Optical notch filter design based on digital signal processing*

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Based on digital signal processing theory, a novel method of designing optical notch filter is proposed for Mach-Zehnder interferometer with cascaded optical fiber rings coupled structure. The method is simple and effective, and it can be used to implement the designing of the optical notch filter which has arbitrary number of notch points in one free spectrum range (FSR). A design example of notch filter based on cascaded single-fiber-rings is given. On this basis, an improved cascaded double-fiber-rings structure is presented to eliminate the effect of phase shift caused by the single-fiber-ring structure. This new structure can improve the stability and applicability of system. The change of output intensity spectrum is finally investigated for each design parameter and the tuning characteristics of the notch filter are also discussed.
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In dense wavelength division multiplexing (DWDM) and optical add-drop multiplexer (OADM) systems, optical filters play an important role. As a special case, recently, optical notch filter has become a focus topic. The output of notch filter has an ultra-narrow band, and it can filter all the unwanted signals but do nothing to the useful signal, so it is widely used in signal controlling area and other fields[1-3]. In fact, the design of optical notch filter has been studied for several years. By cascading birefringent crystals[4], a notch response can be realized, but if the 3-dB bandwidth of the notch filter is narrow, a large number of crystals are needed, and the structure is complicated. Ningsi You[5] designed a novel tunable microwave optical notch filter, the tuning of which is based on changing optical variable attenuators, but the output has poor performance. Mahmoud S. Rasras[6] mentioned a way to design optical notch filter by cascading single-fiber-rings, whose whole structure is simple but the solving of optical parameters is complex and the structure is sensitive to the environment. Compared with them, fiber ring is characterized by compact size, simple parameter and high performance, therefore often used in the design of optical filters.

In this paper, based on digital signal processing theory, the optical all-pass filter (OAF) is used as the platform to construct the optical notch filter. As a designing example, a notch filter with two notch points is realized by cascading single-fiber-rings in the Mach-Zehnder interferometer arms first. Since the phase shift in single-fiber-rings is not a stable factor, an improved structure based on cascading double-fiber-rings is introduced. The influence on the output spectrum is also discussed when the parameters deviate from the designed values, and the tuning characteristics of the notch filter are also investigated.

The design of notch filter can be transformed into all-pass filter in the frequency domain[7]. In fact, such a principle can also be applied to design filters in the optical domain. The basic theory is given in Fig.1.

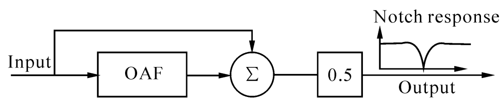

Fig.1 Principle of optical notch filter based on optical all-pass filter

Based on the principle shown in Fig.1, the transfer function of the system can be summarized as follows:

$$H(z)=\frac{1}{2}[1+H_{\text{all-pass}}(z)], \tag{1}$$

where $H_{\text{all-pass}}(z)$ is the transfer function of optical all-pass

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filter. The principle shown in Fig.1 can be accomplished through the optical structure in Fig.2.

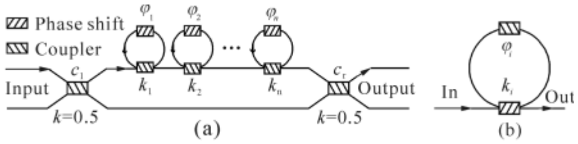

Fig.2 (a) Physical structure and (b) basic fiber ring unit of the optical notch filter

In Fig.2, the optical all-pass filter is composed of *n* cascaded single-fiber-rings. The up arm length is equal to the down arm length, the couplers of Mach-Zehnder C_l and C_r both have a coupling coefficient of 0.5, and the coefficients of *n* different fiber rings are respectively $k_1, k_2, \ldots, k_n$. In fact, after introducing the *Z*-transform into the structure in Fig.2, the transfer function of the system can be calculated through the transfer matrix shown as follows:

$$\boldsymbol{H} = \boldsymbol{M}_r \boldsymbol{T} \boldsymbol{M}_l = \begin{bmatrix} \cos(\varphi_r) & -\mathrm{j}\sin(\varphi_r) \\ -\mathrm{j}\sin(\varphi_r) & \cos(\varphi_r) \end{bmatrix} \times \begin{bmatrix} H_{\text{cascade}}(z) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\varphi_l) & -\mathrm{j}\sin(\varphi_l) \\ -\mathrm{j}\sin(\varphi_l) & \cos(\varphi_l) \end{bmatrix}, \quad (2)$$

where $\boldsymbol{M}_i$ (i=l, r) is the scattering matrix of C_l and C_r, and $\boldsymbol{T}$ is the delay matrix of two arms in Fig.2. $z^{-1}=\mathrm{e}^{-\mathrm{j}2\pi fLn/c}$, where *f* is the frequency of light, *c* is the speed of light in the vacuum, *n* is the refractive index of the fiber rings, and *L* is the length of fiber ring. Since the coefficients of C_l and C_r are both 0.5, the transfer function of the system after calculating is

$$H(z) = \frac{1}{2}[1 + H_{\text{cascade}}(z)],$$
$$H_{\text{cascade}}(z) = \prod_{i=1}^{n} H_i(z) = \prod_{i=1}^{n} \frac{\mathrm{e}^{-\mathrm{j}\varphi_i}(\rho_i \mathrm{e}^{+\mathrm{j}\varphi_i} - z^{-1})}{1 - \rho_i \mathrm{e}^{-\mathrm{j}\varphi_i} z^{-1}}, \quad (3)$$

where $H_{\text{cascade}}(z)$ is the transfer function of *n* different single-fiber-rings, $H_i(z)$ is the transfer function of the *i*-th single-fiber-ring, $\rho_i = \sqrt{1-k_i}$, k_i is the coupling coefficient of the *i*-th single-fiber-ring, and φ_i is the phase shift of the *i*-th single-fiber ring. It can be seen that Eq.(3) is in accordance with Eq.(1), so it can get a notch response. The whole process to get the parameters can be seen in Fig.3.

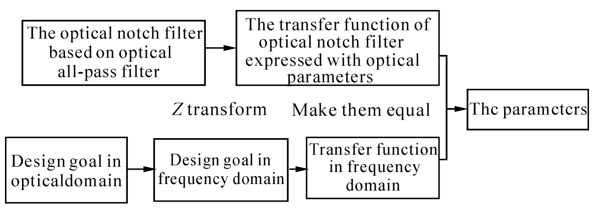

Fig.3 Solution flow chart of optical parameters

As an example, a notch filter with two notch points in one FSR is designed. The locations of two notch points are λ_1=1550.2 nm and λ_2=1550.3 nm respectively, the 3-dB bandwidth of two notch points is $\Delta\lambda_1 = \Delta\lambda_2 = 0.008$ nm, and FSR = 0.8 nm. Four single-fiber-rings are needed to construct such an optical notch filter, and the transfer function of 4 different single-fiber-rings in Fig.2 is

$$H(z) = \frac{1}{2}[1 + \prod_{i=1}^{4} \frac{\mathrm{e}^{-\mathrm{j}\varphi_i}(\rho_i \mathrm{e}^{+\mathrm{j}\varphi_i} - z^{-1})}{1 - \rho_i \mathrm{e}^{-\mathrm{j}\varphi_i} z^{-1}}] \ . \quad (4)$$

However, based on the optical parameters described above, the transfer function of the filter to be designed in frequency domain is:

$$H(z) = \frac{1}{2}[1 + \frac{0.8795 - 1.5861z^{-1} + 2.1682z^{-2} - 1.6936z^{-3} + z^{-4}}{1 - 1.6936z^{-1} + 2.1682z^{-2} - 1.5861z^{-3} + 0.8795z^{-4}}] \ . \quad (5)$$

Compared with Eqs.(4) and (5), we can obtain the parameters of all single-fiber-rings in Tab.1. The spectral transmittance obtained is shown in Fig.4.

Tab.1 Optical parameters of all single-fiber-rings

	i=1	i=2	i=3	i=4
ρ_i	0.9693	0.9675	0.9693	0.9675
φ_i(rad)	0.6874	1.4697	5.5958	4.8135

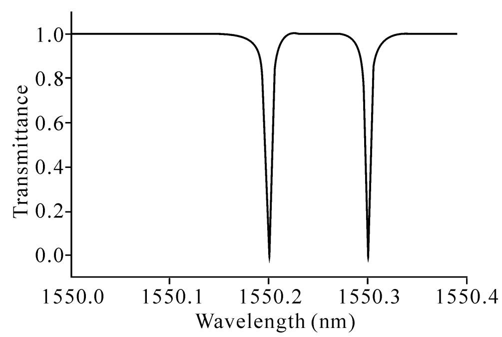

Fig.4 Output spectrum of the optical notch filter

It can be seen from Tab.1 that in order to get the optical notch response, the phase shift of the rings is also needed to be designed, which is a sensitive variable to the environment, and also greatly reduces the stability of the whole system. Hence a modified structure is introduced. In the new structure, the cascading of single-fiber-rings is replaced with the cascading of double-fiber-rings, as a basic unit. The model of a single-double-fiber ring is shown in Fig.5(a). Fig.5(b) is the *Z*-transform model of Fig.5(a).

From Fig.5(b), the transfer function of double-fiber-ring can be deduced as follows:

$$H_{\rm dou}^{i}(z)=\frac{c_{i2}-c_{i1}(c_{i2}+1)z^{-1}+z^{-2}}{1-c_{i1}(1+c_{i2})z^{-1}+c_{i2}z^{-2}}\ , \qquad (6)$$

where $z^{-1}=e^{-j2\pi fLn/c}$, f is the frequency of light, c is the speed of light in the vacuum, n is the refractive index of the fiber rings, L is the length of fiber ring, $c_{ij}=\sqrt{1-k_{ij}}\,(j=1,2)$, and k_{ij} is the coupling coefficient of the double-fiber-ring. If double-fiber-rings are used to design the optical notch filter, two double-fiber-rings are needed. Fig.6 is the structure for such a filter. The transfer function of the system in Fig.6 is

$$H(z)=\frac{1}{2}[1+\prod_{i=1}^{2}\frac{c_{i2}-c_{i1}\left(c_{i2}+1\right)z^{-1}+z^{-2}}{1-c_{i1}\left(1+c_{i2}\right)z^{-1}+c_{i2}z^{-2}}]\ . \qquad (7)$$

Compared with Eqs.(5) and (7), we can obtain c_{12}= 0.9395, c_{11}= 0.7726, c_{22}= 0.9361, c_{21}= 0.1008.

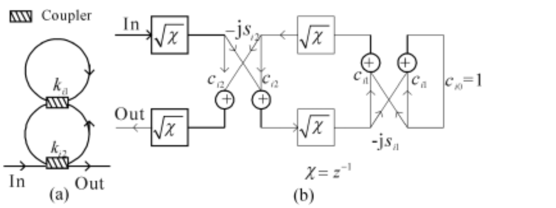

Fig.5 (a) Physical structure and (b)digital model of the double-fiber-ring all-pass filter

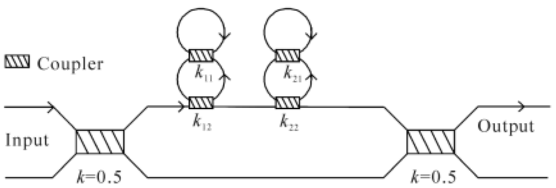

Fig.6 Physical structure for a notch filter with two notch points

In practical applications, the structural parameters can not always be accurate as the designed values. Hereinafter, taking the above design as an example, we will discuss the change of output spectrum when the parameters deviate from the designed values. In order to discuss conveniently, we take c_{12}= 0.9395, c_{11}= 0.7726, c_{22}= 0.9361, c_{21}= 0.1008 into the coupling coefficients of double-fiber-rings. It can be obtained that the coupling coefficient of the first double-fiber-ring is k_{12}= 0.2460 and k_{11}= 0.4769, and that of the second double-fiber-ring is k_{22}= 0.2528 and k_{21}= 0.9483.

The output of the optical notch filter is given in Fig.7 when k_{12}, k_{11}, k_{22} or k_{21} shifts alone. From Fig.7, it can be concluded that k_{12} and k_{11} determine the characteristics of the first notch point (1550.2 nm), and k_{22} and k_{21} determine the characteristics of the second notch point (1550.3 nm). k_{12} and k_{22} respectively tune the 3-dB bandwidths of the first

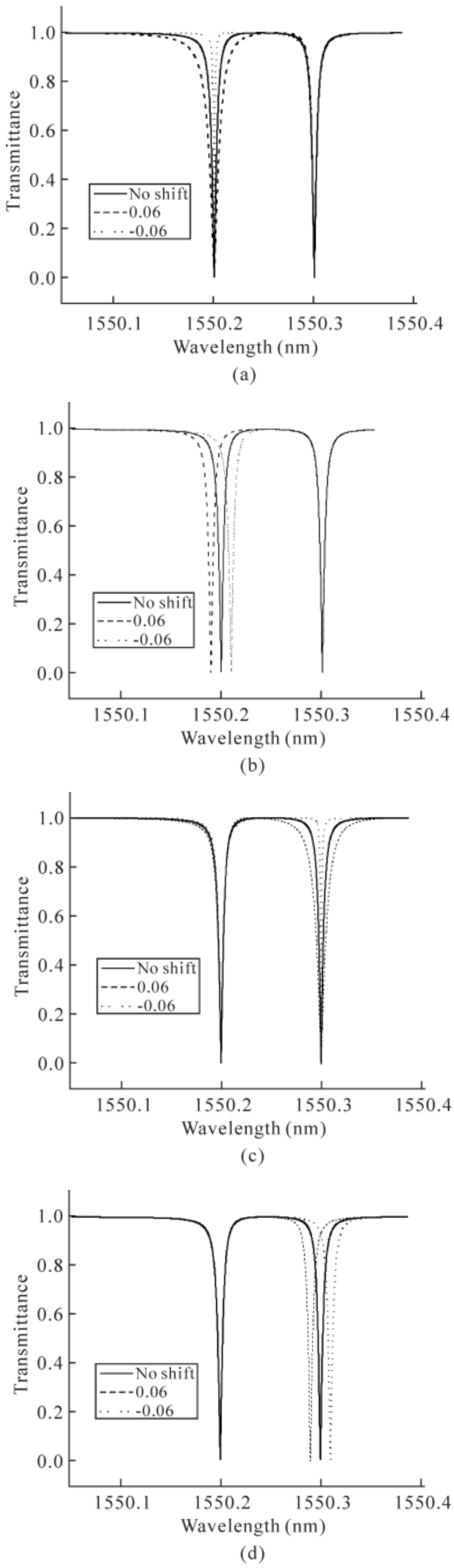

Fig.7 Notch response when (a)k_{12}, (b)k_{11}, (c)k_{22} or (d)k_{21} shifts alone

and second notch points. Meanwhile with the decreasing of k_{12} and k_{22}, the 3-dB bandwidths of notch filter become narrower. k_{11} and k_{21} control the notch positions of the first and second notch points respectively, and with the decreasing of k_{11} and k_{21}, the notch positions of notch filter shift to the long wavelength.

From Fig.8, it can be seen that with the shift of k_{12}, the 3-dB bandwidth of 1550.2 nm is nearly linearly modulated, while that of 1550.3 nm is almost unchanged; with the shift of k_{11}, the position of 1550.2 nm is linearly modulated, while that of 1550.3 nm is almost unchanged, which is consistent with the discussion shown in Fig.7.

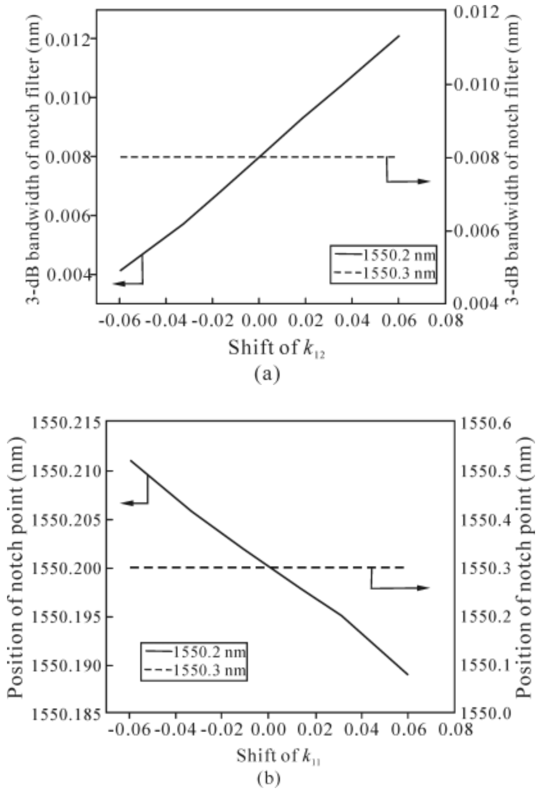

Fig.8 (a) 3-dB bandwidth as a function of the shift of k_{12}; (b) Notch position as a function of the shift of k_{11}

From all the discussion above, it provides us a convenient way to tune the optical notch filter. If the 3-dB bandwidths of the first and second notch points need to be tuned, we can adjust k_{12} and k_{22} accordingly; if the notch positions of the first and second notch points need to be tuned, we can adjust k_{11} and k_{21} accordingly. Meanwhile with the increasing of k_{12} (k_{22}), the 3-dB bandwidth of the first (second) notch point is nearly linearly increased, and with the increasing of k_{11} (k_{21}), the notch position of the first (second) notch point nearly linearly shifts to the short wavelength.

In conclusion, based on digital signal processing theory, a novel method of designing optical notch filter is proposed for Mach-Zehnder interferometer with cascaded optical fiber rings coupled structure. A designing example with two notch points is given based on the cascaded single-fiber-rings. Since the phase shift in single-fiber-ring is sensitive to the environment, a modified structure based on double-fiber-ring is introduced to construct the filter. The influence on the output spectrum is also discussed when the parameters deviate from the designed values and the tuning characteristics of the notch filter are also investigated.

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