Controlled teleportation of an unknown 3D two-particle state via 3D partially entangled states*

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A scheme is presented to realize the controlled teleportation of an unknown three dimensional (3D) two-particle state by using a non-maximally entangled two-particle state and a non-maximally entangled three-particle state in the 3D space as the quantum channels, and one of the particles in the channels is used as the controlled particle. Analysis shows that when the quantum channels are of maximal entanglement, namely the channels are composed of a 3D Bell state and a 3D GHZ state, the total success probability of the controlled teleportation can reach 1. And this scheme can be expanded to control the teleportation of an unknown *D*-dimensional two-particle state.
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Since the seminal work of Bennett et al[1], the quantum teleportation has been paid much attention due to its important applications in quantum computation and quantum communication. A lot of protocols about teleportation have been experimentally demonstrated in optical discrete-variable and continuous-variable systems[2-6], liquid nuclear magnetic resonance technique[7], and trapped ion system[8,9], etc. in recent years. Zhang et al[10] have reported the experimental teleportation of a quantum state composed of a two-particle composite system. But the quantum teleportation of the *N*-particle *D*-dimensional state is very pop for this state is useful for a large-scale realization of quantum information processing. By far, a large number of theoretical protocols for teleporting an unknown state via various kinds of entangled channels have been presented[11-22]. However, no scheme has been reported for controlled teleportation of an unknown two-particle 3D entangled state by using partially entangled states in the 3D space as the quantum channels.

In this paper, we propose a scheme to realize the controlled teleportation of an unknown 3D two-particle state by performing two generalized Bell-state measurements (BSMs) and a separated measurement (SM) on the controlled particle in the basis $\{|\pi_0\rangle_3, |\pi_1\rangle_3, |\pi_2\rangle_3\}$ by introducing an assisted three-state particle[23] and two unitary transformations $U_{\mathrm{tel.1}}$ and $U_{\mathrm{tel.2}}$. Besides, we obtain the success probability of the controlled teleportation in the teleportation process. It is shown that the total success probability can arrive at 1 if the quantum channels consist of the 3D Bell state and the 3D GHZ state.

Suppose that the sender Alice possesses an unknown two-particle in the 3D entangled state and wants to teleport it to the receiver Charlie with the cooperation of the controller Bob. Alice begins with the unknown state

$$|\psi\rangle_{12} = (\alpha|00\rangle + \beta|11\rangle + \gamma|22\rangle)_{12}, \tag{1}$$

where the unknown coefficients α, β and γ satisfy $|\alpha|^2+|\beta|^2+|\gamma|^2=1$. In order to realize the teleporting without loss of generality, we consider to establish the 3D partially entangled states of particles (3, 4) and (5, 6, 7), i.e., the non-maximally entangled states $|\psi\rangle_{34}$ and $|\psi\rangle_{567}$ in 3D space as the quantum channels, which can be expressed respectively as:

$$|\psi\rangle_{34} = (a|00\rangle + b|11\rangle + c|22\rangle)_{34}, \tag{2}$$

$$|\psi\rangle_{567} = (d|000\rangle + f|111\rangle + g|222\rangle)_{567}, \tag{3}$$

where the real coefficients a, b, c, d, f and g satisfy $|a|^2+|b|^2+|c|^2=1$ and $|d|^2+|f|^2+|g|^2=1$. Without loss of generality, we make a further assumption that the coefficients satisfy $\min\{|b|,|c|\}\geq|a|$ and $\min\{|f|,|g|\}\geq|d|$. Particles 4 and 5 be-

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long to the sender Alice, while particles 6 and 7 belong to the receiver Charlie, and the controller Bob holds particle 3. Therefore, the total state $|\psi\rangle_{\text{total}}$ of the seven particles is

$$|\psi\rangle_{\text{total}} = |\psi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{567} = (\alpha|00\rangle + \beta|11\rangle + \gamma|22\rangle)_{12} \otimes (a|00\rangle + b|11\rangle + c|22\rangle)_{34} \otimes (d|000\rangle + f|111\rangle + g|222\rangle)_{567}. \tag{4}$$

The schematic diagram for the controlled teleportation of the unknown state $|\psi\rangle_{12}$ with the use of the partially entangled states of particles 3, 4, 5, 6 and 7 is presented in Fig.1. Alice, Bob and Charlie share particles 3, 4, 5, 6 and 7. Alice and Bob send the outcomes of two BSMs on particles (1, 4), (2, 5) and a SM on particle 3 to Charlie who reincarnates the original state by performing the corresponding unitary transformations.

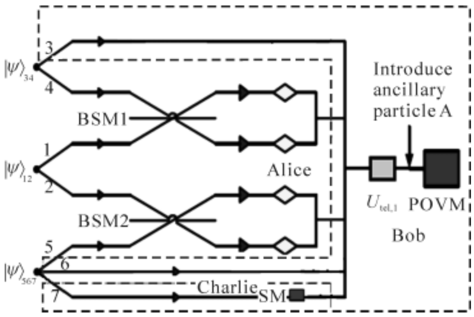

Fig.1 Controlled teleportation of the unknown state $|\psi\rangle_{12}$ with partially entangled states of particles 3, 4, 5, 6 and 7

We will give the controlled teleportation of the unknown state $|\psi\rangle_{12}$ to the particles 6 and 7 at Charlie's side. Firstly, Alice performs two generalized BSMs on particles (1,4) and (2,5) at her side. A generalized BSM can be written as $|\phi_{xy}\rangle = \frac{1}{\sqrt{3}}\sum_z e^{2\pi i xz/3}|z\rangle \otimes |(y+z) \bmod 3\rangle$, where x, y, z=0, 1, 2 and $|\phi\rangle_{xy} = \langle\phi_{xy}|^*$. It is obvious that there are 9 possible outcomes in all after a BSM[12]. If Alice's measurement begins with the particles (1, 4), the total state $|\psi\rangle_{\text{total}}$ will be collapsed into ${}_{14}\langle\phi_{xy}|\psi\rangle_{\text{total}} = \frac{1}{\sqrt{3}}\sum_z e^{-2\pi i xz/3}\langle z,(y+z)|\psi_{\text{total}}\rangle$.

With the same BSM again on particles (2, 5), the system state will be further collapsed into the state

$$\left\langle\phi_{x'y'}^{25}\middle|\phi_{xy}^{14}\middle|\psi\right\rangle_{\text{total}} = \frac{1}{3}\sum_{z'z} e^{-2\pi i(xz+x'z')/3}\left\langle z',(y'+z')\middle|z,(y+z)\middle|\psi_{\text{total}}\right\rangle.$$

Next, the controller Bob makes a separated measurement on particle 3 in the basis $\{|\pi_0\rangle_3, |\pi_1\rangle_3, |\pi_2\rangle_3\}$ which are related to the old basis{|0>, |1>, |2>}. So they can be defined by $|\pi_m\rangle = \frac{1}{\sqrt{3}}\sum_n e^{2\pi i mn/3}|n\rangle$, where m = 0, 1, 2, n = 0, 1, 2 and $|\pi_m\rangle = \langle\pi_m|^*$. After Bob's measurement, the original enment between particles 1 and 2 disappears and a new entanglement is established between particles 6 and 7. Then Alice and Bob inform the receiver Charlie of their measuring results via a classical channel. According to their classical information, Charlie can re-establish the original state $|\psi\rangle_{12}$ by performing the corresponding unitary transformations on particles (6, 7). For example, if Alice's two generalized Bell-state measuring results on particles (1, 4) and (2, 5) are $xy = 20$ and $x'y' = 21$, and Bob's measurement result on particle 3 is $|\pi_1\rangle_3$, the whole system state $|\psi\rangle_{\text{total}}$ will be collapsed into the following state

$$|\psi\rangle_{67} = \left\langle\pi_1^3\middle|\phi_{21}^{25}\middle|\phi_{20}^{14}\middle|\psi\right\rangle_{\text{total}} = \frac{1}{3\sqrt{3}}\sum_{z,z',n} e^{-2\pi i(2z+2z'+n)/3}\left\langle n\middle|z',(1+z')\middle|z,z\middle|\psi_{\text{total}}\right\rangle = \frac{1}{3\sqrt{3}}(\alpha af|11\rangle + e^{-4\pi i/3}\beta bg|22\rangle + e^{-2\pi i/3}\gamma cd|00\rangle)_{67}. \tag{5}$$

Then, Charlie establishes the corresponding unitary transformations $U_{\text{tel.1}}$ and $U_{\text{tel.2}}$ on the state $|\psi\rangle_{67}$ to make the coefficients α, β and γ correspond to $|00\rangle_{67}, |11\rangle_{67}$ and $|22\rangle_{67}$, respectively as shown in Eq.(1). The first unitary transformation $U_{\text{tel.1}}$ can be described by

$$U_{\text{tel},1} = U_{xy,x'y'}^m = \sum_z e^{2\pi i z(x+x'+m)/3}|z\rangle \otimes |z\rangle \times \langle(m+y+y') \bmod 3| \otimes \langle(m+y+y') \bmod 3| . \tag{6}$$

For the measuring results are $|\phi_{20}^{14}\rangle, |\phi_{21}^{25}\rangle, |\pi_1^3\rangle$, $xy = 20$ $x'y' = 21$, m=1 and the correspondence unitary transformation is $U_{\text{tel},1}$=$U_{20,21}^1$=$|00\rangle\langle 11|$+$e^{4\pi i/3}|11\rangle\langle 22|$+$e^{2\pi i/3}|22\rangle\langle 00|$ which is on the state $|\psi\rangle_{67}$, Eq.(5) can be transformed into

$$|\psi^0\rangle_{67} = U_{20,21}^1|\psi\rangle_{67} = \frac{1}{3\sqrt{3}}(\alpha af|00\rangle + \beta bg|11\rangle + \gamma cd|22\rangle)_{67} \cdot \tag{7}$$

According to Alice and Bob's measuring outcomes on particles (6, 7), Charlie introduces an auxiliary three-state particle A in the initial state $|0\rangle_{\text{A}}$ and performs the correspondence unitary transformation $U_{\text{tel},2}$ on particles 6, 7 and A. The state of particles (6, 7, A) can be written as

$$|\psi\rangle_{67\text{A}} = |\psi^0\rangle_{67} \otimes |0\rangle_{\text{A}} \longrightarrow$$

$$|\psi^0\rangle_{67\text{A}} = \frac{1}{3\sqrt{3}}(\alpha af|00\rangle + \beta bg|11\rangle + \gamma cd|22\rangle)_{67} \otimes |0\rangle_{\text{A}} \cdot \tag{8}$$

$U_{\mathrm{tel},2}=\begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \boldsymbol{0} \\ \boldsymbol{A}_2 & -\boldsymbol{A}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}$ is a 9 × 9 matrix where **0** is the 3×3 zero matrix and ***I*** is a 3×3 identity matrix. $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are the 3 × 3 matrices, of which the elements are zero except for the diagonal elements, so they can be written as $\boldsymbol{A}_1=\mathrm{diag}\,(a_0, a_1, a_2)$ and $\boldsymbol{A}_2=\mathrm{diag}(\sqrt{1-a_0^2},\sqrt{1-a_1^2},\sqrt{1-a_2^2})$, respectively, where $a_i (i = 0, 1, 2)$ satisfies $|a_i|\leqslant 1$ and the value of a_i depends on the state of particles (6, 7). For every kind of state of particles (6, 7) in the controlled teleportation, Charlie can perform the corresponding unitary transformation $U_{\mathrm{tel}.2}$ on particles 6, 7 and A in the state $|\psi\rangle_{67\mathrm{A}}$ to reconstruct the state $|\psi\rangle_1$. So there are a set of local unitary operations $\{U_{\mathrm{tel},2}^j\}$ as shown in Tab.1. By using the unitary operators $\{U_{\mathrm{tel},2}^j\}$ Charlie can reincarnate the original state under the basis $\{|000\rangle_{67\mathrm{A}},|110\rangle_{67\mathrm{A}},|220\rangle_{67\mathrm{A}},|001\rangle_{67\mathrm{A}}, |111\rangle_{67\mathrm{A}},|221\rangle_{67\mathrm{A}},|002\rangle_{67\mathrm{A}},|112\rangle_{67\mathrm{A}},\text{and } |222\rangle_{67\mathrm{A}}\}$

Tab.1 Corresponding operation of $U_{\mathrm{tel}.1}$ and the coefficients a_0, a_1 and a_2 for $\{U_{\mathrm{tel},2}^j\}$ in all situations

States of particles (6,7)	$U_{\mathrm{tel},1}$	a_0	a_1	a_2
$\langle\pi_m^3\|\langle\phi_{x'0}^{25}\|\langle\phi_{x0}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'0,x0}^m$	$\frac{bf}{ad}$	$\frac{cg}{bf}$	1
$\langle\pi_m^3\|\langle\phi_{x'1}^{25}\|\langle\phi_{x0}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'1,x0}^m$	$\frac{bg}{af}$	1	$\frac{bg}{cd}$
$\langle\pi_m^3\|\langle\phi_{x'2}^{25}\|\langle\phi_{x0}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'2,x0}^m$	1	$\frac{ag}{bd}$	$\frac{ag}{cf}$
$\langle\pi_m^3\|\langle\phi_{x'0}^{25}\|\langle\phi_{x1}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'0,x1}^m$	$\frac{cf}{bd}$	$\frac{ag}{cf}$	1
$\langle\pi_m^3\|\langle\phi_{x'1}^{25}\|\langle\phi_{x1}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'1,x1}^m$	$\frac{cg}{bf}$	1	$\frac{cg}{ad}$
$\langle\pi_m^3\|\langle\phi_{x'2}^{25}\|\langle\phi_{x1}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'2,x1}^m$	1	$\frac{bg}{cd}$	$\frac{bg}{af}$
$\langle\pi_m^3\|\langle\phi_{x'0}^{25}\|\langle\phi_{x2}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'0,x2}^m$	$\frac{af}{cd}$	$\frac{bg}{af}$	1
$\langle\pi_m^3\|\langle\phi_{x'1}^{25}\|\langle\phi_{x2}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'1,x2}^m$	$\frac{ag}{cf}$	1	$\frac{ag}{bd}$
$\langle\pi_m^3\|\langle\phi_{x'2}^{25}\|\langle\phi_{x2}^{14}\|\psi\rangle_{\mathrm{total}}$	$U_{x'2,x2}^m$	1	$\frac{cg}{ad}$	$\frac{cg}{bf}$

For particles (6, 7) in the state $|\psi^0\rangle_{67}$ (xy=20, $x'y'$ = 21, m=1), we choose $(a_0, a_1, a_2)=(\frac{bg}{af}, 1, \frac{bg}{cd})$ and perform the corresponding unitary transformation $U'_{\mathrm{tel},2}$ on the state $|\psi\rangle_{67\mathrm{A}}$, namely

$$U'_{\mathrm{tel},2}|\psi\rangle_{67\mathrm{A}}=\frac{1}{3\sqrt{3}}\begin{pmatrix} \frac{bg}{af} & 0 & 0 & \frac{\sqrt{a^2f^2-b^2g^2}}{af} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{bg}{dc} & 0 & 0 & \frac{\sqrt{c^2d^2-b^2g^2}}{dc} & 0 & 0 & 0 \\ \frac{\sqrt{a^2f^2-b^2g^2}}{af} & 0 & 0 & -\frac{bg}{af} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{c^2d^2-b^2g^2}}{dc} & 0 & 0 & -\frac{bg}{dc} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha af \\ \beta bg \\ \gamma cd \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{67\mathrm{A}}=$$

$$\frac{bg}{3\sqrt{3}}(\alpha|00\rangle+\beta|11\rangle+\gamma|22\rangle)_{67}\otimes|0\rangle_{\mathrm{A}}+\frac{1}{3\sqrt{3}}(\sqrt{a^2f^2-b^2g^2}\alpha|00\rangle_{67}+\sqrt{d^2c^2-b^2g^2}\gamma|22\rangle_{67})\otimes|1\rangle_{\mathrm{A}}. \tag{9}$$

Finally, Charlie measures the state of auxiliary particle A and his measuring result decides whether the controlled teleportation will be successful or not. If the result $|0\rangle_{\mathrm{A}}$ is measured, the state in Eq.(9) will collapse into

$$|\psi'\rangle_{67}=\frac{bg}{3\sqrt{3}}(\alpha|00\rangle+\beta|11\rangle+\gamma|22\rangle)_{67}, \tag{10}$$

and it means that we have realized the controlled teleportation with the success probability of $p^{\mathrm{succ}}=\frac{b^2g^2}{27}$. If Charlie obtains $|1\rangle_{\mathrm{A}}$, it means that the controlled teleportation scheme has failed. In our scheme, there are possible 9 results after a generalized Bell-state measurement and 3 results after a separated measurement on particle 3. For Alice performs two generalized Bell-state measurements on her particles (1, 4) and (2, 5), respectively, and Bob performs a separated measurement on particle 3, by straightforward calculations, there are 9 × 9 × 3 = 243, kinds of states of particles (6, 7) in all. Based on Refs.[18, 24-27], we can prove that every kind of the state of particles (6, 7) in the controlled teleportation can be realized successfully with the equal probability p^{succ} of $\frac{b^2g^2}{27}$, so the total success probability of the controlled teleportation $p_{\text{total}}^{\text{succ}}$ should be $p_{\text{total}}^{\text{succ}}=\frac{b^2g^2}{27}\times 243=9b^2g^2$.

Note that if the quantum channels consist of the maximally entangled Bell state of particles (3, 4) and the maximally entangled GHZ state of particles (5, 6, 7) in a 3D Hilbert space, the success probability of the controlled teleportation will be $\frac{b^2g^2}{27}=\frac{1}{27\times 9}$, and the total success probability is equal to 1, which are in agreement with the results by Cao et al[12].

In summary, we have proposed a scheme for the controlled teleportation of an unknown 3D entangled state $|\psi\rangle_{12}$ by using the non-maximally entangled two-particle state and the non-maximally entangled three-particle state as the quantum channels. Furthermore, if the quantum channels are composed of the maximally entangled Bell state of particles and the maximally entangled GHZ state in a 3D Hilbert space, the total success probability $p_{\text{total}}^{\text{succ}}$ is equal to 1 and the probabilistic controlled teleportation can reduce the faithful controlled teleportation.

It should be mentioned that in our scheme the controller Bob can be a receiver if and only if Charlie exchanges with Bob and wants to help him. If Bob as the controller does not cooperate with Alice and Charlie as the communicatees, the controlled teleportation scheme will fail. Analysis shows that the controller can hold particle 4 to replace particle 3, as the controlled particle is equivalent to the operation on particle 3 in the controlled teleportation. Of course, the two unitary transformations $U_{\text{tel},1}$ and $U_{\text{tel},2}$ can be incorporated into a transformation $U=U_{\text{tel},1}\otimes U_{\text{tel},2}$. Moreover, it is verified that the present scheme can be expanded to control teleportation of an unknown two-particle state $|\psi\rangle'_{12}$ in D-dimensional space via using a non-maximally entangled two-particle state $|\psi\rangle'_{34}$ and a non-maximally entangled three-particle state $|\psi\rangle'_{567}$ in the D-dimensional space as the quantum channels, i.e.,

$$|\psi\rangle'_{12}=\sum_{j=0}^{D-1}\alpha_j|jj\rangle_{12}, \tag{11}$$

$$|\psi\rangle'_{34}=\sum_{j=0}^{D-1}\beta_j|jj\rangle_{34}, \tag{12}$$

$$|\psi\rangle'_{567}=\sum_{j=0}^{D-1}\gamma_j|jjj\rangle_{567}, \tag{13}$$

where the unknown coefficients α_i, β_i and γ_i (i = 0, 1, 2, ⋯ D-1) satisfy $\sum_{j=0}^{D-1}|\alpha_j|^2=1$, $\sum_{j=0}^{D-1}|\beta_j|^2=1$ and $\sum_{j=0}^{D-1}|\gamma_j|^2=1$. Besides, it should be noticed that if considering the amplitude damping effects, we can further research the generalized amplitude damping effects on the controlled teleportation[28, 29], which will be more important in understanding the practical effects of noise on quantum systems.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K, Phys. Rev. Lett. **70**, 1895 (1993).

[2] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A, Nature **390**, 575 (1997).

[3] Furusawa A, SArensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S, Science **282**, 706 (1998).

[4] Kim Y H, Kulik S P and Shih Y, Phys. Rev. Lett. **86**, 1370 (2001).

[5] Kraus B, Hammerer K, Giedke G and Cirac J I, Phys. Rev. A **67**, 042314 (2003).

[6] Zhang T C, Goh K W, Chou C W, Lodahl P and Kimble H J, Phys. Rev. A **67**, 03380 (2003).

[7] Nielsen M A, Knill E and Lafkamme R, Nature **396**, 52 (1998).

[8] Konner M, Nature **429**, 705 (2004).

[9] Li J, Chen Y R, Xin Y, Zhou M C and Xu S X, Optics Commun. **283**, 3105 (2010).

[10] Li J, Chen Y R and Xu S X, Optoelectron. Lett. **7**, 77 (2011).

[11] Dai H Y, Kuang L M and Li C Z, Phys. Lett. A **323**, 360 (2004).

[12] Cao H J and Song H S, Phys. Scri. **75**, 747 (2007).

[13] Wang J, Wan X, Zhang H F, Gao Y, Chen T Y and Liang H, Optoelectron. Lett. **5**, 380 (2009).

[14] Yeo Y and Chua W K, Phys. Rev. Lett. **96**, 060502 (2006).

[15] Lee J and Kim M S, Phys. Rev. Lett. **84**, 4236 (2000).

[16] Joo J, Park Y J, Oh S and Kim J, New. J. Phys. **5**, 136 (2003).

[17] Liu J M, Zhang Y S and Guo G C, Chin. Phys. **12**, 251 (2003).

[18] Liu H, Xiao X Q and Liu J M, Commun. Theor. Phys. **50**, 69 (2008).

[19] Liu J M and Guo G C, Chin. Phys. Lett. **19**, 456 (2002).

[20] Yang C P and Guo G C, Chin. Phys. Lett. **16**, 628 (1999).

[21] Zheng Y Z, Gu Y J and Guo G C, Chin. Phys. **11**, 537 (2002).

[22] Cabello A, Phys. Rev. Lett. **89**, 100402 (2003).

[23] Li J, Chen Y R, Zhao Q and Zhou M C, Journal of Optical Society of America A **26**, 2121 (2009).

[24] Yuan H C and Qi K G, Chin. Phys. **14**, 898 (2005).

[25] Shi B S and Tomita A, Phys. Lett. A **296**, 161 (2002).

[26] Joo J and Park Y J, Phys. Lett. A **300**, 324 (2002).

[27] Kitaev A Y, Russlan Mathematical Surveys **52**, 1191 (1997).

[28] Braunstein S L, Phys. Rev. Lett. **80**, 4084 (1998).

[29] Beny C, Kempf A and Kribs D W, Phys. Rev. A **76**, 042303 (2007).