Temporal behavior of low-amplitude two-photon screening-photovoltaic grey spatial solitons*

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The time-dependent formation of one-dimensional two-photon screening-photovoltaic (PV) grey spatial solitons under low-amplitude conditions is presented theoretically. The time-dependent propagation equation of two-photon screening-photovoltaic solitons is obtained by the numerical method. The results indicate that as the time evolves, the intensity width of grey screening-photovoltaic spatial solitons decreases monotonously to a minimum value towards the steady state. The higher the ratio of soliton peak intensity to the dark irradiation intensity, the narrower the width of grey solitons within the propagation time.
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Photorefractive (PR) spatial solitons have attracted much attention because of their possible applications for optical switching and routing. To date, there are four types of PR spatial solitons, i.e., quasi-steady-state solitons[1], screening solitons[2], photovoltaic (PV) solitons[3-5] and screening-photorefractive (SP) solitons[6]. All of the above-mentioned solitons are about the steady-state propagation of the solitons. In 2003, Chauvet M[7] reported the temporal behavior of the dark PV solitons under open-circuit. Later, the temporal characteristics of bright PV solitons and SP solitons were investigated by Lu et al[8-10]. At the same time, Castro-Camus and Magana[11] introduced a new model for PR spatial solitons, which involved two-photon PR effect. This model includes a valance band (VB), a conduction band (CB) and an intermediate allowed level (IL). A gating beam is used for maintaining a fixed number of excited electrons from the VB, which are then excited to the CB by the signal beam. Based on the model of two-photon PR spatial solitons, the screening solitons[12], PV solitons[13] and SP solitons[14-16] were predicted one after another. However, the temporal behavior of two-photon PR solitons has not been fully investigated yet. In this paper, we present the time-dependent nonlinear wave equation of the two-photon SP spatial solitons and discuss the temporal characteristics of the normalized intensity profiles and width for grey solitons. The numerical results show that the width of solitons decreases monotonically to a minimum value towards the steady state. The temporal behaviors of the screening solitons and PV solitons can also be obtained from our results.

To start, we consider an optical beam that propagates in a biased two-photon PV-PR crystal along the z-axis and is permitted to diffract only along the x direction. The crystal with the optical c-axis along the x direction is illuminated by the gating beam. Moreover, it is assumed that the optical beam is linearly polarized along the x direction. As usual, we express the optical field of the incident beam in terms of slowly varying envelope ϕ, i.e., $\boldsymbol{E}=\boldsymbol{x}\phi(x,z)\exp(\mathrm{i}kz)$, where $k=k_0n_\mathrm{e}=(2\pi/\lambda_0)n_\mathrm{e}$, n_e is the unperturbed extraordinary index of refraction, and λ_0 is the free-space wavelength. Under these conditions the optical beam satisfies the following envelope evolution equation:

$$\mathrm{i}\phi_z+\frac{1}{2k}\phi_{xx}-\frac{k_0n_\mathrm{e}^3r_{33}E_\mathrm{sc}}{2}\phi=0\,, \tag{1}$$

where $\phi_z=\partial\phi/\partial z$, $\phi_{xx}=\partial^2\phi/\partial x^2$, γ_{33} is the electro-optic coefficient, and $\boldsymbol{E}_\mathrm{sc}=E_\mathrm{sc}\boldsymbol{x}$ is the space-charge field in the crystals. Following Refs.[7-10], the space-charge field in Eq. (1) can be obtained from the time-dependent band-transport

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model. In a non-photovoltaic crystal, the model is represented by the following set of equations[14]

$$\frac{\partial N^+}{\partial t}=(s_1I_1+\beta_1)(N-N^+)-\gamma_1 n_1 N^+-\gamma n N^+ \ , \quad (2)$$

$$\frac{\partial n_1}{\partial t}=(s_1I_1+\beta_1)(N-N^+)+\gamma_2 n(n_{01}-n_1)-\gamma_1 n_1 N^+-(s_2I_2+\beta_2)n_1 \ , \quad (3)$$

$$\frac{\partial n}{\partial t}=(s_2I_2+\beta_2)n_1+\frac{1}{e}\frac{\partial J}{\partial x}-\gamma n N^+-\gamma_2 n(n_{01}-n_1) \ , \quad (4)$$

$$\varepsilon_0\varepsilon_{\mathrm r}\frac{\partial E_{\mathrm{sc}}}{\partial x}=\rho=e(N^+-n-n_1-N_{\mathrm A}) \ , \quad (5)$$

$$J=e\mu n E_{\mathrm{sc}}+\mu k_{\mathrm B}T\frac{\partial n}{\partial x}+\kappa s_2(N-N^+)I_2 \ , \quad (6)$$

$$\frac{\partial J}{\partial x}+\frac{\partial \rho}{\partial t}=0 \ , \quad (7)$$

where N is the donor density, N^+ is the ionized density, $N_{\mathrm A}$ is the acceptor or trap density, and n is the density of the electrons in the CB; n_1 is the density of the electron in the IL; n_{01} is the density of traps in the IL; s_1 and s_2 are photoexcitation cross sections; β_1 and β_2 are the thermoionization probability constants for the transitions of VB-IL and IL-CB, respectively; γ, γ_1 and γ_2 are the recombination factors of the CB-VB, IL-VB, and CB-IL transitions, respectively; D is the diffusion coefficient; μ and e are the electron mobility and charge, respectively, and κ is the photovoltaic constant; ε_0 and $\varepsilon_{\mathrm r}$ are the vacuum and relative dielectric constants, respectively; J is the current density; I_1 is the intensity of the gating beam, which can be considered as a constant; I_2 is the intensity of the soliton beam. According to Poynting's theorem, I_2 can be expressed in terms of ϕ, that is $I_2=(n_{\mathrm e}/2\eta_0)\times|\phi|^2$, where $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$. To establish a time-dependent relation between the space-charge field and the optical intensity, let us recall that in typical PR crystals, n, $n_1 << N^+$, $N_{\mathrm A}$ and $(n_{01}-n_1) << N^+$[10-12]. Simultaneously, we make the hypothesis that the steady-state regime is reached for Eqs.(2) and (3). In such conditions, from Eqs.(2) and (3) we have

$$n_1=\frac{\gamma N^+ n}{s_2I_2+\beta_2} \ . \quad (8)$$

Substituting Eq.(8) into Eq.(2) we get

$$n=\frac{(s_1I_1+\beta_1)(s_2I_2+\beta_2)(N-N^+)}{\gamma N^+(s_2I_2+\beta_2+\gamma_1N^+)} \ . \quad (9)$$

In this case, from Eqs.(5)-(7) we have

$$-\varepsilon_0\varepsilon_{\mathrm r}\frac{\partial^2 E_{\mathrm{sc}}}{\partial x\partial t}=e\mu\frac{\partial(nE_{\mathrm{sc}})}{\partial x}+\mu k_{\mathrm B}T\frac{\partial^2 n}{\partial x^2}+\kappa s_2(N-N^+)\frac{\partial I_2}{\partial x}. \quad (10)$$

Substituting Eq.(9) into Eq.(10) we get

$$T_{\mathrm d}\eta\frac{\partial^2 E_{\mathrm{sc}}}{\partial x\partial t}+\frac{\partial}{\partial x}\left(\frac{I_2+I_{2\mathrm d}}{I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2}E_{\mathrm{sc}}\right)+\frac{k_{\mathrm B}T\gamma_1N_{\mathrm A}}{es_2(I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)}\frac{\partial^2 I_2}{\partial x^2}+\frac{s_2E_{\mathrm p}}{s_1I_1+\beta_1}\frac{\partial I_2}{\partial x}=0 \ , \quad (11)$$

where $T_{\mathrm d}=(\varepsilon_0\varepsilon_{\mathrm r}/e\mu)[\gamma N_{\mathrm A}/\beta_2(N-N_{\mathrm A})]$, $\eta=\beta_2/(s_1I_1+\beta_1)$, $I_{2\mathrm d}=\beta_2/s_2$ is dark irradiance, and $E_{\mathrm p}=\kappa\gamma N_{\mathrm A}/e\mu$ is photovoltaic field.

The integral of Eq.(11) leads to

$$T_{\mathrm d}\eta\frac{\partial E_{\mathrm{sc}}}{\partial t}+\frac{I_2+I_{2\mathrm d}}{I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2}E_{\mathrm{sc}}+\frac{k_{\mathrm B}T\gamma_1N_{\mathrm A}}{es_2(I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)}\frac{\partial I_2}{\partial x}+\frac{s_2E_{\mathrm p}}{s_1I_1+\beta_1}I_2=C_1 \ , \quad (12)$$

where C_1 is integration constant. In the steady state and $x\to\pm\infty$, we have $J(x\to\pm\infty)=J_\infty$ and $E_{\mathrm{sc}}(x\to\pm\infty)=E_0$. From these conditions, we can obtain $C_1=E_0\left(\frac{I_{2\infty}+I_{2\mathrm d}}{I_{2\infty}+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2}\right)+\frac{s_2E_{\mathrm p}}{s_1I_1+\beta_1}I_{2\infty}$. By ignoring the diffusion effect and integrating Eq. (12) we get

$$E_{\mathrm{sc}}=\left[E_0\frac{(I_{2\infty}+I_{2\mathrm d})(I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)}{(I_2+I_{2\mathrm d})(I_{2\infty}+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)}+E_{\mathrm p}\frac{s_2(I_{2\infty}-I_2)(I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)}{(s_1I_1+\beta_1)(I_2+I_{2\mathrm d})}\right]\times\left\{1-\exp\left[-\frac{I_2+I_{2\mathrm d}}{(I_2+I_{2\mathrm d}+\gamma_1N_{\mathrm A}/s_2)\eta}\frac{t}{T_{\mathrm d}}\right]\right\} \ . \quad (13)$$

Substitute Eq.(13) into Eq.(1), and adopt the following dimensionless coordinates and variables: $s=x/x_0$, $\xi=z/(kx_0^2)$, and $U=(2\eta_0I_{2\mathrm d}/n_{\mathrm e})^{-1/2}\phi$, where x_0 is an arbitrary spatial width. Under these conditions, the following time-dependent dynamical evolution equation can be obtained:

$$\mathrm iU_\xi+\frac{1}{2}U_{ss}-\left[\beta\frac{(1+\rho)(1+\sigma+|U|^2)}{(1+\sigma+\rho)(1+|U|^2)}+\alpha\eta\frac{(\rho-|U|^2)(1+|U|^2+\sigma)}{1+|U|^2}\right]U\times\left[1-\exp\left(-\frac{1+|U|^2}{1+\sigma+|U|^2}\frac{\tau}{\eta}\right)\right]=0 \ , \quad (14)$$

where $\alpha=(k_0x_0)^2(n_{\mathrm e}^4r_{33}/2)E_{\mathrm p}$, $\beta=(k_0x_0)^2(n_{\mathrm e}^4r_{33}/2)E_0$, $\sigma=\gamma_1N_{\mathrm A}/\beta_2$, and $\tau=t/T_{\mathrm d}$. In the low amplitude regime, that is $|U|^2<<1$, Eq.(14) can be simplified as

$$\mathrm{i}U_{\xi}+\frac{1}{2}U_{ss}-\{[\frac{\beta(1+\rho)}{(1+\rho+\sigma)}+\alpha\eta\rho](1+\sigma-\sigma|U|^2)-\alpha\eta(1+\sigma)|U|^2\}U\times\{1-\exp[-\frac{1+\sigma+\sigma|U|^2}{(1+\sigma)^2}\frac{\tau}{\eta}]\}=0\ . \quad (15)$$

For grey solitons, the wave power density attains a constant value $I_{2\infty}$ at infinity, resulting in a finite ρ. To obtain the solutions, U is expressed in the following form[2]

$$U(s,\xi)=\rho^{1/2}y(s)\exp[\mathrm{i}(\gamma\xi+\int\frac{Q\,\mathrm{d}s}{y^2(s)})]\ , \quad (16)$$

where Q is a real constant to be determined, and the normalized amplitude $y(s)$ is an even function of s and it satisfies the condition $y(s\to\pm\infty)=1$. All the derivatives of y are zero at infinity. Moreover, we assume that $y^2(s=0)=m$ is the grayness of the solitons and $y'(0)$. Substituting Eq.(16) into Eq.(15), the following equation is obtained

$$\frac{\mathrm{d}^2y}{\mathrm{d}s^2}=2\gamma y+\frac{Q^2}{y^3}+\{[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}](1+\sigma-\sigma\rho y^2)-2\alpha\eta(1+\sigma)\rho y^2\}y\times\{1-\exp[-\frac{1+\sigma+\sigma\rho y^2}{(1+\sigma)^2}\frac{\tau}{\eta}]\}\ . \quad (17)$$

Using the boundary conditions at infinity, we can deduce from Eq.(17) that

$$Q^2=-2\gamma-\{[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}](1+\sigma-\sigma\rho)-\alpha\eta(1+\sigma)\rho\}\times\{1-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}. \quad (18)$$

Eq.(17) can be integrated and we get

$$(\frac{\mathrm{d}y}{\mathrm{d}s})^2=2\gamma(y^2-\frac{1}{y^2}-2)+\{[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}](1+\sigma-\sigma\rho)-2\alpha\eta\rho\}\times\{1-\exp[-\frac{1+\sigma+\sigma\rho}{\eta(1+\sigma)^2}\tau]\}(\frac{1}{y^2}-1)+[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}][(1+\sigma)(y^2-1)-\frac{\sigma\rho}{2}(y^4-1)]-\alpha\eta\rho(1+\sigma)(y^4-1)-\{[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}]\frac{(1+\sigma)^2\eta}{\tau}+2\alpha\frac{\eta^2}{\tau}\frac{(1+\sigma)^3}{\sigma}\}\times\{y^2\exp[-\frac{1+\sigma+\sigma\rho y^2}{(1+\sigma)^2}\frac{\tau}{\eta}]-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}+\{[2\alpha\eta\rho+\frac{2\beta(1+\rho)}{1+\rho+\sigma}]\frac{(1+\sigma)^3\eta}{\sigma\rho\tau^2}[\tau-(1+\sigma)\eta]-2\alpha\frac{\eta^3}{\tau^2}\frac{(1+\sigma)^5}{\sigma^2\rho}\}\times\{\exp[-\frac{1+\sigma+\sigma\rho y^2}{(1+\sigma)^2}\frac{\tau}{\eta}]-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}\ . \quad (19)$$

Using the boundary conditions $y^2(0)=m$ and $y'(0)=0$, we have

$$\gamma=\frac{1}{m-1}\{[\alpha\eta\rho+\frac{\beta(1+\rho)}{1+\rho+\sigma}](1+\sigma-\sigma\rho)-\alpha\eta(1+\sigma)\rho\}\times\{1-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}-\frac{m}{m-1}[\alpha\eta\rho+\frac{\beta(1+\rho)}{1+\rho+\sigma}]\times[(1+\sigma)-\frac{\sigma\rho}{2}(m+1)]+\frac{1}{2}\alpha\eta\rho(1+\sigma)\frac{m+1}{m-1}+\frac{m}{(m-1)^2}\{[\alpha\eta\rho+\frac{\beta(1+\rho)}{1+\rho+\sigma}]+\alpha\frac{\eta^2}{\tau}\frac{(1+\sigma)^3}{\sigma}\}\times\{m\exp[-\frac{1+\sigma+\sigma\rho m}{(1+\sigma)^2}\frac{\tau}{\eta}]-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}-\frac{m}{(m-1)^2}\{[\alpha\eta\rho+\frac{\beta(1+\rho)}{1+\rho+\sigma}]\frac{\eta(1+\sigma)^3}{\sigma\rho\tau^2}[\tau-(1+\sigma)\eta]-\alpha\frac{\eta^3}{\tau^2}\frac{(1+\sigma)^5}{\sigma^2\rho}\}\times\{\exp[-\frac{1+\sigma+\sigma\rho m}{(1+\sigma)^2}\frac{\tau}{\eta}]-\exp[-\frac{1+\sigma+\sigma\rho}{(1+\sigma)^2}\frac{\tau}{\eta}]\}\ . \quad (20)$$

From Eq.(19) we can obtain the normalized field profile $y(s)$ of grey solitons by numerical integration. To illustrate our results, we take the following parameters[14]: $n_e=2.2$, $r_{33}=30\times10^{-12}\ \mathrm{mV^{-1}}$, $E_0=-2\times10^6\ \mathrm{Vm^{-1}}$, $E_p=-4\times10^6\ \mathrm{Vm^{-1}}$, $s_1=s_2=3\times10^{-4}\ \mathrm{m^2W^{-1}s^{-1}}$, $\beta_1=0.05\ \mathrm{s^{-1}}$, $\beta_2=0.05\ \mathrm{s^{-1}}$, $\gamma_1=3.3\times10^{-17}\ \mathrm{m^3s^{-1}}$, $N_A=10^{22}\ \mathrm{m^{-3}}$, $\lambda_0=0.5\ \mu\mathrm{m}$, $x_0=10\ \mu\mathrm{m}$, $I_1=1\times10^6\ \mathrm{Wm^{-2}}$, And then we can obtain $\alpha=-22.2$, $\beta=-11.1$, $\sigma\sim10^6$, $\eta=1.67\times10^{-4}$. The dark irradiance I_{2d} can be modulated by using incoherent uniform illumination[3], so that σ can be adjusted. Here, we take $\sigma=10^4$. Fig.1 shows the soliton normalized intensity profiles when $\rho=0.01$, $\alpha=-22.2$, $\beta=-11.1$, and $m=0.5$. Fig.2 is the full width of half maximum (FWHM) of intensity for $m=0.5$ under different ρ. It can be shown that the FWHM decreases monotonically to a minimum value towards the steady state for a low-amplitude regime. The higher the value of ρ is, the narrower the width of grey solitons is within propagation time.

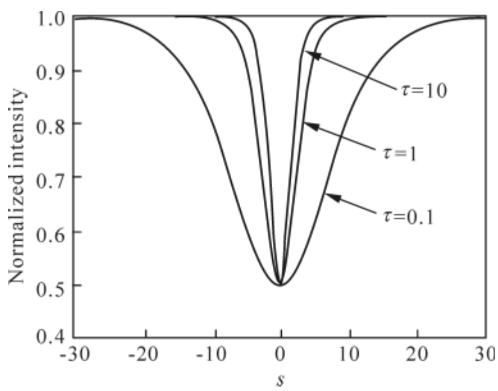

Fig.1 Normalized intensity profiles of grey solitons under different τ values

Fig.3 is the FWHM of grey solitons as a function of τ for different values of m. Fig.3 reveals that within the same propagation time, the smaller the m value is, the narrower the FWHM of solitons is.

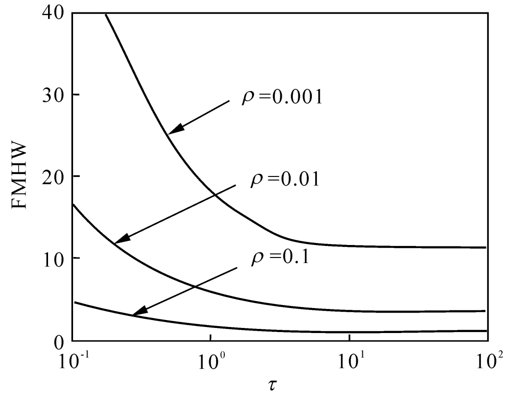

Fig.2 FWHM of intensity for screening-photovoltaic grey spatial solitons versus τ under ρ = 0.001, 0.01 and 0.1

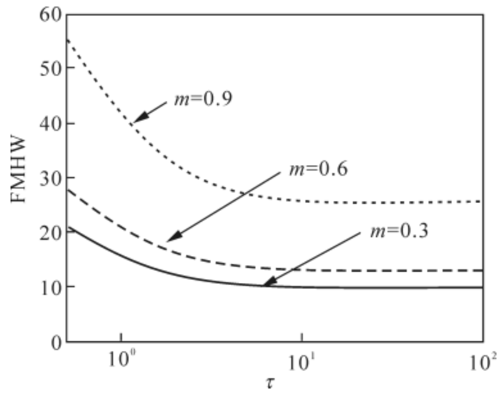

Fig.3 FWHM of intensity for screening-photovoltaic grey spatial solitons versus τ under different m values

Fig.4 depicts the temporal behavior of closed-circuit PV grey solitons at α= -22.2, β=0, and m = 0.5. Fig.5 depicts the temporal behavior of screening grey solitons at α= 0, β= -11.1, and m = 0.5. Those figures show that the temporal behaviors of screening grey solitons and PV grey solitons are similar to that of SP grey solitons.

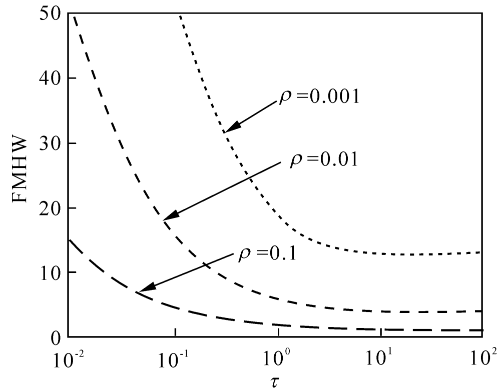

Fig.4 FWHM of intensity for photovoltaic grey spatial solitons versus τ under ρ = 0.001, 0.01 and 0.1

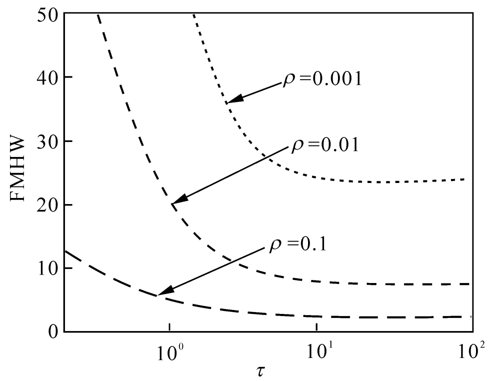

Fig.5 FWHM of intensity for screening grey spatial solitons versus τ under ρ = 0.001, 0.01 and 0.1

In conclusion, the time-dependent propagation equation in biased two-photon PV-PR crystals is obtained. We have demonstrated the temporal behavior of grey SP spatial solitons in low-amplitude regime. We find that the FWHM decreases monotonically to a minimum value towards the steady state. Within the same propagation time, the bigger the value of ρ or the smaller the value of m is, the narrower the intensity width of the grey solitons is. Moreover, the temporal behaviors of grey screening solitons and grey PV solitons can also be obtained from our theory.

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