Physical conditions for sources radiating a cosh-Gaussian model beam

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(Received 29 November 2010)

Based on the coherence theory of diffracted optical field and the model for partially coherent beams, analytical expressions for the cross-spectral density and the irradiance spectral density in the far zone are derived, respectively. Utilizing the theoretical model of radiation from secondary planar sources, the physical conditions for sources generating a cosh-Gaussian (CHG) beam are investigated. Analytical results demonstrate that the parametric conditions strongly depend on the coherence property of sources. When almost coherence property is satisfied in the source plane, the conditions are the same as those for fundamental Gaussian beams; when partial coherence or almost incoherence property is satisfied in the spatial source plane, the conditions are the same as those for Gaussian-Schell model beams. The results also indicate that the variance of cosine parameters has no influence on the conditions. Our results may provide potential applications for some investigations such as the modulations of cosh-Gaussian beams and the designs of source beam parameters.
Document code: A **Article ID:** 1673-1905(2011)04-0308-5
DOI 10.1007/s11801-011-0145-5

In recent years, the quantitative description and quality control of laser beams have attracted much attention, because of the invaluable applications to many fields such as remote sensing, objective tracking and laser ladar. On the other hand, experimental methods for generating some special distributed beams have been reported and have been applied in some works, for example, optical measurements, free space optical communications, etc. Based on the Helmholtz equations, Casperson et al[1,2] derived one particular solution of Hermite-sine-Gaussian model beams, by disposing and solving the wave equations. In the same report, they further pointed out that this class of beam could be produced in the experimental aspect, by performing the optical systems involving a sine-Gaussian aperture and a laser cavity[3]. Among the existing solutions of such laser beams, the cosh-Gaussian (CHG) beam has shown many advantages, and the flat-topped Gaussian and hollow Gaussian beams with certain spatial intensity distributions at the output plane have been obtained by modulating parameters of CHG beams[4,5], in order to trap small particles. Based on the second-order statistical moments of apertured laser beams, B. Lu et al[6,7] studied the beam factor of CHG beams. In free-space optical communications, the issue that how to control the quality of laser beams in atmosphere has become a hot-spot of recent researches[8,9]. In addition, the sources radiating highly directional lasers have found wide applications, such as laser guidance, projective aiming, remote sensing and free-space optical communications. Wolf et al[10] have taken detailed researches on the topic. Subsequently, Korotkova et al[11] further extended the analytical condition for generating scalar Gaussian Schell-model beams into the one for radiating the stochastic electromagnetic beams with arbitrary distributed polarization. They also discussed their condition according to different separated analytical forms. G. Wu et al[12] derived the limit condition for sources radiating electromagnetic J_0-correlated Schell-model beams. In this paper, based on the radiation model of secondary planar sources and the theory of statistical coherence of optical field, the far-field cross-spectral density function and the radiation spectral density of CHG beams are derived in analytical forms, respectively. Finally, the parametric condition for generating CHG beams is obtained.

It is assumed that a planar and secondary source is located at the initial reference plane z=0, radiating a laser beam which propagates towards the half plane of z>0. Here, the planar and secondary source can be approximately regarded as the frequently-used experimental laser source. It requires that this laser source should involve a certain aperture, and

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its generated beam directly exits or is modulated by complex optical systems[10]. Characteristics of the radiated beam can be weighed by the cross-spectral density function at two points [10-12].

$$W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\left\langle E^*(\boldsymbol{\rho}_1,\omega)E(\boldsymbol{\rho}_2,\omega)\right\rangle \quad , \tag{1}$$

where ω is the circular frequency of beams, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ denote the position vectors located at the source plane z=0, E is the electric field of scalar waves, the asterisk denotes the complex conjunction and the bracket represents taking the ensemble average of the optical field. The cross-spectral density function of initial sources radiating partially coherent CHG beams can be expressed as[13]

$$W^{(0)}(x_1,y_1,x_2,y_2,\omega)=I_0\exp\left(-\frac{x_1^2+y_1^2}{2\sigma^2}-\frac{x_2^2+y_2^2}{2\sigma^2}\right)\times$$
$$\exp\left[-\frac{(x_1-x_2)^2+(y_1-y_2)^2}{2\delta_0^2}\right]\times$$
$$\cosh(\Omega_0 x_1)\cosh(\Omega_0 y_1)\cosh(\Omega_0 x_2)\cosh(\Omega_0 y_2) \quad , \tag{2}$$

where σ is the waist width corresponding to the Gaussian part, δ_0 denotes the spatial coherence length, Ω_0 represents the parameter related with the cosine characteristics of beams. Eq.(2) is based on the assumption that the coherence property of CHG beams obeys the Gaussian Schell-model distribution. When the waist width is larger enough compared with the wavelength λ, the far-field cross-spectral density function of light from the planar and secondary source can be represented as[10]

$$W^{(\infty)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)=\left(\frac{2\pi}{k}\right)^2\int_\sigma\int_\sigma W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\frac{\exp[\mathrm{i}k(\boldsymbol{R}_2-\boldsymbol{R}_1)]}{\boldsymbol{R}_1\boldsymbol{R}_2}\times$$
$$\cos\theta_1'\cos\theta_2'\,\mathrm{d}^2\boldsymbol{\rho}_1\,\mathrm{d}^2\boldsymbol{\rho}_2\ , \tag{3}$$

where $k=2\pi/\lambda$ is the wave number, and $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ denote two position vectors in the far field.

$$\boldsymbol{R}_1=|\boldsymbol{r}_1-\boldsymbol{\rho}_1|,\quad \boldsymbol{R}_2=|\boldsymbol{r}_2-\boldsymbol{\rho}_2|, \tag{4}$$

represent the distances between points at source plane $S_1(\boldsymbol{\rho}_1)$, $S_2(\boldsymbol{\rho}_2)$ and points in the far field $P(\boldsymbol{r}_1)$, $P(\boldsymbol{r}_2)$, respectively. θ_1' and θ_2' denote the radial angles between S_1P_1, S_2P_2 and the z axis, respectively. The geometrical relations of the far field radiated from sources are depicted in Fig.1.

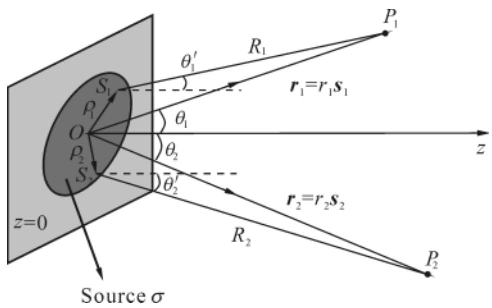

Fig.1 Schematic diagram for geometrical relations of the far field radiated from a finite source

In the far-field region, the scale or distance of the propagation line is larger enough compared with λ, so the factors $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ in Eq.(3) can be further approximated as[10]

$$\boldsymbol{R}_1\approx\boldsymbol{r}_1-\rho_1\cdot\boldsymbol{s}_{1\perp},\quad \boldsymbol{R}_2\approx\boldsymbol{r}_2-\rho_2\cdot\boldsymbol{s}_{2\perp}, \tag{5}$$

where $\boldsymbol{s}_{i\perp}=s_{ix}\boldsymbol{e}_x+s_{iy}\boldsymbol{e}_y$ (i=1, 2) denotes the projective components of unit vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ onto the source plane z=0. Furthermore, the denominator part in Eq.(3) can be approximately replaced by $\boldsymbol{R}_1\approx r_1$, $\boldsymbol{R}_2\approx r_2$, $\theta'_1\approx\theta_1$ and $\theta'_2\approx\theta_2$. Based on above treatments, the cross-spectral density function of the radiated far-field can be rearranged as[10-12]

$$W^{(\infty)}(r_1\boldsymbol{s}_1,r_2\boldsymbol{s}_2,\omega)=(2\pi k)^2\cos\theta_1\cos\theta_2\times$$
$$\left\{\widetilde{W}^{(0)}(-k\boldsymbol{s}_{1\perp},k\boldsymbol{s}_{2\perp},\omega)\frac{\exp[\mathrm{i}k(r_2-r_1)]}{r_1r_2}\right\}, \tag{6}$$

where

$$\widetilde{W}^{(0)}(\boldsymbol{f}_1,\boldsymbol{f}_2,\omega)=\frac{1}{(2\pi)^4}\iint W^{(0)}(\rho_1,\rho_2,\omega)\times$$
$$\exp[-\mathrm{i}(\boldsymbol{f}_1\cdot\rho_1+\boldsymbol{f}_2\cdot\rho_2)]\mathrm{d}^2\rho_1\,\mathrm{d}^2\rho_2, \tag{7}$$

is called as the four-dimensional spatial Fourier transformation of cross-spectral density function $W^{(0)}$. $\boldsymbol{f}_i=f_{ix}\boldsymbol{e}_x+f_{iy}\boldsymbol{e}_y$ (i = 1,2) represents the vector components in the frequency domain in accordance with two reference points. Substituting Eq.(2) into Eq.(7), after tedious calculations, the four-dimensional spatial Fourier transformation of $W^{(0)}$ can be expressed as

$$\widetilde{W}^{(0)}(f_{1x},f_{2x},f_{1y},f_{2y},\omega)=\frac{I_0}{2\pi^2}\frac{\sigma^2}{\frac{1}{8\sigma^2}+\frac{1}{2\delta_0^2}}\left\{\exp\left[-\frac{(f_{1x}-f_{2x})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\times\right.$$

$$\exp\left[2\sigma^2\left(\Omega_0-\mathrm{i}\frac{f_{1x}+f_{2x}}{2}\right)^2\right]+$$

$$\exp\left[-\frac{(f_{1x}-f_{2x})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0+\mathrm{i}\frac{f_{1x}+f_{2x}}{2}\right)^2\right]+$$

$$\exp\left[-\frac{(f_{1x}+f_{2x})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0-\mathrm{i}\frac{f_{1x}-f_{2x}}{2}\right)^2\right]+$$

$$\left.\exp\left[-\frac{(f_{1x}+f_{2x})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0+\mathrm{i}\frac{f_{1x}-f_{2x}}{2}\right)^2\right]\right\}\times$$

$$\left\{\exp\left[-\frac{(f_{1y}-f_{2y})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0-\mathrm{i}\frac{f_{1y}+f_{2y}}{2}\right)^2\right]+\right.$$

$$\exp\left[-\frac{(f_{1y}-f_{2y})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0+\mathrm{i}\frac{f_{1y}+f_{2y}}{2}\right)^2\right]+$$

$$\exp\left[-\frac{(f_{1y}+f_{2y})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0-\mathrm{i}\frac{f_{1y}-f_{2y}}{2}\right)^2\right]+$$

$$\left.\exp\left[-\frac{(f_{1y}+f_{2y})^2}{4\left(\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}\right)}\right]\exp\left[2\sigma^2\left(\Omega_0+\mathrm{i}\frac{f_{1y}-f_{2y}}{2}\right)^2\right]\right\}. \tag{8}$$

Subsequently substituting Eq.(8) into Eq.(6) and letting $\boldsymbol{s}_1=\boldsymbol{s}_2=\boldsymbol{s}$, after mathematical treatments, the far-field spectral density can be represented by

$$S^{(\infty)}(\boldsymbol{r},\omega)=W^{(\infty)}(r\boldsymbol{s},\omega)=\frac{k^2\cos^2\theta}{r^2}I_0\frac{8\sigma^2}{\frac{1}{8\sigma^2}+\frac{1}{2\delta_0^2}}\times$$

$$\left\{\exp\left(-\frac{k^2}{\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}}s_x^2\right)+\exp\left(2\Omega_0^2\sigma^2\right)\exp\left(-2\sigma^2k^2s_x^2\right)\times\right.$$

$$\left.\cos\left(4\Omega_0\sigma^2k^2s_x^2\right)\right\}\times\left\{\exp\left(-\frac{k^2}{\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}}s_y^2\right)+\exp\left(2\Omega_0^2\sigma^2\right)\times\right.$$

$$\left.\exp\left(-2\sigma^2k^2s_y^2\right)\cos\left(4\Omega_0\sigma^2k^2s_y^2\right)\right\}. \tag{9}$$

Observing Eq.(9) and utilizing the inequality of cosine functions

$$\cos\left(4\Omega_0\sigma^2k^2s_x^2\right)\le 1,\quad \cos\left(4\Omega_0\sigma^2k^2s_y^2\right)\le 1, \tag{10}$$

and substituting Eq.(10) into Eq.(9), it yields

$$S^{(\infty)}(\boldsymbol{r},\omega)\le\frac{k^2\cos^2\theta}{r^2}I_0\frac{8\sigma^2}{\frac{1}{8\sigma^2}+\frac{1}{2\delta_0^2}}\left\{\exp\left(-\frac{k^2}{\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}}\right)+\right.$$

$$\exp\left(-\frac{k^2}{\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}}s_x^2-2\sigma^2k^2s_y^2\right)\exp\left(2\Omega_0^2\sigma^2\right)+$$

$$\exp\left(-\frac{k^2}{\frac{1}{2\sigma^2}+\frac{2}{\delta_0^2}}s_y^2-2\sigma^2k^2s_x^2\right)\exp\left(2\Omega_0^2\sigma^2\right)+$$

$$\left.\exp\left(4\Omega_0^2\sigma^2\right)\exp\left(-2\sigma^2k^2\right)\right\}. \tag{11}$$

Subsequently, we mainly focus on the coherence property at the source plane z=0, in order to obtain the parametric condition for sources which can generate CHG beams.

When source parameters satisfy the relationship $\delta >> \sigma$, i.e., the coherence length is large enough compared with the waist width at the source plane, this class of source can be regarded as almost coherent in its spatial domain. Under this condition, the far-field spectral density can be further simplified as

$$S^{(\infty)}(\boldsymbol{r},\omega) \le \frac{64k^2}{r^2} I_0 \sigma^4 \{\exp(-2\sigma^2 k^2) + \exp(-2\sigma^2 k^2 s_x^2 - 2\sigma^2 k^2 s_y^2)\exp(2\Omega_0^2\sigma^2) + \exp(-2\sigma^2 k^2 s_y^2 - 2\sigma^2 k^2 s_x^2)\exp(2\Omega_0^2\sigma^2) + \exp(4\Omega_0^2\sigma^2)\exp(-2\sigma^2 k^2)\} = \frac{64k^2}{r^2} I_0 \sigma^4 [1+\exp(2\Omega_0^2\sigma^2)]^2 \exp(-2\sigma^2 k^2) \ . \tag{12}$$

In the procedure of Eq.(12), a relation is utilized, which further depends on two unit vectors

$$s_x^2 + s_y^2 = |\boldsymbol{s}^2| = 1. \tag{13}$$

It is known that only when the unit vector $\boldsymbol{s}$ is in a narrow solid angle about the z axis, the far-field spectral density $S^{\infty}(\boldsymbol{r},\omega)$ can have appreciable values. Comparably, when the angle between the unit vector $\boldsymbol{s}$ and the z axis reaches a certain magnitude, the far-field spectral density tends to zero, just in order to ensure high directional property of radiated laser beams. Therefore, an approximation can be made in Eq.(12): $\cos\theta \approx 1$. By observing the last step of derivations in Eq.(12), it seems not hard for us to understand that in order to generate the CHG beam in the far field, the following relation should be satisfied for source parameters

$$\frac{1}{2\sigma^2} << k^2, \to \ \delta_0^2 >> \sigma^2 >> \frac{\lambda^2}{8\pi^2}. \tag{14}$$

When source parameters satisfy the relationship $\delta_0 << \sigma$, i.e., the coherence length is small enough compared with the waist width at the source plane, this class of source can be regarded as almost incoherent in its spatial domain. Under this condition, the following approximation can be further made to the spectral density in Eq.(11)

$$S^{(\infty)}(\boldsymbol{r},\omega) \le \frac{16k^2}{r^2} I_0 \delta_0^2 \sigma^2 \left\{ \exp\left(-\frac{\delta_0^2}{2}k^2\right) + \exp\left(-\frac{\delta_0^2}{2}k^2 s_x^2 - 2\sigma^2 k^2 s_y^2\right)\exp(2\Omega_0^2\sigma^2) + \exp\left(-\frac{\delta_0^2}{2}k^2 s_y^2 - 2\sigma^2 k^2 s_x^2\right)\exp(2\Omega_0^2\sigma^2) + \exp(4\Omega_0^2\sigma^2)\exp(-2\sigma^2 k^2)\right\} \approx$$

$$\frac{16k^2}{r^2} I_0 \delta_0^2 \sigma^2 \left\{ \exp\left(-\frac{\delta_0^2}{2}k^2\right) + \exp(-2\sigma^2 k^2 s_y^2)\exp(2\Omega_0^2\sigma^2) + \exp(-2\sigma^2 k^2 s_x^2)\exp(2\Omega_0^2\sigma^2)\right\} \ . \tag{15}$$

In order to generate the CHG beam in the far-field, source parameters should obey the following conditions simultaneously

$$\frac{2}{\sigma^2} << k^2, \ \frac{1}{2\sigma^2 k^2 s_x^2} << k^2, \ \frac{1}{2\sigma^2 k^2 s_y^2} << k^2. \tag{16}$$

Eq.(16) can be further simplified by utilizing the relation between unit vectors of Eq.(13)

$$\sigma^2 >> \delta_0^2 >> \frac{\lambda^2}{2\pi^2}. \tag{17}$$

It is not hard for us to conclude that deductions of Eqs.(14) and (17) depend on the assumption that the source is almost coherent and incoherent in its spatial domain, respectively. Generally speaking, if only either Eq.(14) or Eq.(17) is satisfied, the CHG beam can be generated in the far field. Therefore, one can obtain the following equation by incorporations of both Eq.(14) and Eq.(17)

$$\frac{1}{4\sigma^2} + \frac{1}{\delta_0^2} << \frac{4\pi^2}{\lambda^2}. \tag{18}$$

Eq.(18) is the major result of this paper. Its physical meaning can be explained like this: when a light source can generate a CHG beam in the far field, the waist width σ and the coherence length δ_0 should satisfy certain analytical conditions. The conditions incorporate solutions for two limit cases: the source is almost coherent and incoherent. Therefore, this condition can be applicable to either the coherent or partially coherent cases. Eq.(14) indicates that when the laser source is regarded as almost coherent, conditions for generating the CHG beam are the same as those for generating fundamental Gaussian beams[10]. Comparing Eq.(18) with Ref.[11], one can know that when the source is partially coherent or almost incoherent, our condition is the same as that for generating Gaussian Schell-model beams. Besides, the analytical form of Eq.(18) has no relevance with the cosine parameter Ω_0 of the CHG beam. As a result, one can conclude that variances of the cosine parameter Ω_0 have no impact on the structure of our condition for generating CHG laser beams.

In this paper, by utilizing the coherence theory of a planar and secondary source, we obtain the analytical condition for sources generating a partially coherent cosh-Gaussian

(CHG) beam. The particular cases related with almost coherent and incoherent source conditions are discussed in detail, respectively. The cosine parameter of CHG beams has no relevance with this condition. Results in the paper may provide flexible approach and significance in experimental modulations of CHG beams or objective detections by utilizing CHG beams, etc.

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