Satellite image blind restoration based on surface fitting and multivariate model*

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Owing to the blurring effect from atmosphere and camera system in the satellite imaging, a blind image restoration algorithm is proposed which includes the modulation transfer function (MTF) estimation and the image restoration. In the MTF estimation stage, based on every degradation process of satellite imaging-chain, a combined parametric model of MTF is given and used to fit the surface of normalized logarithmic amplitude spectrum of degraded image. In the image restoration stage, a maximum *a posteriori* (MAP) based edge-preserving image restoration method is presented which introduces multivariate Laplacian model to characterize the prior distribution of wavelet coefficients of original image. During the image restoration, in order to avoid solving high nonlinear equations, optimization transfer algorithm is adopted to decompose the image restoration procedure into two simple steps: Landweber iteration and wavelet thresholding denoising. In the numerical experiment, the satellite image restoration results from SPOT-5 and high resolution camera (HR) of China & Brazil earth resource satellite (CBERS-02B) ane compared, and the proposed algorithm is superior in the image edge preservation and noise inhibition.

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In fact blind image restoration is a feasible method which estimates original image only from degraded image with only a little prior information about original image, blurring function and noise. Iterative blind restoration algorithm is a kind of prevalent methods in which original image and blurring function are estimated simultaneously via alternating minimization (AM) method[1,2]. However the solutions of iterative blind restoration are usually non-unique, unstable and even divergent[1,3]. So in this paper a novel blind image restoration algorithm is proposed, in which a parametric model about MTF is given and the parameters are determined via surface fitting, then with the estimated MTF a MAP-based edge-preserving image restoration algorithm is presented using multivariate Laplacian model as the prior distribution of wavelet coefficients of original image.

The blurring caused by atmospheric turbulence, the scattering of aerosols, optical system and sensor can be respectively approximated by Gaussian function[4,5]. If the effect of electronic components and A/D transformation are neglectable and the motion of camera is uniform linear, then the combined parametric model about MTF can be approximated by the following anisotropic Gaussian function in the frequency domain

$$H(\xi,\eta)=\exp[-(\alpha|\xi|^2+\beta|\eta|^2)] \ , \tag{1}$$

where α and β are respectively model parameters to be estimated, ξ and η are spatial frequency.

In most cases, image degradation can be modeled by a linear shift-invariant blur and an additive Gaussian white noise as

$$g(x,y)=h(x,y)*f(x,y)+n(x,y) \ , \tag{2}$$

where g , f, h and n denote degraded image, original image, point spread function (PSF) and additive noise respectively, * denotes 2-D convolution, (x, y) denotes spatial coordinate. The corresponding expression in the frequency domain is

$$\hat{g}(\xi,\eta)=H(\xi,\eta)\hat{f}(\xi,\eta)+\hat{n}(\xi,\eta) \ , \tag{3}$$

where $\hat{g}$, $\hat{f}$ and $\hat{n}$ are the Fourier spectra of g , f and n

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respectively, H denotes MTF in eq.(1).

Based on Fourier transform theory, we let $\delta \equiv \hat{g}(0,0) = \int_{R^2} g(x,y)\,\mathrm{d}x\,\mathrm{d}y$, normalize eq.(3) with δ and take the nature logarithm of the normalized result in eq. (3).

$$\ln\left|\hat{g}^{\dagger}(\xi,\eta)\right| = \ln\left|H(\xi,\eta)\hat{f}^{\dagger}(\xi,\eta) + \hat{n}^{\dagger}(\xi,\eta)\right| , \quad (4)$$

where normalized quantity $q^{+}(\xi,\eta) = q(\xi,\eta)/\delta$. Usually there exists $\Omega = \{(\xi,\eta) \mid \xi^2 + \eta^2 \le \omega^2\}$ that is a neighborhood of the frequency origin such that $H(\xi,\eta)\hat{f}^{+}(\xi,\eta) >> \hat{n}^{+}(\xi,\eta)$ always holds, eq.(1) is subsbituted into eq.(4) .

$$\ln\left|\hat{g}^{\dagger}(\xi,\eta)\right| \approx -(\alpha|\xi|^2 + \beta|\eta|^2) + \ln\left|\hat{f}^{\dagger}(\xi,\eta)\right| .(\xi,\eta) \in \Omega . \quad (5)$$

In addition, amplitude spectrum of natural image can be modeled as $\left|\hat{f}(\xi,\eta)\right| = A/(\xi^2+\eta^2)^{\gamma/2}$, where A and γ are respectively scale parameter and shape parameter in most cases[6,7]. So eq.(5) can be rewritten as

$$\ln\left|\hat{g}^{\dagger}(\xi,\eta)\right| \approx -(\alpha|\xi|^2 + \beta|\eta|^2) - \gamma/2\ln(\xi^2+\eta^2) + \ln A \quad (\xi,\eta) \in \Omega . \quad (6)$$

From eq.(6) the model parameters α and β in MTF can be estimated by fitting the surface of normalized logarithmic amplitude spectrum of degraded image via the following 2-D parameterized function

$$\psi_{\alpha,\beta,\gamma,A}(\xi,\eta) = -(\alpha|\xi|^2 + \beta|\eta|^2) - \gamma/2\ln(\xi^2+\eta^2) + \ln A . \quad (7)$$

We adopt least-square fitting, then model parameters can be estimated via the following optimization problem

$$\min_{\alpha,\beta,\gamma,A} \sum_{(\xi,\eta)\in\Omega} (\ln|\hat{g}^{+}(\xi,\eta)| - \psi_{\alpha,\beta,\gamma,A}(\xi,\eta))^2 , \quad (8)$$

which is solved via Nelder-Mead simple search method[8].

For expression convenience, g , f and n in eq.(2) denote lexicographically ordered vectors and H denotes blurring matrix associated with PSF h. Then the degradation model in eq.(2) can be rewritten as in matrix-vector type

$$g = Hf + n = HW\theta + n , \quad (9)$$

where W denotes inverse orthogonal wavelet transform, θ denotes wavelet coefficients of f.

Based on the degradation model in eq.(9), the log-likelihood estimation can be formulated as $\ln p_{g|\theta}(g|\theta) \propto -1/(2\sigma_n^2) \|g - HW\theta\|^2$, where σ is standard deviation of noise. Because image restoration belongs to an ill-posed linear inverse problem, it is very necessary to impose some prior knowledge on the solution. So the prior statistical model of image plays an important role under Bayesian framework. General statistical model, e.g. generalized Gaussian distribution, Cauchy distribution, student-t distribution and so on[7,9], is usually univariate model which is based on the assumption that each wavelet coefficient is independent. However, although orthonormal wavelet transform is found to be fairly well-decorrelated, a residual dependency structure always remains among nature image wavelet coefficients[7,10,11]. To capture the statistical dependency, a multivariate Laplacian model is introduced which perfectly characterizes the dependency of the neighboring wavelet coefficients in the interscale and intrascale and has been successfully applied to MAP-based image denoising[10]. The multivariate Laplacian model with parameter λ can be expressed as[10]

$$p_\theta(\theta) = (2\pi)^{-N/2} \frac{2}{\lambda} \frac{K_{N/2-1}(\sqrt{2\theta^T\rho^{-1}\theta})}{\left(\sqrt{\frac{\lambda^2}{2}\theta^T\rho^{-1}\theta}\right)^{N/2-1}} , \quad (10)$$

where $K_\nu(\bullet)$ denotes the modified Bessel function of the second kind, ρ denotes the covariance matrix of θ, N is the length of θ.

With the log-likelihood estimation $\log p_{g|\theta}(g|\theta)$ and the multivariate statistical model in eq.(10), the logarithmic MAP estimation $\log p_{g|\theta}(g|\theta) \propto \log p_{g|\theta}(g|\theta) + \log p_\theta(\theta)$ about θ can be formulated as the minimization of the following objective functional

$$J(\theta) = \frac{1}{2}\|g - HW\theta\|^2 - \sigma_n^2 \log p_\theta(\theta) . \quad (11)$$

Optimization transfer method[12] is very suitable to solve the optimization problem in eq.(11). The method first introduces a surrogate function that is an upper bound on the objective function to be minimized, and then minimizes the surrogate function to indirectly obtain the goal of minimization of the objective function. As for eq.(11), a feasible surrogate functional can be given as

$$\begin{aligned} Q(\theta \mid \theta^k) = {} & \frac{1}{2}\|g - HW\theta\|^2 - \sigma_n^2 \ln p_\theta(\theta) - \\ & \frac{1}{2}\|HW\theta - HW\theta^k\|^2 + \frac{1}{2}\|W\theta - W\theta^k\|^2 = \\ & \frac{1}{2}\|\theta - [\theta^k + W^T H^T(g - HW\theta^k)]\|^2 - \sigma_n^2 \ln p_\theta(\theta) + \\ & \|g\|^2 + \|W\theta^k\|^2 - \|HW\theta^k\|^2 . \end{aligned} \quad (12)$$

Let $$\phi^k = \theta^k + W^T H^T(g - HW\theta^k) , \quad (13)$$

which is well-known Landweber iteration, and

$$C=\|g\|^2+\|W\theta^k\|^2-\|HW\theta^k\|^2,$$

which is constant with respect to θ, then eq. (12) can be rewritten as

$$Q(\theta\,|\,\theta^k)=\frac{1}{2}\|\theta-\phi^k\|^2-\sigma_n^2\ln p_\theta(\theta)+C\ . \qquad (14)$$

According to optimization transfer method, the image restoration problem in eq.(11) is transformed into the minimization of surrogate functional $Q(\theta\,|\,\theta^k)$ in eq.(14). Owing to the absence of convolution matrix H which is “hidden” in the temporary vector ϕ^k (also seen in eq.(13)), minimization of eq.(14) is actually a MAP-based wavelet denoising problem (i.e. $\phi^k=\theta+n$)

$$\theta^{k+1}=\arg\min_\theta\frac{1}{2}\|\theta-\phi^k\|^2-\sigma_n^2\ln p_\theta(\theta)+C$$
$$=\arg_\theta zero\left[\phi^k-\theta+\sigma_n^2\frac{d}{d\theta}\log p_\theta(\theta)\right]\ . \qquad (15)$$

Note that in the remainder denoising stage of this section, vector θ denotes the wavelet coefficients associated with local neighborhood in the interscale and intrascale and that ϕ^k has the same meaning. As shown in Ref.[10], eq.(15) can be approximately solved as the following element-wise Multishinkage denoising

$$\theta_i^{k+1}=\frac{\frac{1}{2}L+[\frac{1}{4}L^2-2\sigma_n^2(\frac{N}{2}-1)]^{1/2}}{\sqrt{(\phi^k)^T\phi^k}}\phi_i^k\ , \qquad (16)$$

where $L=\sqrt{(\phi^k)^T\phi^k}-\sqrt{2}\sigma_n^2/\sigma$, σ^2 is estimated as[11]

$\hat{\sigma}^2=\max(\sigma_\phi^2-\rho_n^2,0)$, $\sigma_{\phi_i}^2$ is estimated using

$\sigma_{\phi_i}^2=\frac{1}{M}\sum_{\phi_i\in N(i)}\phi_i^2$, where M is the size of the neighborhood $N(i)$, i denotes element index. As for noise variance σ_n^2, the robust median estimator is used:
$\hat{\sigma}_n^2=\text{median}(|\phi_{diag}|)/0.6745$, where ϕ_{diag} denotes the finest scale wavelet coefficients of diagonal subband.

The whole framework of the proposed algorithm in the paper is as following,

1) MTF estimation step

a) Choose a suitable size of frequency origin neighborhood Ω, which is usually not sensitive to the last fitting results in our numerical experiments.

b) Implement least-square surface fitting via Nelder-Mead simplex search method in eq.(8). The existing C codes in Ref. [8] are used in the paper.

c) PSF is determined via the inverse Fourier transform of MTF.

2) Image restoration step

a) Implement Landweber iteration in eq.(13). The edges of images are assumed to be of period extension, then the convolution matrix H is block circulant matrix that can be diagonalized via fast Fourier transform (FFT). So the whole Landweber iteration is calculated in the frequency domain using FFT and the wavelet transform is carried out on the last results.

b) Implement wavelet thresholding denoising in eq.(16).

c) If specific iterative number is completed or a terminative criterion is satisfied, then stop iteration, otherwise return to 2) a).

To evaluate the validity of the proposed algorithm, we compare with well-known TV-based iterative non-parametric blind image deconvolution by Tony Chan[2,13]

$$\min_{f,h}J(f,h)=\min_{f,h}\frac{1}{2}\|g-f^*h\|^2+$$
$$\alpha_1\int_\Omega|\nabla f|\,dx\,dy+\alpha_2\int_\Omega|\nabla h|\,dx\,dy\ , \qquad (17)$$

and Mumford-Shah regularization based iterative parametric blind image deconvolution by L. Bar[14]

$$\min_{f,\sigma_1,\sigma_2}J(f;\sigma_1,\sigma_2)=\min_{f,\sigma_1,\sigma_2}\frac{1}{2}\|g-f^*h\|^2+$$
$$\lambda_1G_\in(f,v)+\lambda_2\int_\Omega|\nabla h_{\sigma_1,\sigma_2}|^2\,dx\,dy\ , \qquad (18)$$

where the parametric model on PSF in spatial domain is

$$h_{\sigma_1,\sigma_2}(x,y)=\frac{1}{2\pi\sigma_1\sigma_2}\exp[-(\frac{x^2}{\sigma_1^{\ 2}}+\frac{y^2}{\sigma_2^{\ 2}})].$$

$G_\in$ denotes Mumford-Shah regularization. Note that in Ref.[14] the parametric blind deconvolution is called as semi-blind deconvolution. Both of the two kinds of iterative blind restoration are solved via AM method and corresponding Euler-Lagrange equations are solved via lagged diffusivity fixed point scheme[2,13]. In our experiments, the regularization parameters α_1 and α_2 in eq.(17) and λ_1 and λ_2 in eq.(18) are adaptively determined via least square method[15]. Note that the regularization parameter λ_1 is included in $G_\in$ and that there only exists a model parameter σ to be estimated in Ref.[14].

In the following two experiments, we employ Daubechies-2 wavelet base. The iterative numbers in the image restoration are both 150. The neighborhoods of the frequency origin in eq.(5) are both chosen 40 × 40. In the thresholding denoising stage, we use a 3×3+1 neighborhood where 3×3+1 denotes the neighborhood containing a 3×3 local window in the intrascale and a parent coefficient of the central coefficient of the window in the interscale.

In the first experiment, the estimated parameters α, β, γ and A in eq.(8) are respectively 0.091,0.083,1.13 and 3.22 × 10^{-6}. The profile of surface fitting across frequency domain origin is shown in Fig.1(a). Fig.2(a) is a 128 × 128 real degraded panchromatic SPOT-5 satellite image with spatial resolution 5 m. Fig.2(b-d) are respectively the restored images by Tony Chan's, L. Bar's and our algorithms. Fig.2 (e) is the panchromatic SPOT-5 satellite image with spatial resolution 2.5 m from the same scene in Fig.2(a). Comparison of Fig.2(a) with Fig.2(b-d) shows that some textures, which are too blurred to identify in the Fig.2(a), are restored. Furthermore the restored textures can be found in Fig.2(e). But Fig.2(d) is superior to Fig.2(b-c) in the case of edge preservation and noise inhibition.

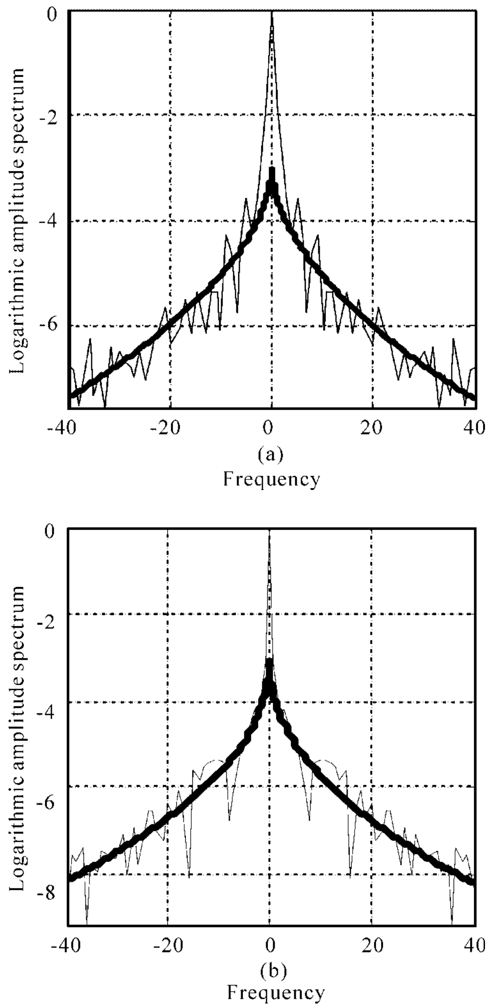

Fig.1 The profile of surface fitting across frequency origin in the two numerical experiments above. The bold lines correspond to fitting curves and thin lines correspond to normalized logarithmic amplitude spectrum of degraded image. (a) the first experiment; (b) the second experiment

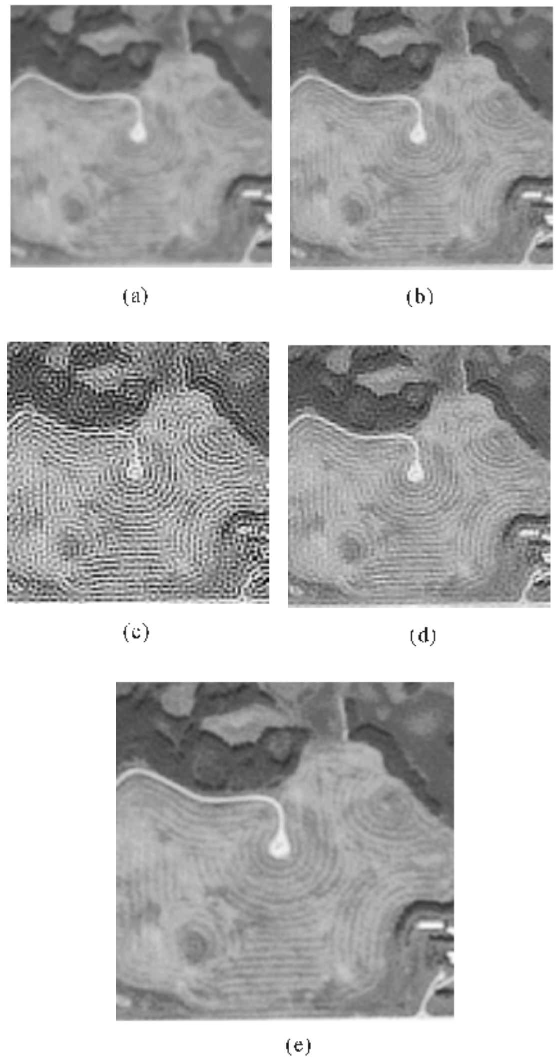

Fig.2 Restoration of a real degraded panchromatic SPOT-5 (5 m) image. (a) original image; (b) restored image by Tony Chan's; (c) restored image by L. Bar's; (d) restored image by ours; (e) panchromatic SPOT-5 (2.5 m) image with the same scene

In the second experiment, the estimated parameters α, β, γ and A in eq. (8) are respectively 0.130, 0.176, 0.95 and 4.14×10^{-6}. The profile of surface fitting across frequency domain origin is shown in Fig.1(b). Fig.3(a-d) are respectively 128×128 real degraded CBERS-02B HR satellite image and corresponding restored images by Tony Chan's, L. Bar's and our algorithms. Comparison of the degraded image with the restored images shows that Fig.3(d) is also superior to Fig.3(b-c) in the case of edge preservation and noise inhibition.

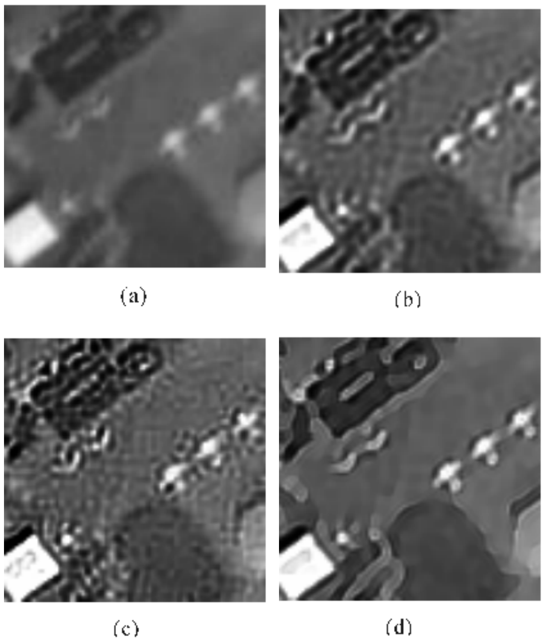

Fig.3 Restoration of a real degraded CBERS-02B HR image. (a) original image; (b) restored image by Tony Chan's; (c) restored image by L. Bar's; (d) restored image by our method

In conclusion, we develop a surface fitting based estimation method of blurring function, with which a MAP-based edge-preserving image restoration algorithm is given which uses multivariate Laplacian model as the prior distribution of wavelet coefficients of original image. With optimization transfer method, the image restoration procedure is transformed into simple Landweber iteration and wavelet thresholding denoising. The restoration results show that compared with Tony Chan's and L. Bar's algorithms the proposed method can significantly retrieve some lost detail information and effectively inhibit noise.

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