] is adopted

$$\varepsilon(\omega)=\varepsilon_0(1-\omega_{\mathrm{pe}}^2/\omega^2),\mu(\omega)=\mu_0(1-\omega_{\mathrm{pm}}^2/\omega^2), \quad (2)$$

where ε_0 and μ_0 are the vacuum permittivity and magnetic permeability, and ω_{pe} and ω_{pm} are the electric and magnetic plasmas frequencies, respectively. According to Eq.(2) and the definitions of the equation coefficients in Eq.(1), the refractive index $n(\omega_0)$ at the carrier frequency ω_0 and the other dimensionless equation coefficients can be deduced as follows

$$n(\omega_0)=\sqrt{1-\omega_{\mathrm{pe}}^2/\omega_0^2}\sqrt{1-\omega_{\mathrm{pm}}^2/\omega_0^2}, \quad (3)$$

$$\beta_2'=\beta_2 c/T_0=\frac{s}{n}\frac{\omega_{pe}}{\omega_0}\times\left[\left(1+3\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2/\omega_0^4\right)-\frac{1}{n^2}\left(1-\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2/\omega_0^4\right)^2\right], \quad (4)$$

$$\gamma_0'=\gamma_0 cT_0/\chi^{(3)}=\frac{1}{2ns}\frac{\omega_0}{\omega_{\mathrm{pe}}}\left(1-\omega_{\mathrm{pm}}^2/\omega_0^2\right), \quad (5)$$

$$s_1=\frac{S_1}{T_0}=s\frac{\omega_{\mathrm{pe}}}{\omega_0}\left(\frac{\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2-\omega_0^4}{n^2\omega_0^4}-\frac{2\omega_0^2}{\omega_{\mathrm{pm}}^2-\omega_0^2}\right), \quad (6)$$

$$s_2=\frac{S_2}{T_0^2}=s^2\frac{\omega_{\mathrm{pe}}^2}{\omega_0^2}\times\left[\frac{\omega_0^2}{\omega_0^2-\omega_{\mathrm{pm}}^2}-\frac{1}{4n^2}\left(1+\frac{3\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2}{\omega_0^4}\right)+\frac{1}{4n^4}\left(1-\frac{\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2}{\omega_0^4}\right)^2\right], \quad (7)$$

where T_0 is the pulse width, $s=1/(\omega_{\mathrm{pe}}T_0)$ is the dimensionless parameter, c is the light speed in vacuum, and $\chi^{(3)}$ is the third-order electric susceptibility. $\chi^{(3)}>0$ and $\chi^{(3)}<0$ stand for the self-focusing and self-defocusing nonlinearity, respectively. The variations of coefficients above with the normalized dimensionless angular frequency $\omega_0/\omega_{\mathrm{pe}}$ are shown in Fig.1. The parameters are set as s=0.25 and $\omega_{\mathrm{pm}}/\omega_{\mathrm{pe}}$=0.8 during calculations. It can be seen from Fig.1 that the frequency region of $0<\omega_0/\omega_{\mathrm{pe}}<0.8$ should be the negative refractive region. In this region, the parameters β_2', s_1 and s_2 can take positive, 0 or negative value, while the frequency region of $\omega_0/\omega_{\mathrm{pe}}>1$ should be the positive refractive region, and the frequency region of $0.8<\omega_0/\omega_{\mathrm{pe}}<1$ should be the forbidden band region. In addition, the dimensionless parameters are related to the normalized angular frequency $\omega_0/\omega_{\mathrm{pe}}$, and can not vary independently.

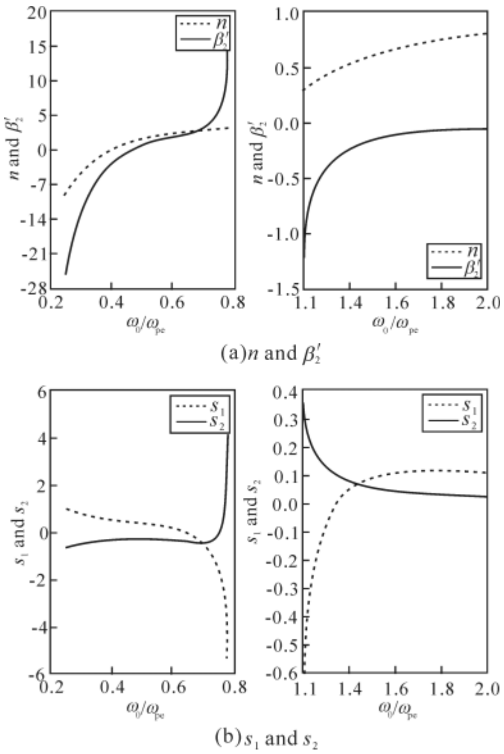

Fig.1 Variations of the refractive index and dimensionless equation coefficients with the normalized angular frequency ω_0/ω_{pe}

Perturbing the stable solution of Eq.(1) by inserting the perturbation $a(z, T)$ with the angular frequency Ω and the wave vector $\boldsymbol{K}$, and adopting the standard linear stability analysis, the following linearized nonlinear propagation equation is obtained

$$\frac{\partial a}{\partial z}=\mathrm{i}e_1(a+a^*)-e_2\frac{\partial a}{\partial T}-e_3\frac{\partial a^*}{\partial T}-\mathrm{i}e_4\frac{\partial^2 a}{\partial T^2}-\mathrm{i}e_5\frac{\partial^2 a^*}{\partial T^2}, \quad (8)$$

where the parameters e_j (j = 1, 2, 3, 4, 5) are defined as $e_1=\gamma_0 P_0/x^2$, $e_2=\gamma_0 S_1 P_0\left(1/x+1/x^2\right)$, $e_3=\gamma_0 S_1 P_0/x^2$, $e_4=0.5\beta_2+\gamma_0 S_2 P_0\left(1/x+1/x^2\right)$, and $e_5=\gamma_0 S_2 P_0/x^2$, where $x=1+\Gamma P_0$, and P_0 is the incident power. According to the processions and steps of many references, the following dispersion relation of MI is deduced as

$$K=e_2\,\Omega\pm\mathrm{i}\,\Omega\sqrt{\left[2e_1\left(e_5-e_4\right)-e_3^2\right]-\left(e_4^2-e_5^2\right)\Omega^2}\ . \quad (9)$$

When the inner part of the squared root takes the positive value, K is complex, and then the MI occurs. And the corresponding instable condition, the cut-off frequency, and the power gain should be of the following forms

$$\left[2e_1\left(e_5-e_4\right)-e_3^2\right]-\left(e_4^2-e_5^2\right)\Omega^2>0\ , \quad (10)$$

$$\Omega_c^2=\left|\left[2e_1\left(e_5-e_4\right)-e_3^2\right]/\left(e_4^2-e_5^2\right)\right|\ , \quad (11)$$

$$g(\Omega)=2\,\mathrm{Im}(K)=2\left|\Omega\right|\times\sqrt{\left[2e_1\left(e_5-e_4\right)-e_3^2\right]-\left(e_4^2-e_5^2\right)\Omega^2}\ . \quad (12)$$

For convenient calculation, the perturbation angular frequency, the cut-off angular frequency and the power gain can be transformed into the dimensionless forms as

$$\Omega_c^2T_0^2=B/C,\ g\,L_D=\left|\Omega T_0\right|\sqrt{B-C\Omega^2T_0^2}\ , \quad (13)$$

where $L_D=T_0^2/\left|\beta_2\right|$ is the dispersion length, and the parameters B and C are defined as

$$B=-\frac{4s_1^2N^2}{\left(1+\Gamma P_0\right)^4}-\frac{8s_2N^2}{\left(1+\Gamma P_0\right)^3}-\frac{4N\,\mathrm{sgn}\left(\gamma_0\beta_2\right)}{\left(1+\Gamma P_0\right)^2}\ , \quad (14)$$

$$C=\frac{8s_2^2N^2}{\left(1+\Gamma P_0\right)^3}+\frac{4s_2^2N^2}{\left(1+\Gamma P_0\right)^2}+\frac{4s_2N\,\mathrm{sgn}\left(\gamma_0\beta_2\right)}{\left(1+\Gamma P_0\right)^2}+\frac{4s_2N\,\mathrm{sgn}\left(\gamma_0\beta_2\right)}{1+\Gamma P_0}+1\ , \quad (15)$$

where $N=L_D/L_{NL}$ is the soliton order, $L_{NL}=1/\left|\gamma_0P_0\right|$ is the nonlinear length, and sgn stands for the signal function.

As analyzed above, the parameters s_1 and s_2 are related to the normalized angular frequency ω_0/ω_{pe}, and can not vary independently. Thus, the variations of dimensionless gain spectra with the normalized angular frequency and the incident optical power are calculated and discussed in detail for different dispersion regions as shown in Figs.2 and 3. The common parameters are set as s = 0.25, ω_{pm}/ω_{pe} = 0.8, and N=1.

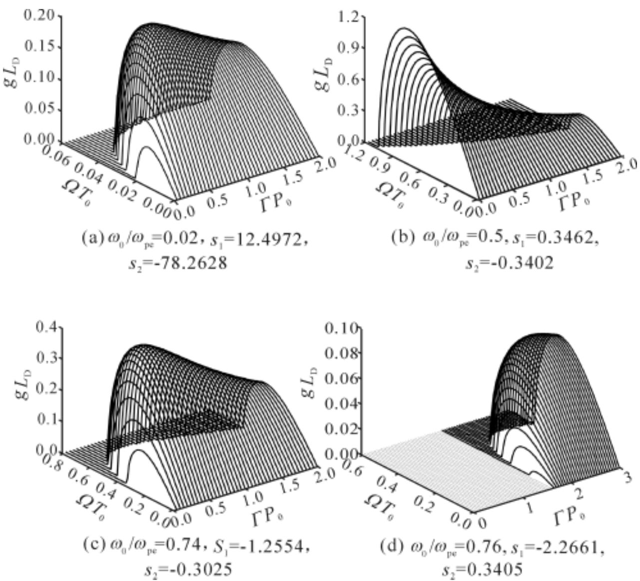

Fig.2 Variations of dimensionless gain spectra with the normalized incident power ΓP_0 for different ω_0/ω_{pe} in the negative refractive index region when sgn($\gamma_0\,\beta_2$) < 0

It can be seen from Fig.2 that in the negative refractive region and sgn($\gamma_0\,\beta_2$) < 0, the peak gain and the width of the gain spectrum increase then decrease with the increase of ΓP_0 for small ω_0/ω_{pe}. And the peak gain and the width of the gain spectrum increase with ω_0/ω_{pe} increasing. If the value of ω_0/ω_{pe} continues to increase, the peak gain and the width of the gain spectrum decrease with the increase of ΓP_0 monotonously. And the peak gain and the width of the gain spectrum decrease with ω_0/ω_{pe} increasing. When the value of ω_0/ω_{pe} continues to increase and approaches the forbidden band region, the peak gain and the width of the gain spectrum increase then decrease with the increase of ΓP_0 again. When ω_0/ω_{pe} increases to a certain large value, MI takes on the threshold behavior. That is to say, MI can only occur when the value of ΓP_0 is larger than a certain critical value. Moreover, the closer to the forbidden band region the value of ω_0/ω_{pe} is, the larger the critical value is. In addition, MI can occur irrespective of the sign relation between the parameters s_1 and s_2.

According to Fig.3, it can be seen that in the negative refractive region and sgn ($\gamma_0\,\beta_2$) > 0, the peak and the width of the gain spectrum increase then decrease with the increase of ΓP_0. But they decrease with the increase of ω_0/ω_{pe}. When the value of ω_0/ω_{pe} increases to a certain large value, MI also takes on threshold behavior. And the larger the value of ω_0/ω_{pe}, the larger the critical value of ΓP_0. Furthermore, MI can only occur within a small region of ΓP_0 for large ω_0/ω_{pe}. MI disappears for a certain larger value of ω_0/ω_{pe}. In comparison of Fig.2, MI here can only occur when ω_0/ω_{pe} is small.

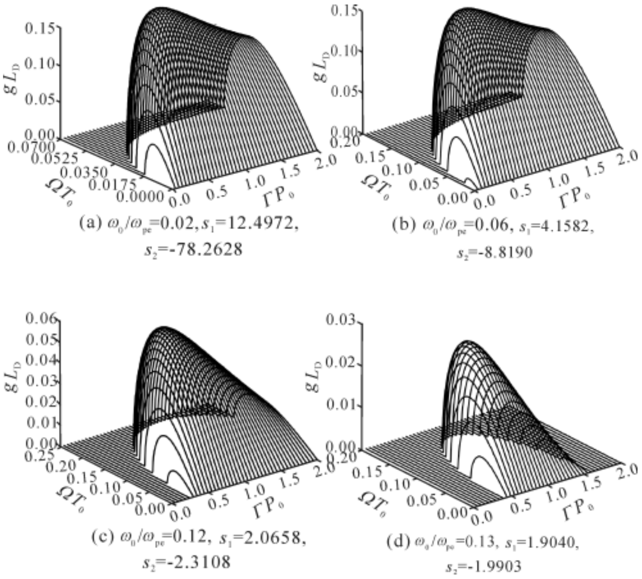

Fig.3 Variations of dimensionless gain spectra with the normalized incident power ΓP_0 for different ω_0/ω_{pe} in the negative refractive index region when sgn($\gamma_0\beta_2$) > 0

In summary, MI is analyzed and calculated in the negative refractive materials with saturable nonlinearity, the first- and the second-order nonlinear dispersion effects in this paper. The results show that in the negative refractive index region, MI can occur irrespective of the sign relation between the linear dispersion and the third-order nonlinear coefficients. And depending on different dimensionless angular frequencies and different sign relations, the variations of the dimensionless gain spectra with incident power take on several different forms. Namely, the peak gain and the cut-off frequency of MI may increase then decrease with the increase of the incident power, or decrease monotonously. Moreover, MI may even have a threshold incident power for some cases. Furthermore, depending on different normalized angular frequencies and different sign relations between the linear dispersion and the third-order nonlinear coefficient, the threshold incident power is also different. In comparison, when the linear dispersion and the third-order nonlinear coefficient are common in sign, MI can only occur when the normalized angular frequency is small.

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