Tunneling properties of electromagnetic wave in slab superconducting material

Khem B. Thapa[1]*, Sanjay Srivastava[2], Alka Vishwakarma[1], and S. P. Ojha[3]

1. Department of Physics, Institute of Engineering & Technology, Chhatrapati Shahu ji Maharaj University, Kanpur 208024, India

2. Material Sciences & Metallurgy Engineering, Maulana Azad National Institute of Technology, Bhopal 462051, India

3. Department of Applied Physics, Institute of Technology, Banaras Hindu University, Varanasi 221005, India

(Received 13 April 2011)

When the electromagnetic wave propagates through a slab superconducting material in microwave ranges, tunneling properties of the electromagnetic wave at critical temperature are investigated theoretically. The transmittance and the reflectance of the slab superconducting material vary with the thickness of material as well as the refractive index of substrates. The high transmittance is found for thin superconductor at low wavelength region. However, optical properties are strongly dependent upon temperature and incidence wavelength. The electromagnetic wave is totally transmitted without loss for incidence wavelength (λ = 5000 nm) due to the zero refractive index and infinite penetration depth of the superconductor at the critical temperature.

Document code: A **Article ID:** 1673-1905(2011)04-0277-5

DOI 10.1007/s11801-011-1043-6

The studies of one dimensional photonic crystals (1D-PCs) including metallic, chiral, negative refractive index (NI) and superconducting materials, etc. have attracted much attention[1-6]. The propagation of electromagnetic waves in periodic media including metal, NI and superconducting materials has been a subject of interest because of its several useful applications in making optical devices like reflectors, super lens, etc[7-10]. One-dimensional metallic photonic crystals (1D-MPCs) have been shown to have high transmittance within a certain controllable spectral range and an enhanced nonlinear optical response in ID-MPC has also been reported[11,12]. The band structure for dielectric–dielectric photonic crystals (DDPCs) displays that the photonic band gap (PBG) between the first and second bands widens as the difference in dielectric permittivity is increased[13,14].

Recently, Thapa et al[15] has studied the enlargement of photonic band gap in heterostructure of metallic photonic and superconducting photonic crystals. They have shown that the heterostructure of the metallic-dielectric and superconductor-dielectric can have omni-directional behavior. The photonic crystals composed of superconductor and metallic layers have also been studied by Aly et al[16-18]. The electromagnetic wave at THz in the one-dimensional superconducting metallo-dielectric superlattice is allowed to propagate through the structure when its frequency is higher than the cutoff frequency. However, it will be totally reflected at a frequency below the cutoff frequency. This cutoff frequency is strongly dependent on the thicknesses of the superconductor and dielectric layers, and the temperature as well. Raymond Ooi and Kam[19] have studied the transient pulse propagation in 1D-PC. The two new effects are identified: double reflection and slow-light ringing of transmitted and reflected pulses. The angle and thickness dependent photonic band structures in a one-dimensional superconducting photonic crystal have been studied by Wu et al[20-22] based on the framework of the Bloch theorem together with the transfer matrix method in a multilayer structure. Additionally, the effects coming from the oblique incident angle for both TE and TM modes also have been numerically elucidated by them. They have also analyzed the thickness-dependent optical properties in a one-dimensional superconducting photonic crystal consisting of alternating superconductor and dielectric layers by using the transfer matrix method in a stratified structure. Clark et al[23] have studied the superconducting gap and determined the direction of nodes in the gap of the organic superconductors in nanoscale. It has been shown that ultrathin layers of metal can display superconductivity, but any limit on the size of superconducting systems remains a mystery.

* E-mail: khem.phy@gmail.com

In this paper, we study the optical properties of a slab superconductor layer versus thickness, wavelength and temperature. Nb (T_c = 9.2 K, λ_0 = 83.4 nm) is taken as a slab superconductor. The tunneling properties of electromagnetic wave in the slab superconducting material in microwave range are investigated by using the translation matrix method. The transmittance and the reflectance are calculated with varying the thickness of the slab superconductor and the incidence wavelength. The superconducting material is strongly dependent upon the temperature and wavelength. So we calculate the refractive index, transmittance and reflectance of the slab superconductor versus temperature. Besides, we also study the transmittance and the reflectance of the superconducting materials with different substrates.

Firstly, we study the index of refraction of the superconductor based on the conventional two-fluid model[12,15-17]. The two-fluid model is used to describe the electromagnetic characteristics of superconductor at non-zero temperature. Normally, the conductivity in the superconductor is complex and is given as

$$\sigma(\omega)=\frac{-\mathrm{i}e^2 n_{\mathrm{sup}}}{m\omega} \ , \tag{1}$$

which is satisfied only when the conduction meets $n_n\tau < \mathrm{i}n_{\mathrm{sup}}/\omega$. $\sigma(\omega)$ is the conductivity of the superconductor, e and m are the charge and mass of electron, respectively, n_{sup} is the density of electron in superconductor state and ω is the frequency of external electromagnetic wave. Eq.(1) can be expressed in term of the London-penetration depth λ_L

$$\lambda_L^2=\frac{m}{\mu_0 n_{\mathrm{sup}} e^2} \ . \tag{2}$$

So the conductivity of the superconductor can be written as

$$\sigma(\omega)\approx-\frac{\mathrm{i}}{\mu_0\omega\lambda_L^2} \ . \tag{3}$$

From the results in Refs.[15-17], $\frac{n_{\mathrm{sup}}}{n_n}=\left(\frac{T_c}{T}\right)^4-1$, and the temperature dependent London-penetration depth is given as

$$\lambda_L(T)=\frac{\lambda_L(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}} \ . \tag{4}$$

Now the conductivity equation for slab superconductor becomes temperature dependent.

The propagation of electromagnetic wave in the superconductor layer is considered to be with no external source current and no charge, and then the Maxwell's equation becomes

$$\nabla\cdot\boldsymbol{E}=0 \ , \ \nabla\cdot\boldsymbol{B}=0, \ \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t} \ \text{and}$$

$$\nabla\times\boldsymbol{B}=\mu_0\left[\sigma\boldsymbol{E}+\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}\right] \ . \tag{5}$$

By taking the curl on both sides of Eq.(5) and using the convention plane wave $\mathrm{e}^{-\mathrm{i}(\omega t+\boldsymbol{k}.\boldsymbol{r})}$, we have

$$\nabla\times\nabla\times\boldsymbol{B}=\left[\frac{\omega^2}{c^2}-\mathrm{i}\omega\mu_0\sigma(\omega)\right]\boldsymbol{B} \ \text{or} \ \nabla^2\boldsymbol{B}+k_{\mathrm{sup}}^2\boldsymbol{B}=0 \ . \tag{6}$$

By substituting $\sigma(\omega)$ from Eq.(3) into Eq.(6), we obtain k_{sup}^2 which is given as

$$k_{\mathrm{sup}}^2=\left(\frac{\omega^2}{c^2}-\frac{1}{\lambda_L^2}\right) \ . \tag{7}$$

Then from the Snell's law, the length of the tangential wave vector component $k_{\mathrm{sup}x}$ is conserved. So $k_{\mathrm{sup}y}=(\omega/c)\sin\theta=\beta$, where θ is incident angle of the electromagnetic wave in vacuum. Then, we have the frequency dependent normal vector component

$$k_{\mathrm{sup}x}=\sqrt{\frac{\omega^2}{c^2}\cos^2\theta-\frac{1}{\lambda_L^2}}=\frac{\omega}{c}n_{\mathrm{sup}}(\omega) \quad , \tag{8}$$

where the refractive index of the superconducting material is given as

$$n_{\mathrm{sup}}(\omega)=\sqrt{\cos^2\theta-\frac{c^2}{\omega^2}\frac{1}{\lambda_L^2}} \ . \tag{9}$$

To calculate optical properties of the slab superconductor, we consider a superconducting material (n_{sup}) sandwiched between two semi-infinite dielectric media. The refractive index distribution is given as below in Fig.1:

$$n(x)=\begin{cases} n_0, & x<0 \quad \text{(Air)} \\ n_{\mathrm{sup}}, & 0<x<d \quad \text{(Superconductor)} \\ n_s, & d<x \quad \text{(Substrate)} \end{cases} \ . \tag{10}$$

The electromagnetic wave is incident in the x-y plane, so the plane wave solution of wave equation of the electric field is defined as

$$E(x,t)=E(x)\exp[\mathrm{i}(\omega t-\beta y)] \ , \tag{11}$$

where β is the y-component of the propagation wave vector. By considering the wave incident from the left side, the electric field for the three regions (Fig.1) is given as

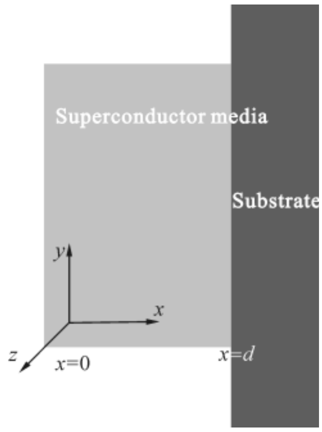

Fig.1 Thin single superconductor sandwiched between two media with dielectric constants n_0=1.0 (air) and n_s (substrate), respectively

$$E(x)=\begin{cases} A\,e^{-ik_{0x}x}+B\,e^{ik_{0x}x}, & x<0 \\ C\,e^{-ik_{\sup x}x}+D\,e^{ik_{\sup x}x}, & 0<x<d \\ F\,e^{-ik_{sx}(x-d)}, & d<x \end{cases}\quad, \tag{12}$$

where *A*, *B*, *C*, *D* and *F* are constants and k_{0x}, $k_{\sup x}$ and k_{sx} are the *x*-components of wave vector in n_0, $n_{\sup}$ and n_s, respectively. The wave vector k_{ix} is defined by $k_{ix}=[(\frac{n_i\omega}{c})^2-\beta^2]^{\frac{1}{2}}=\frac{\omega}{c}n_i\cos\theta_i$, where *i*=0, sup, s and θ_i is the ray angle measured from *x*-axis. The magnetic field of the corresponding electric field is defined by

$$\boldsymbol{H}=\frac{i}{\omega\mu}\nabla\times\boldsymbol{E}\ . \tag{13}$$

Now by applying boundary conditions on the tangential component of electric fields (TE-mode) and the tangential component of magnetic fields (TM-mode), there is no coupling between these components in the whole medium. The boundary condition at $x = 0$ and $x = d$ is applied in Eqs.(12,13), and we can obtain the transmission coefficient:

$$t=F/A=$$

$$\frac{4k_{0x}k_{\sup x}\,e^{-ik_{\sup x}d}}{(k_{1x}+k_{\sup x})(k_{\sup x}+k_{sx})\left[1+\frac{(k_{0x}-k_{\sup x})(k_{\sup x}-k_{sx})\,e^{-2ik_{\sup x}d}}{(k_{0x}+k_{\sup x})(k_{\sup x}+k_{sx})}\right]},$$

$$\text{or}\quad t=\frac{t_{12}t_{23}\,e^{-ik_{\sup x}d}}{[1+r_{12}r_{23}\,e^{-2ik_{\sup x}d}]}\ . \tag{14}$$

Similarly, the reflection coefficient is

$$r=B/A=$$

$$\frac{[(k_{\sup x}-k_{0x})(k_{\sup x}+k_{sx})+(k_{0x}+k_{\sup x})(k_{sx}-k_{\sup x})\,e^{-2ik_{\sup x}d}]}{[(k_{0x}-k_{\sup x})(k_{sx}-k_{\sup x})\,e^{-2ik_{\sup x}d}-(k_{0x}+k_{\sup x})(k_{\sup x}+k_{sx})]},$$

$$\text{or}\quad r=\frac{r_{12}+r_{23}e^{-2ik_{\sup}d}}{1+r_{12}r_{23}e^{-2ik_{\sup}d}}\ . \tag{15}$$

Secondly, we discuss numerical analysis of the slab superconductor. The two fluid models are considered for calculating the refractive index of the superconducting material. As known the penetration depth of the superconducting material is highly sensitive to the temperature and the applied external magnetic field. But in our calculations, we consider the two fluid models to obtain the penetration depth and also describes the electromagnetic response of a typical superconductor without an external magnetic field. According to two fluid models, the electron in the superconducting material either occupies normal state or superconducting state. The London-penetration depth increases rapidly and approaches infinity as the temperature is either close to the critical temperature (T_c) or above 0.95 T_c. In order to investigate the possible correlations between the penetration depth and temperature, we carry out several analyses for controlling the reflectance and the transmittance of the superconducting material with respect to substrate, thickness of material and wavelength. The expressions for the reflectance and the transmittance are given in Eqs.(14,15). When the thickness of the slab superconducting layer is equal to or less than the relevant skin depth of the superconducting layer, the electromagnetic wave will propagate in the material.

Fig.2 shows the variations of the refractive index and penetration depth of the slab superconducting material with temperature. At the critical temperature, the penetration depth tends to infinity, and the real and imaginary parts of the refractive index are zero. The real part of the refractive index is linearly increased above the critical temperature and it

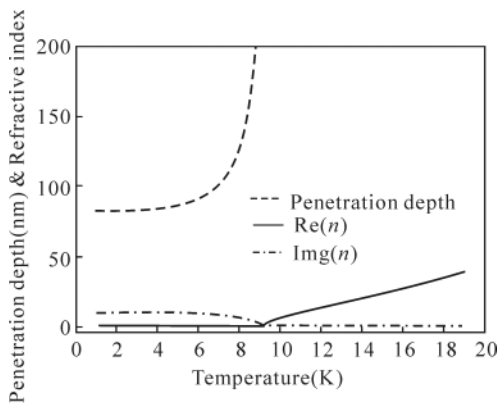

Fig.2 Penetration depth and refractive index of slab superconductor versus temperature

becomes zero below the critical temperature. Below the critical temperature, the imaginary part of the refractive index is slightly decreased with increasing temperature. However, it is zero at the critical temperature as well as above the critical temperature.

Fig.3 shows the variations of the reflectance and transmittance of the superconducting material versus the thickness (*d*/wavelength) with the refractive index of substrate n=1.0 . The reflectance and transmittance versus thickness of the slab superconductor on the substrate with the refractive index n= 3.162 are represented in Fig.4. Figs.5 and 6 show the variations of reflectance and transmittance of the superconductor versus wavelength with different substrates of n=1.0 and n= 3.162. It is observed that the transmittance and the reflectance are approximately 50% at the wavelength of 1900 nm for all substrates which we have considered for our calculations. The high transmittance of the slab superconductor is obtained in the lower range of the incidence wavelength. It reveals that the slab superconductor has low transmittance for higher wavelength and partially high transmittance for lower wavelength. We can say that the transmittance of the slab superconductor may be enhanced when a thin superconductor interacts with a low wavelength source on air substrate. Such material for a device is not available in laboratory.

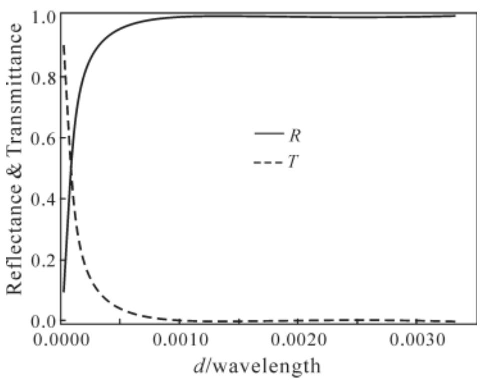

Fig.3 Transmittance and reflectance of slab superconductor versus thickness with *n*=1.0 for substrate

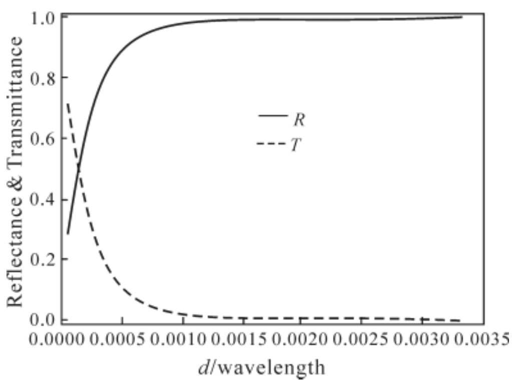

Fig.4 Transmittance and reflectance of slab superconductor versus thickness with *n*=3.162 for substrate

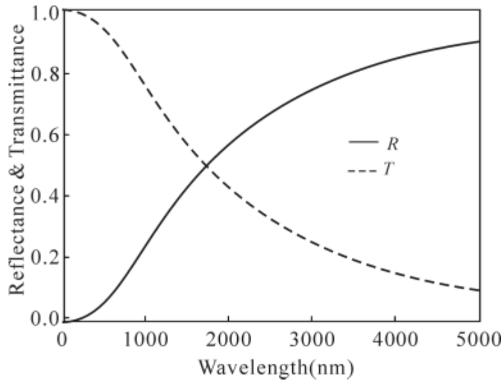

Fig.5 Transmittance and reflectance of slab superconductor versus wavelength with *n*=1.0 for substrate

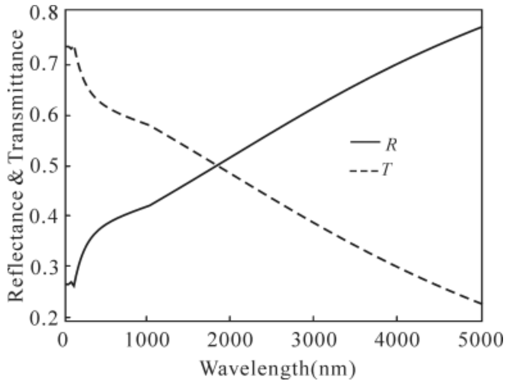

Fig.6 Transmittance and reflectance of slab superconductor versus wavelength with *n*=3.162 for substrate

Figs.7 and 8 show the variations of reflectance and transmittance with temperature on both substrates of n =1.0 and n =3.162. It is shown that the refractive index of the slab superconductor is zero below T_c and positive above T_c. So the transmission is reduced and the absorption is enhanced. The electromagnetic wave with substrate n=1.00 is more transparent than that with substrate n=3.162.

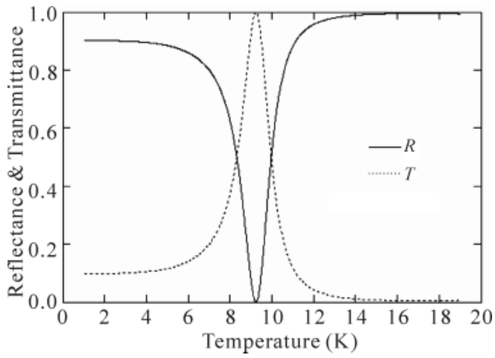

Fig.7 Transmittance and reflectance of slab superconductor versus temperature with *n*=1.0 for substrate

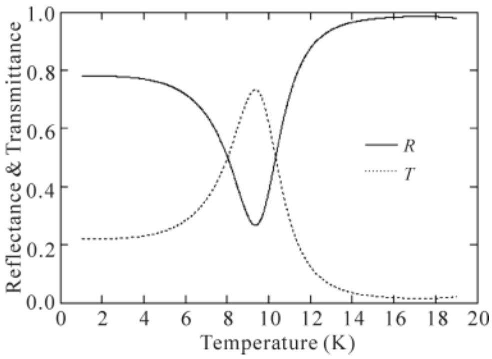

Fig.8 Transmittance and reflectance of slab superconductor versus temperature with n=3.162 for substrate

In this paper, we have theoretically calculated the tunneling properties of the electromagnetic wave at the critical temperature when the electromagnetic wave propagates through a slab superconducting material in microwave ranges. The transmittance and the reflectance of the slab superconducting material vary with thickness of material as well as the refractive index of substrate. The high transmittance is found for thin superconductor in lower wavelength region. It reveals that the slab superconductor has low transmittance for higher wavelength and partially high transmittance for lower wavelength. The transmittance of the slab superconductor may be enhanced when a thin superconductor interacts with a low wavelength source on air substrate. Such material for a device is not available in laboratory. However, the optical properties are strongly dependent upon temperature and incidence wavelength. The electromagnetic wave is totally transmitted for incidence wavelength λ=5000 nm due to the zero refractive index and the infinite penetration depth of the superconductor at the transition temperature. The supporting substrate with the refractive index n=3.162 has less transparent property than that with the refractive index n=1.0 for the electromagnetic wave propagating, which supports the metallic characteristic of the superconducting slab film.

The first author is thankful to Dr. Ajay Tiwari (post doctoral fellow), Department of Electrical and Electronic Engineering, Toyohashi University of Technology, Tempakucho, Hibariga-oka-1-1, Toyohashi 441-8580, Japan, and to Dr. Satendra Pal Singh, IT-BHU, Varanasi for providing technical help to complete this manuscript.

References

[1] P. Filloux and N. Paraire, J. Opt. A: Pure Appl. Opt. **4**, S175 (2002).

[2] A. Polman and P. Wiltzius, Materials Science Aspects of Photonic Crystals, MRS Bulletin, 2001.

[3] M. Bertolotti, J. Opt. A: Pure Appl. Opt. **8**, S9 (2006).

[4] S. A. Dyakov, V. A. Tolmachev, E. V. Astrova, S. G. Tikhodeev, V. Yu Timoshenko and T. S. Perova, International Proc. of SPIE **7521**, 75210G (2010).

[5] K. B. Thapa, Optical Properties of Photonic Crystals, Ph. D. thesis, IT-BHU, Varanasi, 2006.

[6] N. Richard, Eur. Phys. J. B **17**, 11 (2000).

[7] Y. Chen, Z. Huang, Q. Wang, C. Li and J. Shi, J. Opt. A: Pure Appl. Opt. **7**, 519 (2005).

[8] C. Cheng, C. Xu, T. Zhou, X. Zhang and Y. Xu, J. Phys.: Condens. Mat. **20**, 275203 (2008).

[9] V. Shklover, L. Braginsky, G. Witz, M. Mishrikey and C. Hafner, J. Comp and Theo. Nanoscience **5**, 862 (2008).

[10] I. L. Lyubchanskii, N. N. Dadoenkova, A. E. Zabolotin, Y. P. Lee and Th. Rasing, J. Opt. A: Pure Appl. Opt. **11**, 114014 (2009).

[11] S. Chen, Y. Wang, D. Yao and Z. Song, Optica Applicata **39**, 473 (2009).

[12] A. Alejo-Molina, J. J. Sa´nchez-Mondrago´n, D. A. May-Arrioja, D. Romero, J. Escobedo-Alatorre and A. Zamudio-Lara, J. Microelectronics **40**, 459 (2009).

[13] F. Szmulowicz, Phys. Lett. A **345**, 469 (2005).

[14] J. Lekner, J. Opt. Soc. Am. A **11**, 2892 (1994).

[15] K. B. Thapa, Sanjay Srivastava and Sarika Tiwari, J. Supercond. Nov. Magn. **23**, 4517 (2010).

[16] A. H. Aly, J. Supercond. Nov. Magn. **21**, 421 (2008).

[17] A. H. Aly, S. Ryu, H. Hsu and C.-J Wu, Materials Chemistry and Physics **113**, 382 (2009).

[18] A. H. Aly, Materials Chemistry and Physics **15**, 391 (2009).

[19] C. H. Raymond Ooi and C. H. Kam, J. Opt. Soc. Am. B **27**, 458 (2010).

[20] C-J. Wu, Z-H. Wang and T-J. Yang, J. Supercond. Nov. Magn. **23**, 1395 (2010).

[21] C-J.Wu and C. L. Liu and W. K. Kuo, J. Electromagnetic Waves and Applications **23**, 1113 (2009).

[22] C-J. Wu, C-L. Liu and T-J. Yang, J. Opt. Soc. Am. B **26**, 2089 (2009).

[23] K. Clark, A. Hassanien, S. Khan, K.-F. Braun, H. Tanaka and S-W. Hla, Nature Nanotechnology **5**, 261 (2010).