Training sequence based channel estimation for indoor visible light communication system*

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Channel estimation is a key technology in indoor wireless visible light communications (VLCs). Using the training sequence (TS), this paper investigates the channel estimation in indoor wireless visible light communications. Based on the propagation and signal modulation characteristics of visible light, a link model for the indoor wireless visible light communications is established. Using the model, three channel estimation methods, i.e., the correlation method, the least square (LS) method and the minimum mean square error (MMSE) method, are proposed. Moreover, the performances of the proposed three methods are evaluated by computer simulation. The results show that the performance of the correlation method is the worst, the LS method is suitable for higher signal to noise ratio (SNR), and the MMSE method obtains the best performance at the expense of highest complexity.

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The indoor visible light communiation (VLC) systems[1-3] still have many critical issues. Generally speaking, the indoor VLC environment is a confined space and LED is not only used as a lighting device, but also to be used as a communication device. The optical signals of LED travel via different paths and arrive at the receiver at different instances in time[4]. Experimental results have shown that the multipath propagation may cause the inter-symbol-interference (ISI), which will degrade the VLC system performance significantly when the original data rate is high[5]. In order to eliminate the effects of ISI, the instantaneous channel impulse response must be obtained before the recovery of the transmitted information. Obviously, the channel estimation plays a critical role in VLC systems. Refs.[6,7] described an iterative method for infrared channel estimation. However, the effect of the irregular shape of the objects and the human's activity were ignored, which makes the results not suitable for real indoor communication systems. As far as we know, no similar results for channel estimation in VLC systems are available.

Since the optical propagation environment is complicated and time-varying, this paper proposes a training sequence (TS)-based channel estimation for indoor VLC systems. Considering the propagation property of light and the characteristics of optical modulation, a mathematical channel model for indoor VLC systems is established. Further, based on the established link model, three channel estimation methods, i.e., the correlation method, the least square (LS) method and the minimum mean square error (MMSE) method, are proposed and evaluated.

The intensity modulation and direct detection (IM/DD) modulation is considered for indoor VLC systems, in which the desired waveform is modulated onto the instantaneous power of the carrier and a photo-detector produces a current proportional to the received instantaneous power. The binary information is modulated into "0" and "1". As illustrated in Fig.1, the received signal of the indoor VLC systems is given by[1]

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$$y(t) = a(t)*h(t) + n(t) \quad t \geqslant 0 , \tag{1}$$

where $*$ denotes convolution, $a(t) \in \mathrm{R}^{+} \cup \{0\}$ denotes the instantaneous optical power in the transmitter, $h(t) \in \mathrm{R}^{+} \cup \{0\}$ is the channel impulse response and $n(t) \in \mathrm{R}$ is the optical channel noise. Digitally sampling the continuous signals in Eq.(1), the signal of the ith sampling instant with the sampling period T_s can be expressed as

$$y(iT_s) = \sum_m h(mT_s) a(iT_s - mT_s) + n(iT_s) \quad . \tag{2}$$

In order to simplify the expression, Eq.(2) can be further expressed as

$$y_i = \sum_m h_m a_{i-m} + n_i \quad . \tag{3}$$

Assuming L is the number of received samples, the received sequence of signal samples can be denoted as $\boldsymbol{y} = [y_0, y_1, \cdots, y_{L-1}]^{\mathrm{T}}$. If the channel order is J, let $\boldsymbol{h} = [h_0, h_1, \cdots, h_{J-1}]^{\mathrm{T}}$ be the channel impulse response, and the initially transmitted sequence is $\boldsymbol{a} = [a_{-(J-1)}, \cdots, a_0, \cdots, a_{N-1}]$, where $a_k \in \{0, 1\}$. Eq.(3) can be rewritten as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{h} + \boldsymbol{n} , \tag{4}$$

where $\boldsymbol{n} = [n_0, n_1, \cdots, n_{L-1}]^{\mathrm{T}}$ is the noise vector, and $\boldsymbol{A}$ is an $L \times J$ matrix whose rows correspond to different shifts of the transmitter sequence of symbols. The Toeplitz matrix $\boldsymbol{A}$ can be expressed as follows:

$$\boldsymbol{A} = \begin{bmatrix} a_0 & \cdots & a_{-(J-2)} & a_{-(J-1)} \\ a_1 & \cdots & a_{-(J-3)} & a_{-(J-2)} \\ \vdots & & \vdots & \vdots \\ a_{L-1} & \cdots & a_{L-J+1} & a_{L-J} \end{bmatrix}_{L \times J} . \tag{5}$$

Without loss of generality, the moment that the optical transmitters begin to send information is assumed to be the instant of $i = 0$. In other words, the transmitter has not sent any symbol yet when the condition $i-m<0$ in Eq.(2) holds. Therefore, the set of J-1 symbols $\{a_{-(J-1)}, \cdots, a_{-1}\}$ is required as the precursor.

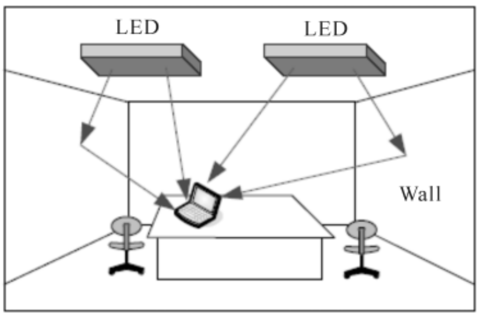

Fig.1 Indoor visible light communication system

In data aided channel estimation, the known information to the receiver is inserted in information symbols so that the current channel state can be estimated. By analyzing the relationship between the known training sequence and the received symbols, the instantaneous channel impulse response can be estimated[8].

One of the simplest ways of estimating channel impulse response is to correlate the received signal to a known training sequence[9]. The mathematical expression of channel coefficient estimation is given by

$$\hat{h}_{\mathrm{COR}}(j) = \frac{1}{N_s} \sum_{l=l_0}^{l_0+N_s-1} y_{l+j} a_l , \; j = 0, 1, \cdots, J-1 , \tag{6}$$

where N_s is the length of the subsequence used in correlation and l_0 is the index of the first symbol used in correlation.

Submitting Eq.(3) into Eq.(6), the correlation channel estimation can be rewritten as

$$\hat{h}_{\mathrm{COR}}(j) = h(j) + \frac{1}{N_s} \sum_{m=0, m \neq j}^{J-1} h(m) R_{j-m} + n'_j$$
$$j = 0, 1, \cdots, J-1 , \tag{7}$$

where $R_l = \sum_{n=l_0}^{l_0+N_s-1} a_n a_{n+l}$ is the deterministic correlation between the portion of the training sequence used for channel estimation and the l th shift of the full training sequence. $n'_j = \sum_{l=l_0}^{l_0+N_s-1} n_j b_l / N_s$ is the equivalent expression of channel noise. From Eq.(7), the accuracy of channel estimation is affected by channel noise and the interference between taps, which depends on the training sequence auto-correlation function. Therefore, some efforts can be devoted to the design of ideal training sequence to obtain better channel estimation performance.

The least square (LS) channel estimation is also commonly used in radio frequency communications when a training sequence is available. The optimization criterion of this method is to minimize the least square errors to find an optimal estimator for the unknown parameters[10]. For the indoor VLC systems described in this paper, the sum of square errors of channel estimation can be defined as

$$\varepsilon(\boldsymbol{h}) = \| \boldsymbol{y} - \boldsymbol{A}\boldsymbol{h} \|^2 = (\boldsymbol{y} - \boldsymbol{A}\boldsymbol{h})^{\mathrm{H}} (\boldsymbol{y} - \boldsymbol{A}\boldsymbol{h}) \quad , \tag{8}$$

where $\|\cdot\|$ denotes the norm of a vector, and $(x)^{\mathrm{H}}$ is the Hermitian transposition of x . Therefore, the optimization criterion can be defined as

$$\hat{\boldsymbol{h}}_{\mathrm{LS}} = \arg\min_{\boldsymbol{h}} \varepsilon(\boldsymbol{h}) \quad . \tag{9}$$

By differentiating with respect to each channel coefficient $\boldsymbol{h}$ and setting the result to zero, i.e.,

$$\frac{\partial}{\partial \boldsymbol{h}} (\boldsymbol{y} - \boldsymbol{A}\boldsymbol{h})^{\mathrm{H}} (\boldsymbol{y} - \boldsymbol{A}\boldsymbol{h}) = 0 \quad , \tag{10}$$

the closed-form expression for the LS channel estimation can be obtained as

$$\hat{\boldsymbol{h}}_{\mathrm{LS}} = (\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{y} \quad . \tag{11}$$

Substituting Eq.(4) into Eq.(11), the LS channel estimation can be expressed as

$$\hat{\boldsymbol{h}}_{\mathrm{LS}} = \boldsymbol{h} + (\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{n} \quad , \tag{12}$$

where $\boldsymbol{h}$ is the objective of the channel estimation, and $(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{n}$ denotes the estimated error. It can be observed that unlike the correlation approach, the interference between taps is eliminated. In other words, the LS channel estimation method is independent of the correlation properties of TS. However, the estimation error variance is largely depending on the channel noise.

Unlike the LS approach, the optimization criterion of the minimum mean square error (MMSE) method is to minimize the mean square errors to find an optimal estimator for the unknown parameters. Define the linear estimator that achieves the MMSE as

$$\hat{\boldsymbol{h}}_{\mathrm{MMSE}} = \boldsymbol{V}\boldsymbol{y} \quad , \tag{13}$$

where $\boldsymbol{V}$ is the linear estimation operator[11]. From Eq.(13), the mean square error of the channel estimation for VLC systems can be described as

$$\delta(\boldsymbol{V}) = \mathrm{E}\left[\| (\boldsymbol{h} - \hat{\boldsymbol{h}}_{\mathrm{MMSE}}) \|^2\right] = \mathrm{E}\left[\| (\boldsymbol{h} - \boldsymbol{V}\boldsymbol{y}) \|^2\right] \quad , \tag{14}$$

where E[•] denotes the expectation operator. To obtain the expression of the channel estimation, it is expected that the estimation operator $\boldsymbol{V}$ must be determined at first. Obviously, the optimal operator $\boldsymbol{V}$ should satisfy the function as follows

$$\hat{\boldsymbol{V}} = \arg\min_{\boldsymbol{V}} \delta(\boldsymbol{V}) \quad . \tag{15}$$

By differentiating with respect to $\boldsymbol{V}$ and setting the result to zero, i.e.,

$$\frac{\partial}{\partial \boldsymbol{V}} = \mathrm{E}\left[(\boldsymbol{h} - \boldsymbol{V}\boldsymbol{y})(\boldsymbol{h} - \boldsymbol{V}\boldsymbol{y})^{\mathrm{H}}\right] = 0 \quad , \tag{16}$$

the closed form of operator $\boldsymbol{V}$ can be written as

$$\hat{\boldsymbol{V}} = \boldsymbol{R}_h\boldsymbol{A}^{\mathrm{H}} + (\boldsymbol{A}\boldsymbol{R}_h\boldsymbol{A}^{\mathrm{H}} + \boldsymbol{R}_n)^{-1} \quad , \tag{17}$$

where $\boldsymbol{R}_h = \mathrm{E}[\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}]$ denotes the channel coefficient covariance matrix, $\boldsymbol{R}_n = \mathrm{E}[\boldsymbol{n}\boldsymbol{n}^{\mathrm{H}}]$ is the covariance matrix of the noise, and $(\bullet)^{-1}$ denotes the inverse operation of a matrix. Therefore, the MMSE channel estimation can be expressed as follows

$$\hat{\boldsymbol{h}}_{\mathrm{MMSE}} = \boldsymbol{V}\boldsymbol{y} = \boldsymbol{R}_h\boldsymbol{A}^{\mathrm{H}}(\boldsymbol{A}\boldsymbol{R}_h\boldsymbol{A}^{\mathrm{H}} + \boldsymbol{R}_n)^{-1}\boldsymbol{y} \quad . \tag{18}$$

From Eq.(18), the MMSE approach can alleviate the effect of channel noise in some degree compared with the LS approach. However, the MMSE channel estimation requires prior knowledge of the channel coefficient covariance matrix $\boldsymbol{R}_h$ and noise covariance matrix $\boldsymbol{R}_n$, which means that the MMSE method is more complicated than the above two approaches.

The performances of the proposed three methods are compared by computer simulation. One way of quantifying channel estimation performance is by the mean square error (MSE) matrix[9], also referred to the error covariance. The MSE matrix can be defined as

$$\boldsymbol{MSE} = \mathrm{E}\{(\boldsymbol{h} - \hat{\boldsymbol{h}})(\boldsymbol{h} - \hat{\boldsymbol{h}})^{\mathrm{H}}\} \quad . \tag{19}$$

Without loss of generality, we assume that the length of TS used in channel estimation is N=26 and the number of channel taps is J=5.

It can be seen from Fig.2 that the MSE performance decreases with the increase of the SNR. In addition, the MMSE channel estimation method obtains the best performance while the correlation method obtains the worst performance. With the SNR increasing, the difference between the performances of MMSE and LS approaches disappears, which means that the LS method is more sensitive to the signal noise power. Moreover, since the auto-correlation property of the training sequence is not deterministic, the correlation method is rarely used in practical systems.

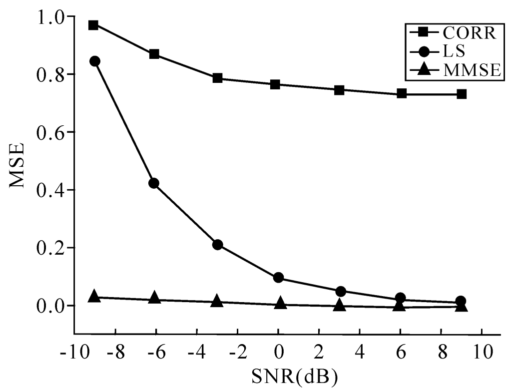

Fig.2 MSE performance versus SNR with different channel estimation methods

In Fig.3, with the LS or correlation channel estimation method, the MSE performance is free from the effects caused by the coefficient covariance matrix $\boldsymbol{R}_h$. However, the MSE increases significantly with the increase of the estimation error of $\boldsymbol{R}_h$ when MMSE channel estimation method is used. Even though the SNR is high, the MSE performance for MMSE estimation is worse than that of the LS estimation when there exits the estimation error for $\boldsymbol{R}_h$.

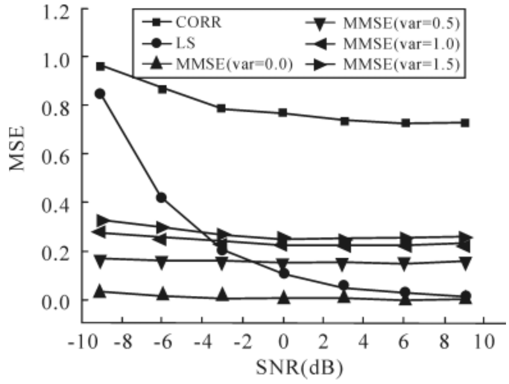

Fig.3 MSE performance versus SNR with different estimation errors for MMSE method

This paper investigates the TS-based channel estimation for indoor VLC systems. Firstly, the mathematical model is established for indoor VLC systems. Based on the model, three channel estimation methods, i.e., the correlation method, the LS method and the MMSE method, are proposed. Moreover, the paper evaluates the performances of the proposed three methods by computer simulation. The results show that the performance of the correlation method is the worst, the LS method is suitable for high SNR due to its sensitivity to high channel noise power, and the MMSE method obtains the best performance at the expense of highest complexity.

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