Spectra of fiber Bragg grating and long period fiber grating undergoing linear and quadratic strain*

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The effects of linear and quadratic strain on the reflective spectrum of FBG and the transmission spectrum of LPFG have been numerically investigated based on the coupled -mode equations. The results show that for the FBG or LPFG undergoing only linear strain, the reflection or transmission spectrum is symmetrical about its central wavelength, the dependence relationships of the central wavelength shift, full-width-at-half-maximum, peak intensity upon the strain gradient have been obtained.

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Using a FBG or LPFG as a single-point sensor, the strain distribution on the sensor is assumed to be uniform, such assumption only considers the 0-order term of Taylor expansion of strain profile, and the space resolution is limited to the grating length. But the nonuniform strain distributions on the FBG or LPFG with certain length exist widely in practical applications and affect the measuring resolution. In order to use the FBG or LPFG sensor in the field of strain sensing with high resolution, it is necessary to investigate the effects of strain profile on the FBG or LPFG[1].

To obtain the strain distribution along the FBG or LPFG sensor from its reflection or transmission spectrum, some reconstruction methods have been proposed[2-4]. But these methods are not suitable to the real-time measurement for strain gradient due to their long cost time. In this paper, based on the numerical simulations for the coupled-mode equations of the FBG or LPFG undergoing linear and quadratic strain, the effects of linear and quadratic strain on their reflection and transmission spectra have been investigated, the dependence relationships of the wavelength shift, full-width-at-half-maximum, and peak intensity upon the strain parameters are also obtained.

For a FBG or a LPFG written in a single-mode fiber, the total index perturbation can be expressed as[5]

$$\Delta n(z) = \Delta n_0[1 + v\cos(\frac{2\pi}{\Lambda}z + \varphi(z))] \ , \qquad (1)$$

where Δn_0 is the "dc" index change that is spatially averaged over a grating period, v is the fringe visibility of the index change, Λ is the nominal period and equals to Λ_B for the FBG or Λ_S for the LPFG. The additional phase $\varphi(z)$ may be chosen as 0 for the uniform grating.

The temperature distribution of the FBG or LPFG endowing in z-direction can be expressed as

$$\varepsilon(z) = \varepsilon_0 + k_1 z + k_2 z^2 \ , \qquad (2)$$

where $z\in[0, L]$, L is the grating length, ε_0 is the uniform strain, $k_1=[d\varepsilon/dz]_{z=0}$ is the strain gradient at z=0, and $k_2=[d^2\varepsilon/dz^2]_{z=0}$ is the 2-order expansion coefficient of Taylor expansion of the strain profile.

As the FBG undergoes a strain, the additional phase can be written as[6]

$$\varphi(z) = -\frac{2\pi z}{\Lambda_B}\frac{(1-p_e)\varepsilon(z)}{1+(1-p_e)\varepsilon(z)} \ , \qquad (3)$$

where p_e is the effective elastooptic parameter, and p_e=0.22 is fixed in this paper. FBG's coupled-mode equations are

$$\frac{dA_{co}}{dz} = i\kappa_{co}\exp(-i2\delta_{co}z + i\varphi)B_{co} \ , \qquad (4)$$

$$\frac{dB_{co}}{dz} = -i\kappa_{co}\exp(i2\delta_{co}z - i\varphi)A_{co} \ , \qquad (5)$$

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A_{co} and B_{co} are the forward and backward amplitudes of LP_{01} core mode, the detuning parameter is

$$\delta_{co} = \frac{1}{2}(2\beta_{co} - \frac{2\pi}{\Lambda_B}) \ , \qquad (6)$$

where $\beta_{co}=2\pi n_{eff}^{co}/\lambda$, β_{co} and n_{eff}^{co} denote the propagation constant and the effective refractive index of LP_{01} core mode, respectively. The coupling coefficient κ_{co} describes the contra-directional coupling between forward propagating core mode (LP_{01}) and backward propagating core mode (LP_{01}). By using the 4-order Runge-Kutta method for the coupled-mode equations, the reflection spectrum can be obtained.

For the LPFG undergoing the axial strain, the additional phase can be written as

$$\varphi(z) = -\frac{2\pi z}{\Lambda_s}\frac{(1+\Gamma_{strain})\varepsilon(z)}{1+(1+\Gamma_{strain})\varepsilon(z)} \ , \qquad (7)$$

$$1+\Gamma_{strain} = \frac{(1-p_{co})n_{eff}^{co} - (1-p_{cl})n_{eff}^{cl}}{n_{eff}^{co} - n_{eff}^{cl}} \ , \qquad (8)$$

where p_{co} and p_{cl} are respectively the effective elastooptic parameters of fiber core and cladding, n_{eff}^{co} and n_{eff}^{cl} are the effective refractive indices of core mode and cladding mode. LPFG's multimode coupled equations can be written as

$$\frac{dA_{01}}{dz} = i\sigma_{01-01}A_{01} + i\sum_{v=1}^{\mu_m}\kappa_{01-1v}\exp(-i2\delta_{01-1v}z + i\varphi)A_{1v} \ , \qquad (9)$$

$$\frac{dA_{1v}}{dz} = i\kappa_{01-1v}\exp(i2\delta_{01-1v}z - i\varphi)A_{01}, v=1,2,\cdots,\mu_m \ , \qquad (10)$$

where A_{co} and A_{1v} are the forward amplitudes of LP_{01} core mode and v-order cladding mode LP_{1v}, respectively. The detuning parameters

$$\delta_{01-1v} = \frac{1}{2}(\beta_{01} + \beta_{1v} - \frac{2\pi}{\Lambda_s}) \ , \qquad (11)$$

where $\beta_j = 2\pi n_{eff}^{j}/\lambda$, $j=(01,1v)$, β_{01} and n_{eff}^{01} are the propagation constant and effective refractive index of LP_{01} core mode, respectively; β_{1v} and n_{eff}^{1v} are the propagation constant and effective refractive index of LP_{1v} cladding mode, respectively. The coupling coefficient σ_{01-01} describes the self-coupling for LP_{01} core modes, and κ_{01-1v} describes the co-directional coupling between LP_{01} core mode and LP_{1v} cladding mode, which are calculated according to Ref. [7].

It is known to all that the reflection spectrum of an initially uniform FBG will shift as the FBG undergoes an uniform strain ε_0, and the central wavelength shift $\Delta\lambda_B=\lambda_B(1-p_e)\varepsilon_0$. So we firstly consider the effect of strain gradient k_1 on the reflection spectrum of the FBG endowing linear strain, where the grating parameters λ_B=1550 nm, L=4 mm are fixed, and ε_0=200 $\mu\varepsilon$ and k_2=0 are chosen. The reflection spectra corresponding to different values of k_1 are shown in Fig.1(a),

Fig.1(b) shows the central wavelength shifts, FWHMs, and the intensities at central wavelengths corresponding to different values of k_1.

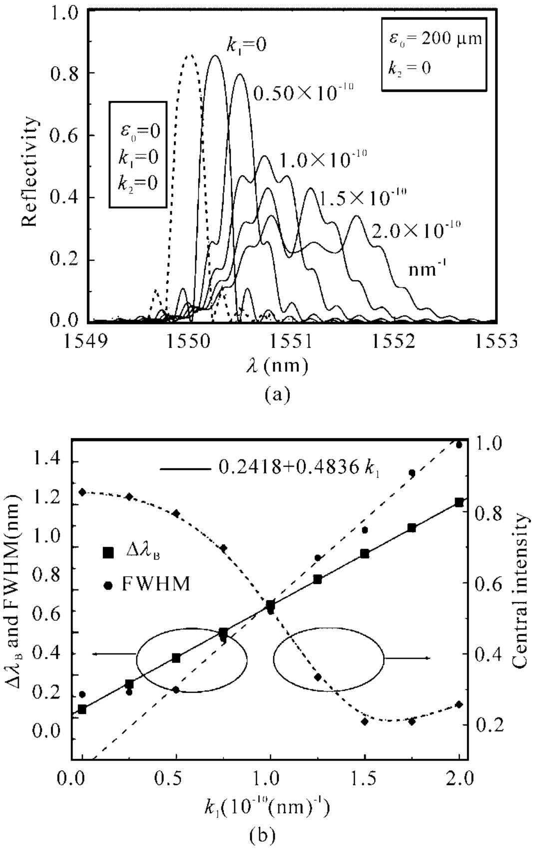

Fig.1 (a) Reflection spectra of FBGs corresponding to different values of k_1, dash curve is the reflection spectrum of FBG without strain. (b) Dependence relationships of wavelength shift, FWHM, and the intensity at central wavelength on strain gradient k_1, solid line is drawn according to the formula $\Delta\lambda_B$=0.2418+0.4836k_1 where ε_0=200 με are chosen.

To understand the equation, we try to substitute $\varepsilon(L) = \varepsilon_0 + k_1 L$ into the wavelength shift formula of FBG endowing uniform strain $\Delta\lambda_B=\lambda_B(1-p_e)\varepsilon$, and obtain

$$\Delta\lambda_B = \lambda_B(1-p_e)\,\varepsilon_0 + \lambda_B(1-p_e)\,k_1 L \ , \qquad (12)$$

where $\lambda_B(1-p_e)\varepsilon_0$= 0.2418 nm, $\lambda_B(1-p_e)\,L = 0.4836\times10^{10}$ nm^2, these data are consistent with the simulated result, such consistence shows Eq.(12) can also be used to describe the wavelength shift of FBG endowing linear strain. In the process of obtaining the formula (12), the grating length L=4 mm is chosen. In fact, we also have finished a series of simulation calculations corresponding to different grating lengths (2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0 mm) while $k_1 = 1.0 \times 10^{-10}$ nm^{-1} is fixed, the simulated result ensures the validity of Eq.(12).

To understand the effect of quadratic strain k_2 on the reflection spectrum of FBG, Fig.2 gives the reflection spectra corresponding to different values of k_2 for $k_1 = 0$.

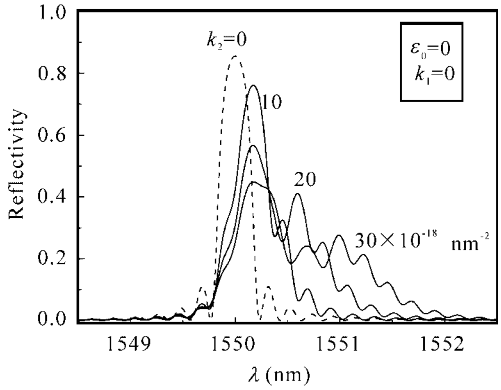

Fig.2 Reflection spectra of FBGs corresponding to different values of k_2, dash curve is the reflection spectrum of FBG without strain, where ε_0=0 is fixed.

In the above parts, Eq. (2) based on a second order Taylor expansion of axial strain profile is used to calculate the reflection spectra of FBGs, where FBG's axial position is in the region of [0, L]. In this case, the linear strain term $k_1 z$ is larger than 0, which leads to that the reflection spectrum shifts toward longer wavelength. Recently, the reflection spectra of FBGs based on the axial strain distribution as Eq.(2) in different region of [-L/2, L/2] are considered[6]. So a progressive discussion is given in the following. We adopt Eq.(2), it is, $\varepsilon(z)=\varepsilon_0 + k_1 z + k_2 z^2$, but the region is defined as $z \in [z_0, z_f]$, where $z_0<0<z_f$, z_0 and $z_f = z_0 + L$ are the endpoint coordinates of FBG, and z_0 is adjustable. Fig.3 gives the reflection spectra of FBGs undergoing quadratic strain within different regions of $[z_0, z_f]$, where ε_0=0, $k_1 = 1.0 \times 10^{-10}$ nm^{-1}, k_2= 0 are chosen. In this figure, the case of [0, L] has been discussed in above section, and the case of [-L/2, L/2] was appeared in Ref.[6]. Using a series of simulations, the dependence relationship of the central wavelength shift $\Delta\lambda_B$ upon the linear strain k_1 can be obtained

$$\Delta\lambda_B = \lambda_B(1-p_e)\,[\varepsilon_0 + k_1(z_0, z_f)] \ . \tag{13}$$

The formula is easy to understand, the contribution of linear strain $k_1 z$ with $z \in [z_0, 0]$ to the central wavelength shift is negative, the contribution of linear strain $k_1 z$ with $z \in [0, z_f]$ to the central wavelength shift is positive, and their algebraic sum decides the contribution of linear strain. As one uses the FBG as a single-point sensor, the measured value of strain ε_m is only the strain of FBG at its midpoint, and the strains of both endpoints of FBG equal to ε_m-$k_1 L$/2 and ε_m+$k_1 L$/2, respectively.

Fig.3 gives the reflection spectra of FBGs undergoing quadratic strain with different regions of $[z_0, z_f]$, where ε_0=0, k_1=0, $k_2 = 30 \times 10^{-18}$ nm^{-2} are chosen. Combining Fig.2 and Fig. 3, it can be seen that the reflection spectra of FBGs undergoing quadratic strain are asymmetrical, and the distorted degree of reflection spectrum is partly decided by $k_2(z_0^2+z_f^2)$, whose minimum is $k_2 L^2/2$ at z_0=-L/2.

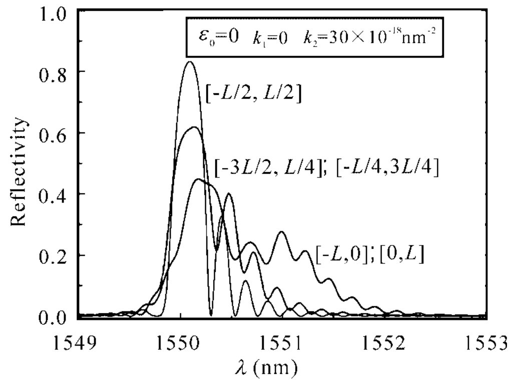

Fig.3 Reflection spectra of FBGs undergoing the strain $\varepsilon(z)=\varepsilon_0+k_1z+k_2z^2$ in different regions of $[z_0, z_0+L]$, where ε_0=0 is fixed.

To understand the effect of temperature on the transmission spectrum of LPFG, without losing generality, a LPFG (core radius a_1=4.15 μm, cladding radius a_2=62.50 μm, refractive indices n_1=1.53345, n_2=1.52793 and n_3=1.0, average index perturbation $\Delta n_0 = 2.5 \times 10^{-4}$, grating period Λ_S= 4.0×10^5 nm, grating length $L = 2.4 \times 10^7$ nm, effective elastooptic parameters p_{co}=0.22 and p_{cl} = 0.23) has been considered. As a preparation, the effective refractive indices n^{01}_{eff} and $n^{1\nu}_{\mathrm{eff}}$, detuning parameters $\delta_{01\text{-}1\nu}$, coupling coefficient $\kappa_{01\text{-}1\nu}$ are firstly calculated[2,10], where ν = 1, 2 ..., 10. Since the emphasis of this paper is to discuss the effects of temperature change on the LPFG, only the 7-order cladding mode LP_{17} is considered in the following.

The spectrum of an initially uniform LPFG will shift as the LPFG undergoes an uniform strain ε_0, the wavelength shift $\Delta\lambda_7$ changes with $\mathring{a}_0$,

$$\Delta\lambda_7 = \frac{\mathrm{d}\lambda_7}{\mathrm{d}\varepsilon}\varepsilon_0 \ , \tag{14}$$

where

$$\frac{\mathrm{d}\lambda_7}{\mathrm{d}\varepsilon} = \lambda_7\gamma_7(1+\Gamma_{\mathrm{strain}}) = \lambda_7\gamma_7\frac{(1-p_{\mathrm{co}})n^{\mathrm{co}}_{\mathrm{eff}} - (1-p_{\mathrm{cl}})n^{\mathrm{cl}}_{\mathrm{eff}}}{n^{\mathrm{co}}_{\mathrm{eff}} - n^{\mathrm{cl}}_{\mathrm{eff}}} \ , \tag{15}$$

At the central wavelength λ_7=1452.0 nm, $n^{\mathrm{co}}_{\mathrm{eff}}$ = 1.530767,

n^{cl}_{eff} = 1.527137, γ_7= 0.714, we may obtain that $d\lambda_7/d\varepsilon$ equals to 5.17 nm/mε, these analyzed data are consistent with the simulated results.

To understand the effect of strain gradient k_1 on the transmission spectrum of LPFG, the strain region [0, L], ε_0= 2.0 mε and k_2=0 are fixed. Fig.4(a) gives the transmission spectra of LPFGs corresponding to different values of k_1 (0, 1.0, 2.0 and 3.0 × 10^{-10} nm^{-1}). Fig.4(b) gives the dependence relationships of the peak intensity and FWHM of loss spectrum $|A_{cl}(L)|^2$ on linear strain $k_1 L$.

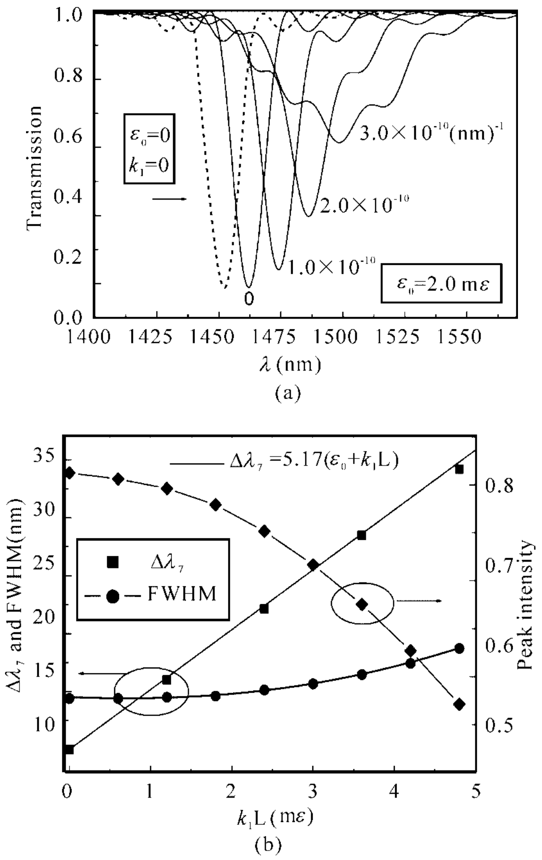

Fig.4 (a) Transmission spectra of LPFGs corresponding to different values of k_1, dash curve shows the transmission spectrum of LPFG without strain. (b) Dependence relationships of the wavelength shift, FWHM and peak intensity of $|A_{cl}(L)|^2$ on the parameter $k_1 L$, where ε_0=2.0 με and k_2=0 are fixed.

The relationship between the central wavelength shift and linear strain $k_1 L$ can be written as

$$\Delta\lambda_7 = \frac{d\lambda_7}{d\varepsilon}(\varepsilon_0 + k_1 L) \ . \tag{16}$$

Inversely, based on the nominal spectrum of LPFG, the shift of central wavelength of new spectrum means that the LPFG undergoes an uniform or (and) linear strain, and the changes of peak intensity and FWHM of $|A_{cl}(L)|^2$ the happen of linear strain, and decide strain gradient $k_1 L$.

Finally, the effect of quadratic strain k_2 on the transmission spectra of LPFGs has been also investigated. Fig.5 gives the transmission spectra of LPFGs with different values of k_2, where ε_0= 0, k_1=1.0 × 10^{-10} nm^{-1}.

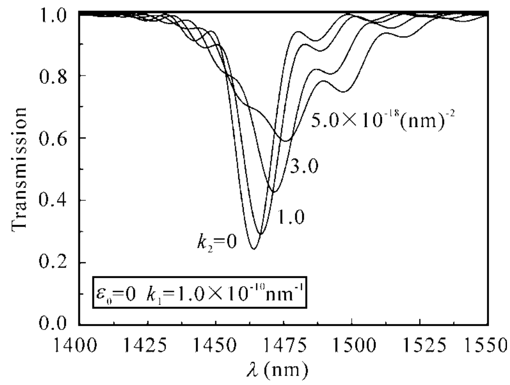

Fig.5 Transmission spectra of LPFGs corresponding to different values of k_2, where ε_0=0.

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