Analysis of error performance on Turbo coded FDPIM

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Due to variable symbol length of digital pulse interval modulation(DPIM), it is difficult to analyze the error performances of Turbo coded DPIM. To solve this problem, a fixed-length digital pulse interval modulation(FDPIM) method is provided. The FDPIM modulation structure is introduced. The packet error rates of uncoded FDPIM are analyzed and compared with that of DPIM. Bit error rates of Turbo coded FDPIM are simulated based on three kinds of analytical models under weak turbulence channel. The results show that packet error rate of uncoded FDPIM is inferior to that of uncoded DPIM. However, FDPIM is easy to be implemented and easy to be combined. with Turbo code for soft-decision because of its fixed length. Besides, the introduction of Turbo code in this modulation can decrease the average power about 10 dBm, which means that it can improve the error performance of the system effectively.

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Because of the advantage of both fiber and mobile communications and good application prospects, optical wireless communication (OWC) has drawn wide attentions[1,2]. However, its performance is badly influenced by attenuation and turbulence. Many researchers have studied the feasibilities of some effective modulations, such as on-off keying (OOK)[2-4], pulse position modulation (PPM) [5,6], and digital pulse interval modulation (DPIM)[7-10]. To further improve the communication quality, Jon Hamkins and Zhou Hai-yan et al.[11-13] introduced Turbo code into the application of OWC system and studied the error performance under PPM modulation, but overlooked the fact that symbol length of DPIM is variable. In this paper, a new modulation, fixed-length digital pulse interval modulation (FDPIM), is given based on DPIM and PPM. Error performance is derived theoretically at given communication model. Error performance of uncoded FDPIM is simulated at weak turbulence channel and compared with that of DPIM. Also, three models are used to analyze the performance of Turbo coded FDPIM.

FDPIM maps binary M bit data into dual-pulse signal in a period of time that composes of $L=2^M+4$ slots. If k is the decimal number of a symbol, the beginning position of every symbol is a pulse of single slot, then a guard slot and k empty slots, which represent information, are followed by a dual-slot pulse as the pulse of useless information and by 2^M-k empty slots, of which the first one is a guard slot. For example, if M=3, the modulation structures of OOK, PPM, DPIM, and FDPIM are shown in Fig.1.

Fig.1 indicates that FDPIM could prevent the emergence of three continuous “1” among or within the symbols from arising by overcoming inter-symbol interference, because it has two guard slots. The average symbol length of this modulation is larger related to that of DPIM, which is $\overline{n}=(2^M+3)/2$. However, its symbol length is fixed, which is very important to further improve error performance when combined with Turbo code.

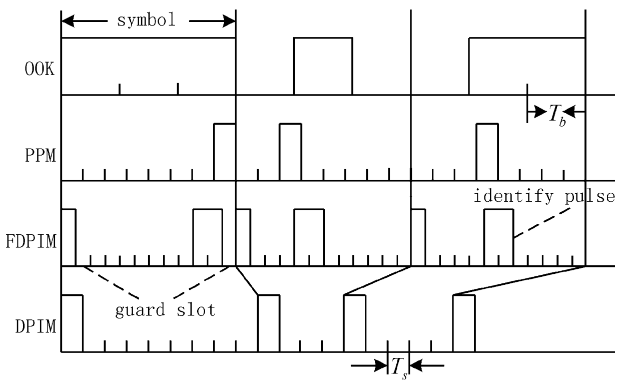

Fig.1 Symbol structures for four kinds of modulations

Since laser beam, that carries data in OWC system, must transmit through the air, scintillation exists when the laser passes through weak turbulence (maybe strong turbulence) channel. Because the radius of receiving antenna is very large,

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the average effect could exist. Within several kilometers, the energy of optical scintillation approximately conforms to log-normal distribution[14,15]. Its probability distribution function (PDF) can be expressed as,

$$P_I(I)=\frac{1}{2\sqrt{2\pi}\sigma_x I}\exp\left[-(\ln\frac{I}{I_0}+2\sigma_x{}^2)^2\Big/8\sigma_x{}^2\right], \quad (1)$$

where σ_x is the index of optical scintillation (log-amplitude standard of optical signal), I_0 and I are the mean and receiving optical intensities, respectively. When the optical intensity is homogeneous, the receiving optical power conforms to log-normal distribution approximately,

$$P_P(P)=\frac{1}{2\sqrt{2\pi}\sigma_x P}\exp\left[-(\ln\frac{P}{P_0}+2\sigma_x{}^2)^2\Big/8\sigma_x{}^2\right], \quad (2)$$

where P and P_0 are the receiving and mean optical powers respectively.

In the process of deriving error rate, we make the following assumption: 1) channel is weak turbulence, 2) prepositive bandwidthes of transmitter and receiver are wide enough, 3) there is no multipath propagation, 4) shot noise induced by background optics is the main influencing element. Then the system will be influenced by scintillation of weak turbulence, path attenuation, and additive Gauss white noise, $n_0(t)$. If $P_e(P)$ is the bit error rate when P is fixed, the average bit error rate will be,

$$BER=\int_0^\infty P_e(P)\frac{1}{2\sqrt{2\pi}\sigma_x P}\exp\left[-(\ln\frac{P}{P_0}+2\sigma_x{}^2)^2\Big/8\sigma_x{}^2\right]\mathrm{d}P. \quad (3)$$

The uncoded error performance is discussed as follows.

Fig.2 gives the system model on above assumptions. In the figure, Because of the output of encoder is 0 or 1, the input of matched filter $x(t)$ is [$\sqrt{gP_t}$ +$n_0(t)$] or $n_0(t)$, whether the optical pulse exists or not, where P_t is the peak power of transmitted pulse, $n_0(t)$ is a Gaussian white noise with 0 mean and σ_n^2 variance. Therefore, the output of matched filter at T=T_s will be,

$$y(t)=\begin{cases}E_p+n(T_s)\,, & 1 \text{ is transmitted,}\\ n(T_s)\,, & 0 \text{ is transmitted.}\end{cases} \quad (4)$$

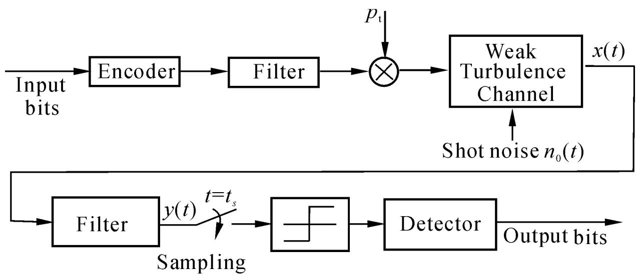

Fig.2 The system model

If the waveform is rectangular, $y(t)$ must be the convolution of $x(t)$ and $\sqrt{gP_t}$ in one bit time, then E_p=gP_tT_s, $n(T_s)$ is still Gauss noise with 0 mean, but the variance now is $\sigma^2=gP_tT_s{}^2\sigma_n^2$. We further assume that the input bits of 1 and 0 have equal probability, and p_0 and p_1 are the propabilities receiving 0 and 1, respectively, then p_0=(L-3)/L, p_1=3/L for FDPIM. If the threshold is β, and p_{e0} and p_{e1} are the propabilities transmiting 0 but receiving 1 or transmiting 1 but receiving 0, respectively, then we have

$$p_{e1}=\Phi((1-\beta)\sqrt{gP_t/\sigma_n^2}),\ \ p_{e0}=\Phi(\beta\sqrt{gP_t/\sigma_n^2}), \quad (5)$$

where $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty\exp(-z^2/2)\,\mathrm{d}z$. Thus the average slot error rate of FDPIM is

$$P_{se\mathrm{FDPIM}}=p_0\Phi\left(\beta\sqrt{gP_t/\sigma_n^2}\right)+p_1\Phi\left((1-\beta)\sqrt{gP_t/\sigma_n^2}\right). \quad (6)$$

For FDPIM, one slot error influences not only the symbol which it lies in, but also the following symbols. It is necessary to analyze the packet error rate, we also name it bit error rate(BER) because it consists of many bits. For a packet of N bits, after modulated by FDPIM, there are NL/M slots, and its BER is[9,10]

$$P_{pe\mathrm{FDPIM}}=1-(1-P_{se\mathrm{FDPIM}})^{NL/M}. \quad (7)$$

According to eqs. (7) and (3), the BER of FDPIM at weak turbulence may be written as

$$P_{p\mathrm{FDPIM}}=\int_0^\infty P_{pe\mathrm{FDPIM}}(P_t)\frac{1}{2\sqrt{2\pi}\sigma_x P_t}\exp\left[-(\ln\frac{P_t}{P_{t0}}+2\sigma_x{}^2)^2\Big/8\sigma_x{}^2\right]\mathrm{d}P_t. \quad (8)$$

If P is the average transmitting power, and for FDPIM, $P_{t\mathrm{FDPIM}}$=LP/3=(2^M+4)P/3, then BER of FDPIM at weak turbulence based on equal average transmitting power can be gotten from eq.(8).

Simulation results of DPIM and FDPIM at Gauss and weak turbulence channel for the values of $M=4$, $\sigma_x=0.25$, $\beta=0.5$, $g=1$, σ_n^2 = 1e-8 and $N=2048$, are shown in Fig.3.

The results show that the BER of weak turbulence is obviously worse than that of Gauss channel. However, BER degrades sharply with the increase of average transmitting power. When BER is 10^{-6}, the average transmitting power required by weak turbulence is about 1 dBm, which is more than the Gauss channel. In the same channel, BER of FDPIM is inferior to that of DPIM. This is because FDPIM is more complicated and has longer average symbol length, i.e. the advantage of FDPIM is at the cost of its BER performance.

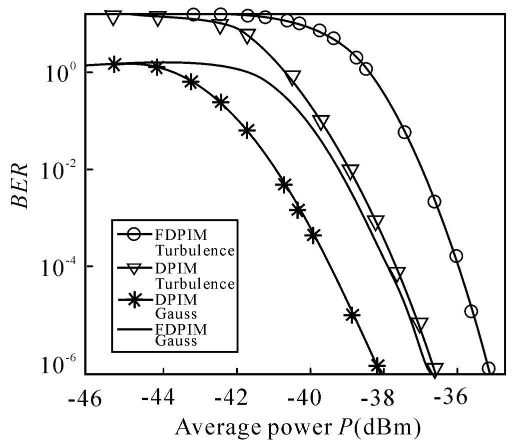

Fig.3 Packet error rate of FDPIM af different channels

The error performance of turbo coded FDPIM is discussed as follows.

To analyze error performance of Turbo coded FDPIM, three models are used in this paper. They are first modulation and later encoder (FMLE), first encoder and later modulation hard decision (FELMHD), and first encoder and later modulation soft decision (FELMSD), respeceively. The first two models are similar to that in Fig.4. The difference between the former and that in Fig.4 lies in the fact that it exchanges the position of modulation/demodulation and encoder/coding. The difference between the later and that in Fig.4 is that it uses hard decision to demodulate.

For the FMLE and the FELMHD, they both use hard decision, we can use the coding algorithm of OOK. As the input is x_k^s=0,1, we can translate the channel bits to

$v_k=(v_k^s,v_k^p)=(x_k^s-1/2,x_k^p-1/2)$, then the receiving bits are $w_k=(w_k^s,w_k^p)=(w_k^s-1/2,w_k^p-1/2)$.

Apriori information is defined as

$$L^a(x_k^s)=\ln\left(P(x_k^s=1)/P(x_k^s=0)\right)= \ln\left(P(v_k^s=1/2)/P(v_k^s=-1/2)\right) \quad . \tag{9}$$

If S_{on} and S_{off} are the state change induced by $x_k^s=1$ and $x_k^s=0$, respectively, the output of iterative decoder can be written as eq.(10)[16,17], where the first item is apriori information, the second item is the measurement of channel, and the third is outer information that is sent to the next order coding.

$$L(\hat{x}_k^s)=L^\alpha(\hat{x}_k^s)+L_c w_k^s+\ln\left(\sum_{S_{on}}\tilde{\alpha}_{k-1}(s')\cdot\gamma_k^{e'}(s',s)\cdot\tilde{\beta}_k(s)/\sum_{S_{off}}\tilde{\alpha}_{k-1}(s')\cdot\gamma_k^{e'}(s',s)\cdot\tilde{\beta}_k(s)\right)= L^a(x_k^s)+L_c(y_k^s-1/2)+L^e(x_k^s) \quad . \tag{10}$$

We can easily get the log-normal formula from eq.(10). If the transmitting information sequence u with length N conforms to random distribution, the final output sequence from above coding algorithm is $\hat{u}$ .They are compared and the number of their difference is recorded as M', then the error rate of this time is $P_e'=M'/N$. Finally P_{eFDPIM} can be worked out by using Monte Carlo algorithm. *BER* of the Turbo coded FDPIM at weak turbulence can be written as

$$P_{ptFDPIM}= \int_0^\infty P_{eFDPIM}(P_t)\frac{1}{2\sqrt{2\pi}\sigma_x P_t}\exp\left[-(\ln\frac{P_t}{P_{t0}}+2\sigma_x{}^2)^2\Big/8\sigma_x{}^2\right]\mathrm{d}(P_t) \tag{11}$$

In the model as shown in Fig.4, we use an avalanche photodiode (APD) to detect soft bit information.We assume that $p_s(\cdot)$ and $p_N(\cdot)$ are the PDF of soft output when there is optical slot or not ,respectively, $c=(c_0, c_1)$ is a probable constituent code, where $k=1,2,\cdots N+\gamma$, N and γ are the interleaver length and the number of constituent code register,

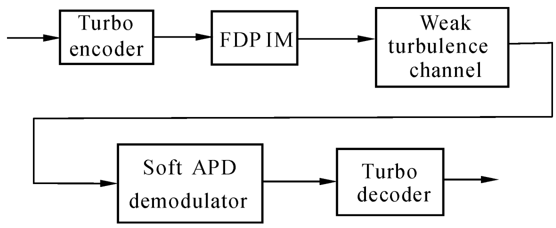

Fig.4 Soft-decision analysis model for FELM

respectively, and $P_k(c,I)$ is the apriori information of k_{th} constituent encoder.

If we use the decoder shown in Fig.5, where $P(u,I)$, $P(c,I)$ and $P(u,O)$ represent the probability of input, checkout, and output sequence, respectively, then the apriori probability $P_k(c,I)$ can be written as[11-13],

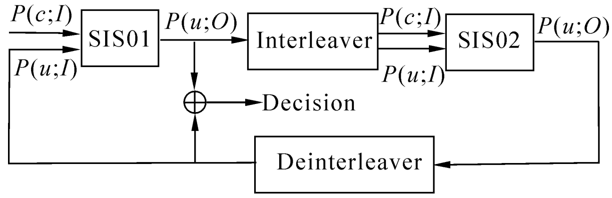

Fig.5 The decoder structure

$$P_k(c,I)=\begin{cases}\left[\dfrac{\sum\limits_{\substack{a=(a_1,\cdots a_4)\\ a_l=c_0}} L_a^{(s)}}{\sum\limits_{i=0}^{19} L_i^{(s)}}\right]\cdot\left[\dfrac{\sum\limits_{\substack{a=(a_1,\cdots a_4)\\ a_m=c_1}} L_a^{(t)}}{\sum\limits_{i=0}^{19} L_i^{(t)}}\right], & c_0,c_1 \text{ are in different FDPIM symbols},\\[2ex] \dfrac{\sum\limits_{\substack{a=(a_1,\cdots a_4)\\ a_l=c_0,a_m=c_1}} L_a^{(s)}}{\sum\limits_{i=0}^{19} L_i^{(s)}}, & c_0,c_1 \text{ are in the same FDPIM symbols},\end{cases} \tag{12}$$

where $L_i^{(j)}$ represents the likelihood function of i_{th} slot of j_{th} FDPIM frame. If the model of photoelectron count conforms to Poisson distribution, and K_b+K and K_b represent the average counts when there is optical pulse slot or not, respectively, and K_s is the average counts induced by optical pulse, then $L_i^{(j)}$ can be expressed as [13]

$$L_i^{(j)}=p_s(y_i)/p_N(y_i)\propto\left(1+K_s/K_b\right)^{y_i}. \tag{13}$$

For comparison, we let the length of information bits be the same, N=2048. According to the three methods above, the simulation results of Turbo coded FDPIM and DPIM with Log-MAP algorithm under weak turbulence are shown in Fig.6 when $M=4$, $\sigma_x=0.25$, $g=1$, $\sigma_n^2=10^{-8}$, code rate = 1/2, constituent code is $g(7,5)$ and iterative number is 5.

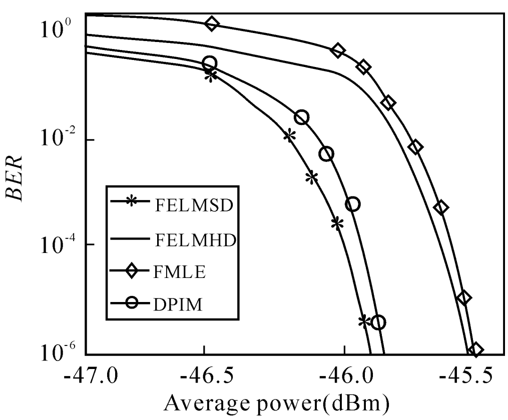

Fig.6 Comparison of the BER for

The results for FDPIM show that the *BER*s using the methods of 1 and 2 are much similar, and that using the third method can save average transmitting power. For example, when BER is 10^{-6}, the third method can save the average transmitting power about 0.4 dBm. So that, the third method is obviously superior to the methods of 1 and 2 when the average transmitting powers are equal. What is more, the BER of Turbo coded FDPIM in the soft decision is better than that in the hard decision. That is, we will use soft decision to improve the error performance.

At the same time, the results in Fig.6 and Fig.3 show that, under weak turbulence, the average transmitting power required by uncoded FDPIM is about -35 dBm, and that required by Turbo coded FDPIM is between -45.4 and -45.8 dBm when *BER* is 10^{-6}. So that, the average transmitting power will be degraded about 10 dBm by using Turbo code.

In this paper, a new modulation scheme, FDPIM, based on DPIM and PPM is presented. By giving its modulation structure, the BERs of uncoded and Turbo coded FDPIM under weak turbulence are analyzed. The results show that, even though the error performance of FDPIM is not very satisfying, it has the fixed symbol length, and is easy to be implemented and to joint the Turbo code. By using the soft decision, the average transmitting power can decrease about 10 dBm. FDPIM could yet be regarded as a modulation scheme with a good application prospect in the OWC systems.

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