Evanescent wave of extraordinary beam at uniaxial crystal surfaces

JIA Guang-yi (贾光一)*
College of Science, Tianjin University, Tianjin 300072, China

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Under the circumstance of optical axis being in the incident plane, the evanescent wave of total reflection is studied when an extraordinary beam is incident from an isotropic medium upon a uniaxial crystal by using the general characteristics of uniaxial crystal and electromagnetic field. This paper presents the propagation directions of equiphase plane and the images of evanescent wave, and reveals that the equiamplitude plane and the equiphase plane are not in quadrature any more, and the phase difference between longitudinal wave and transversal wave does not equal π/2, either. But the reflectivity is still kept at 100%.
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When the light is propagated from an optically denser medium into an optically less dense one, if the incident angle exceeds the critical angle, all the incident light is reflected back into the first medium and we call it as total reflection. But experimental results show that the incident beam still establishes an evanescent electromagnetic field — named evanescent wave — that penetrates into the second medium and decays exponentially with the distance z from the interface.

In recent years, with the rapid development of photonics and telecommunications, it is found that the evanescent wave combined with surface plasmon polaritons (SPPs) can help explore new phenomena and novel mechanisms that occur in the near-field optics, which has generated extensive application demands in nano-photonic devices such as SPPs transducer, SPPs photonic chips and so on[1-3]. Evanescent wave fiber-optic biosensors, which are on the basis of evanescent wave principle, represent the next generation of cutting-edge technology with the potential of rapidly detecting and identifying microorganisms, toxins, allergens, and other analytes of interest[4]. Along with the vast using amount of optical polarization elements, the need of studying the propagation of light in crystals is increasing progressively[5,6]. But the current discussion of light is much more in bireflection and birefringence of uniaxial crystals[7-9] than that in evanescent wave of extraordinary beam at crystal surfaces. Ref.[10] has introduced the analytical expressions of penetration depth and Goos-Hänchen shift and also some discussions when an extraordinary beam with total reflection occurs at uniaxial crystal surfaces, but it did not make a specific study on the function of evanescent wave. In this paper, according to the general characteristics of uniaxial crystal and electromagnetic field, we will do some detailed analyses on the evanescent wave when an extraordinary beam is incident from an isotropic medium upon a uniaxial crystal (the optical axis is in the incident plane) along with total reflection. It presents that the propagating regulations of evanescent wave of extraordinary beam are obviously different from those of isotropic media.

For the case of optical axis in the incident plane and keeping a θ angle to the crystal surface, the plane wave whose electric vector $\boldsymbol{E}$ is parallel to the incident plane is called as extraordinary beam. When an extraordinary beam — its electric vector is $\boldsymbol{E}_1$ — is incident upon the crystal surface, it will be divided into a reflected wave and a refracted wave whose electric vectors are $\boldsymbol{E}_1'$ and $\boldsymbol{E}_2$ respectively and both of them are plane waves parallel to the incident plane. The positive direction of electric vector $\boldsymbol{E}$ and the associated direction of magnetic vector $\boldsymbol{H}$ are shown in Fig.1 ($\boldsymbol{H}$ is perpendicular to the paper and points to the outside). θ_1, θ_1', θ_{2s}, and θ_{2k} are incident angle, reflection angle, refraction angle of light ray and refraction angle of wave normal respectively, and $\boldsymbol{k}$ is the wave vector. For isotropic media, wave vector $\boldsymbol{k}$ is parallel to light ray $\boldsymbol{s}$.

The ray velocity stands for the velocity of light disturbances and associates with the propagation direction of

*E-mail: chwjwh@sina.com

energy, so $\theta_{2s}=\pi/2$ means reaching the critical state of total reflection, and the critical angle meets[10]

$$n_1^2\sin^2\theta_{1c}=n_e^2\sin^2\theta+n_o^2\cos^2\theta\ . \tag{1}$$

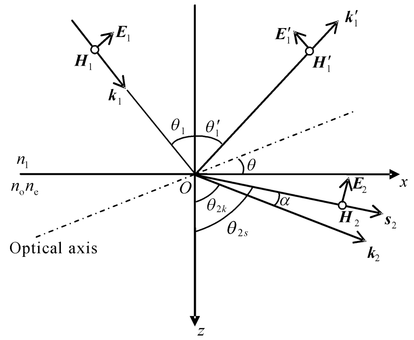

Fig.1 Directions of *E* and *H* when an extraordinary beam is incident upon a crystal from an isotropic medium

In uniaxial crystals, the relationship between light ray direction and wave normal direction meets[11]

$$\tan\varphi=\frac{n_e^2}{n_o^2}\tan\xi\ , \tag{2}$$

where φ denotes the angle between wave normal and optical axis, and ξ denotes that between light ray and optical axis. For Fig.1, $\varphi=\pi/2-\theta_{2k}+\theta$, $\xi=\pi/2-\theta_{2s}+\theta$, so Eq.(2) may be written as

$$n_o^2\tan(\theta_{2s}-\theta)=n_e^2\tan(\theta_{2k}-\theta)\ . \tag{3}$$

When reaching the critical state, $\theta_{2s}=\pi/2$, substituting it into Eq.(3), we obtain the angle between the propagation direction of wave normal (equiphase plane) and the z axis,

$$\tan\theta_{2kc}=\frac{n_e^2\sin^2\theta+n_o^2\cos^2\theta}{(n_e^2-n_o^2)\sin\theta\cos\theta}\ . \tag{4}$$

We can obtain the propagation direction of wave normal through making a tangent face across the intersection point between the ellipsoidal plane of gyration and the light ray of total reflection (the positive direction of x axis) and making a vertical line from the origin O to the tangent face (see Fig.2 and Fig.3). When the light is incident from an isotropic medium upon a uniaxial crystal, the refraction angle of light ray is given by[10]

$$\tan\theta_{2s}=\frac{(n_o^2-n_e^2)\sin\theta\cos\theta+n_on_en_1\sin\theta_1(n_e^2\sin^2\theta+n_o^2\cos^2\theta-n_1^2\sin^2\theta_1)^{-1/2}}{n_e^2\sin^2\theta+n_o^2\cos^2\theta}\ . \tag{5}$$

According to Eqs.(4) and (5), we know that the angle between the propagation direction of equiphase plane and the interface, under the critical state, is equal to the refraction angle of light ray when the light is normally incident upon the crystal surface ($\theta_1=0$). However, when $\theta=0$ or π , $\theta_{2kc}=\pi/2$, it means that the wave normal coincides with the light ray.

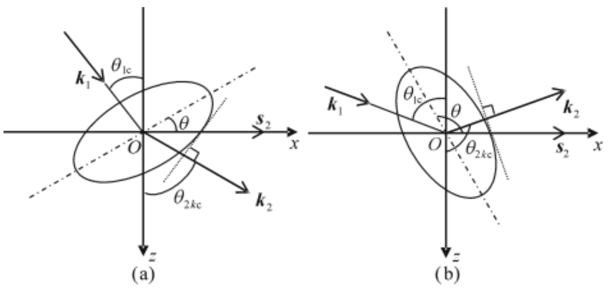

Fig.2 Images of wave normal vector of positive uniaxial crystal ($n_o<n_e$) at the state of total reflection: (a) 0 <θ <π/2; (b) π/2<θ <π

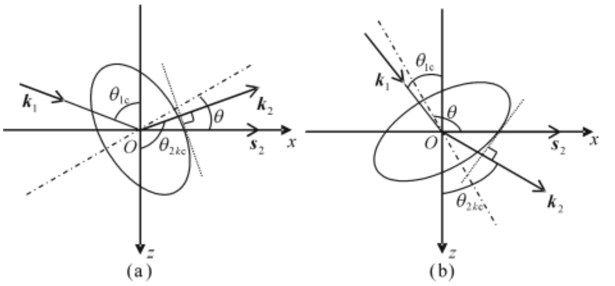

Fig.3 Images of wave normal vector of negative uniaxial crystal ($n_o>n_e$) at the state of total reflection:(a) 0<θ <π/2; (b) π/2<θ <π

The total reflection will occur when $\theta_1>\theta_{1c}$, and the function of transmission wave — the evanescent wave — is[10]

$$\boldsymbol{E}_{20}=\widetilde{\boldsymbol{E}}_{20}\exp\left(-\frac{kn_on_e\sqrt{n_1^2\sin^2\theta_1-n_e^2\sin^2\theta-n_o^2\cos^2\theta}}{n_e^2\sin^2\theta+n_o^2\cos^2\theta}z\right)\times$$

$$\exp\left\{\mathrm{i}k\left[n_1\sin\theta_1x+\frac{(n_e^2-n_o^2)n_1\sin\theta_1\sin\theta\cos\theta}{n_e^2\sin^2\theta+n_o^2\cos^2\theta}z\right]\right\}\times$$

$$\exp(-\mathrm{i}\omega t), \tag{6}$$

where k denotes the wave number in vacuum. In isotropic media, the equiamplitude plane and the equiphase plane of the evanescent wave do not coincide with each other, but they are just in quadrature[11]. But here we find that the equiamplitude plane and the equiphase plane of the evanescent wave are neither coincident nor in quadrature, which is shown in Fig.4.

The complex vector amplitude of the function of evanescent wave includes three components $\widetilde{\boldsymbol{E}}_{20}(\widetilde{E}_{20x},\widetilde{E}_{20y},\widetilde{E}_{20z})$.

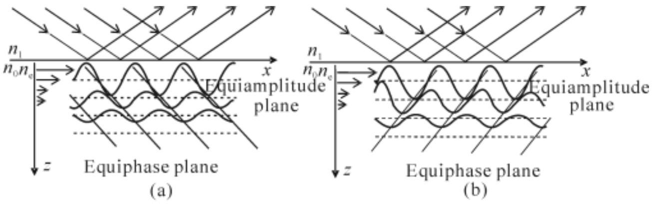

Fig.4 Images of the evanescent wave: (a) $n_o<n_e$, $\pi/2<\theta<\pi$ or $n_o>n_e$, $0<\theta<\pi/2$; (b) $n_o<n_e$, $0<\theta<\pi/2$ or $n_o>n_e$, $\pi/2<\theta<\pi$

It should be noted that though crystals are anisotropic media, they are homogeneous[11]. Based on one of the Maxwell equations, the divergence of electric field equals zero in the space of homogeneous media, which means[11]

$$\nabla\cdot\boldsymbol{E}=0\,,\text{or}\ \frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0\,, \tag{7}$$

then

$$\left[\mathrm{i}kn_1\sin\theta_1\widetilde{E}_{20x}-\frac{kn_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}-\mathrm{i}k(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta}{n_\mathrm{e}^2\sin^2\theta+n_o^2\cos^2\theta}\widetilde{E}_{20z}\right]\times$$
$$\exp\left(-\frac{kn_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right)\exp\left\{\mathrm{i}k\left[n_1\sin\theta_1x+\frac{(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right]\right\}\times$$
$$\exp(-\mathrm{i}\omega t)=0\ , \tag{8}$$

which must be established for any variable of space-time ($\boldsymbol{r}$, t) .

$$\frac{\widetilde{E}_{20x}}{\widetilde{E}_{20z}}=\frac{-(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta-\mathrm{i}\,n_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}{n_1\sin\theta_1(n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta)}\ , \tag{9}$$

which means that the evanescent wave expressed by Eq.(6) and propagating along the x axis includes both the component of longitudinal wave

$$E_{20x}\exp\left(-\frac{kn_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right)\exp\left\{\mathrm{i}k\left[n_1\sin\theta_1x+\frac{(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right]\right\}\exp(-\mathrm{i}\omega t), \tag{10}$$

and the component of transversal wave

$$E_{20z}\exp\left(-\frac{kn_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right)\exp\left\{\mathrm{i}k\left[n_1\sin\theta_1x+\frac{(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta}{n_\mathrm{e}^2\sin^2\theta+n_\mathrm{o}^2\cos^2\theta}z\right]\right\}\exp(-\mathrm{i}\omega t). \tag{11}$$

According to Eq.(9), we have the phase difference between them

$$\Phi=\arctan\frac{n_\mathrm{o}n_\mathrm{e}\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}{(n_\mathrm{e}^2-n_\mathrm{o}^2)n_1\sin\theta_1\sin\theta\cos\theta}. \tag{12}$$

Uniaxial crystals are anisotropic media and there is a walk-off angle α between $\boldsymbol{k}$ and $\boldsymbol{s}$. Since $\boldsymbol{E}$ is perpendicular to $\boldsymbol{s}$, and $\boldsymbol{H}_2$ is considered to be perpendicular to the incident plane, then

$$H_{2y}=H_2=n_2\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos\alpha E_2=n_2^*\sqrt{\frac{\varepsilon_0}{\mu_0}}E_2\ , \tag{13}$$

where $n_2^*=n_2\cos\alpha$ denotes the refractive index of the refracted wave, and n_2 denotes that of the wave normal for the refracted wave[12].

According to Fig.1, we make cross multiplication for the extraordinary beam, so we can obtain the average energy density of the refracted wave.

$$\overline{\boldsymbol{S}}=\frac{1}{2}\mathrm{Re}(\boldsymbol{E}^*\times\boldsymbol{H})=\frac{1}{2}\mathrm{Re}(-E_{2z}^*H_{2y}\boldsymbol{i}+E_{2x}^*H_{2y}\boldsymbol{j}). \tag{14}$$

Considering $\tan\theta_{2s}$ as an imaginary number and[10]

$$\frac{n_2^*}{\cos\theta_{2s}}=-\mathrm{i}\frac{n_\mathrm{o}n_\mathrm{e}}{\sqrt{n_1^2\sin^2\theta_1-n_\mathrm{e}^2\sin^2\theta-n_\mathrm{o}^2\cos^2\theta}}\ , \tag{15}$$

then substituting $E_{2x}=E_2\cos\theta_{2s}$, $E_{2z}=E_2\sin\theta_{2s}$ and Eq.(13) into Eq.(14), we obtain

$$\bar{S}_{2x} = -\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|E_2|^2\,\mathrm{Re}\left(\frac{n_2^*}{\cos\theta_{2s}}\frac{|\sin\theta_{2s}|^2}{\tan\theta_{2s}}\right) \neq 0 \ , \quad (16)$$

$$\bar{S}_{2z} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|E_2|^2\,\mathrm{Re}\left(\frac{n_2^*}{\cos\theta_{2s}}|\cos\theta_{2s}|^2\right) = 0 \ . \quad (17)$$

So we know that the average energy density of the refracted wave only has an x component, and the average energy density which penetrates into the crystal along the z axis equals zero.

In this paper we have made analyses on the transmission wave field of total reflection — evanescent wave when an extraordinary beam is incident from an isotropic medium upon a uniaxial crystal (the optical axis is in the incident plane). The propagation direction of equiphase plane at the critical state of total reflection and the images of evanescent wave for total reflection are given. We also have presented the phase difference between the component of longitudinal wave and the component of transversal wave. From these we achieve obviously different propagating laws compared with those in isotropic media when the extraordinary beam with total reflection occurs at the uniaxial crystal surface: the equiamplitude plane and the equiphase plane of evanescent wave are not in quadrature, and the phase difference between the component of longitudinal wave and the component of transversal wave does not equal π/2. Nevertheless, due to the imaginary number of $n_2^* / \cos\theta_{2s}$, the light energy does not enter the crystal, so the reflectivity is kept at 100%. These provide some other new theoretical base for studying near-field optics, and have potential practical values for applying evanescent wave of extraordinary beams to the fields of photonics and telecommunications.

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