Theoretical analysis of the relationship between the Brillouin gain coefficient and the strain in the optical-fiber sensors*

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(Received 2 January 2008)

The relation between the power of the Brillouin signal and the strain is one of the bases of the distributed fiber sensors of temperature and strain. The coefficient of the Brillouin gain can be changed by the temperature and the strain that will affect the power of the Brillouin scattering. The relation between the change of the Brillouin gain coefficient and the strain is thought to be linear by many researchers. However, it is not always linear based on the theoretical analysis and numerical simulation. Therefore, errors will be caused if the relation between the change of the Brillouin gain coefficient and the strain is regarded as to be linear approximately for measuring the temperature and the strain. For this reason, the influence of the parameters on the Brillouin gain coefficient is proposed through theoretical analysis and numerical simulation.

CLC numbers: TN253 **Document code:** A **Article ID:** 1673-1905(2008)04-0276-4

DOI 10.1007/s11801-008-8007-5

Distributed-fiber temperature and strain sensors have attracted interests of many people since they can be used in wide fields. At present, many papers on the relationship between the Brillouin frequency shift and the strain can be found[1-4]. The relation between the change of the Brillouin power and the strain has been reported. The relation between the change of the Brillouin gain coefficient and the strain is thought to be linear in Ref. [5], which is $\Delta g_B = -2.73\times10^{-10}\Delta\varepsilon$. However, in this paper we find that the relation between the change of the Brillouin gain coefficient and the strain is not always linear based on the theoretical analysis and numerical simulation, and this relation has been observed in experiment[6]. Therefore, errors will be caused if the relation between the change of the Brillouin gain coefficient and the strain is regarded as to be linear approximately for measuring the temperature and the strain. For this reason, the influence of the parameters on the Brillouin gain coefficient is proposed through theoretical analysis and numerical simulation.

In the power testing of Brillouin sensors, we can obtain the change of the temperature and the strain by measuring the relative change of power. However, the coefficient of the Brillouin gain may be changed by the temperature and the strain that will affect the power of the Brillouin scattering. So we must get the relation between the change of the Brillouin gain coefficient and the strain in order to get the coefficient of the change with the change of the Brillouin power. If the difference between the frequency of the pump light and that of the probe light is equal to the Brillouin frequency shift, the coefficient of the Brillouin gain reaches the peak g_B,

$$g_B = \frac{2\pi^2 n^7 p_{12}^2}{c\lambda^2 \Delta\nu_B}\sqrt{\frac{(1+\sigma)(1-2\sigma)}{\rho E(1-\sigma)}}, \quad (1)$$

where λ is the wavelength of the incident light, p_{12} is the longitudinal elasto-optic coefficient, n is the core refractive index, ρ is the mass density, σ is the Poisson ratio, E is the Young's module, and $\Delta\nu_B$ is the width of the Brillouin gain spectrum which is expressed as[6],

$$\Delta\nu_B = \frac{8n^2\eta_s}{3\pi\lambda^2\rho}, \quad (2)$$

where η_s is the shear viscosity coefficient.

If only the strain changes and the temperature is assumed to be invariable, the change of the Brillouin gain coefficient is,

* This work has been supported by Talent Supporting Project of Education Ministry of China (Grant No. NCET-05-0897), and Scientific Research Project for Universities in Xinjiang (Grant No. XJEDU2004 E02 and XJEDU2006110)

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$$\Delta g_B(\Delta\varepsilon) = g_B(T,\Delta\varepsilon) - g_B(T,0) =$$

$$\left[\frac{\partial g_B}{\partial n}\frac{\partial n}{\partial \varepsilon} + \frac{\partial g_B}{\partial \rho}\frac{\partial \rho}{\partial \varepsilon} + \frac{\partial g_B}{\partial \sigma}\frac{\partial \sigma}{\partial \varepsilon} + \frac{\partial g_B}{\partial E}\frac{\partial E}{\partial \varepsilon}\right] \times \Delta\varepsilon, \quad (3)$$

where $\frac{\partial g_B}{\partial n}$, $\frac{\partial g_B}{\partial \rho}$, $\frac{\partial g_B}{\partial \sigma}$ and $\frac{\partial g_B}{\partial E}$ can be obtained from eq. (1).

n, ρ, σ and E will change with the change of the strain, which can be expressed as[6-8],

$$n = n_0\left(1 - \frac{n_0^2}{2}\left[p_{12} - \sigma(p_{11} + p_{12})\right]\varepsilon\right), \quad (4)$$

$$\rho = \frac{\rho_0}{(1+\varepsilon)(1-\sigma\varepsilon)^2}, \quad (5)$$

$$\sigma = \sigma_0(1 + k\varepsilon), \quad (6)$$

$$E = E_0(1 + \alpha\varepsilon), \quad (7)$$

where p_{11} and p_{12} are the lateral and the longitudinal elasto-optic coefficients, respectively, k is the Poisson strain coefficient, and α is the strain coefficient of Young's module.

Therefore, $\frac{\partial n}{\partial \varepsilon}$, $\frac{\partial \rho}{\partial \varepsilon}$, $\frac{\partial \sigma}{\partial \varepsilon}$ and $\frac{\partial \sigma}{\partial \varepsilon}$ can be derived from eqs. (4)-(7), and we can obtain the change of the Brillouin gain coefficient from eq. (3).

We take the parameters in Refs. [6-8], which are p_{11} = 0.12, p_{12} =0.27, n_0 =1.47, E_0 =64.1 × 10^9 Pa, α = 8.7, σ_0= 1.7, ρ_0= 2220 kg/m^3, c = 3 × 10^8 m/s, η_s= 0.11, λ =1.31 × 10^{-9} m, and k = 22.

Fig.1 shows the relation between the change of the Brillouin gain coefficient and the strain, and the trend of the change is the same as the experimental result in Ref. [6]. The result reveals that the trend of the change of the Brillouin gain coefficient is a downward concave curve, but generally it is regarded as to be linear approximately for measuring the temperature and the strain which will cause errors. We can

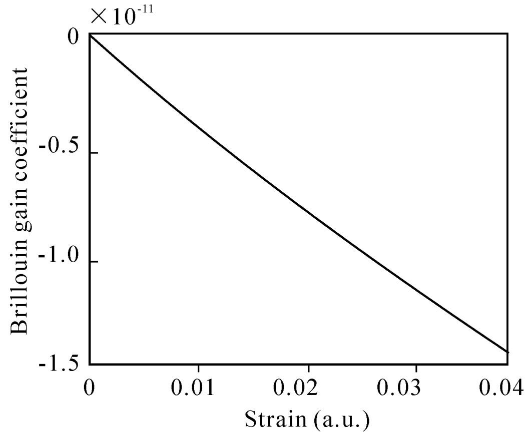

Fig.1 Relation between the Brillouin gain coefficient and the strain

use an appropriate optical fiber to reduce the errors and to improve the precision.

To find the most appropriate optical fiber, we will discuss all parameters that relate to the change of the Brillouin gain coefficient. Because the parameters n_0, E_0, α, σ and k are related to the material of the optical fiber, their values may affect the change of the Brillouin gain coefficient, and we will discuss them below. In this discussion, we calculate the *MSE* (the square error between the exact change of the Brillouin gain coefficient and its approximate linear value).

If only α changes and the other parameters are assumed to be invariable, we can obtain the relation between the

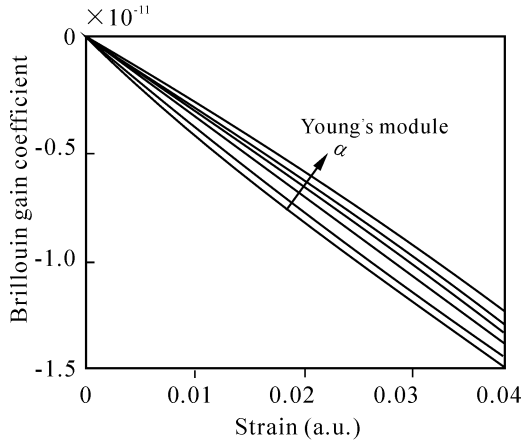

Fig.2 Relation between the change of the Brillouin gain coefficient and the strain for different α values of α = 10, 8.7, 7, 6, 5 and 4, respectively.

Brillouin gain coefficient and the strain as shown in Fig.2. These curves belong fo different α values of 10, 8.7, 7, 6, 5 and 4 from down to top. The curves change with the α from 10 to 4, so we can find a curve that is more like a beeline. Hence, we calculate the *MSE* for different α values (Fig. 3). *MSE* is minimal when α is about 6.06.

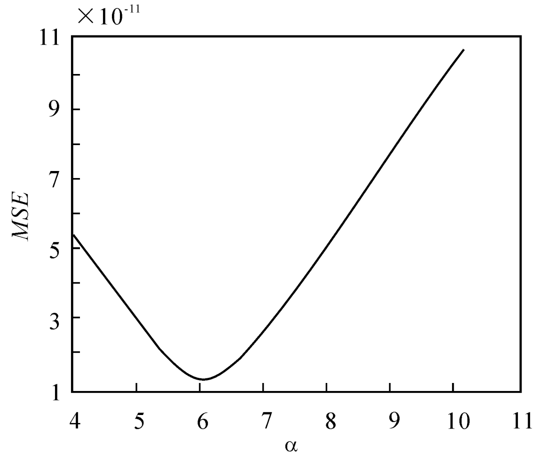

Fig.3 Relation between the *MSE* of the change of the Brillouin gain coefficient and α.

If n_0 changes and the other parameters are assumed to be invariable, the relation between the *MSE* and α can be obtained as shown in Fig.4. It can be seen that the *MSE* becomes bigger when n_0 changes from 1.3 to 1.7.

Fig.5, shows that the *MSE* has the minima with the change of $\boldsymbol{\alpha}$, which are nearly the same for different E_0 values.

As shown in Fig.6, the *MSE* has the minima with the change of α, which are different for different σ values.

As shown in Fig.7, the *MSE* has the minima with the change of α, which increase with the increasing of κ.

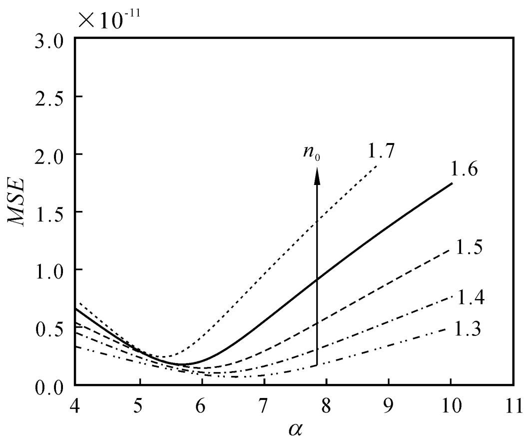

Fig.4 Relation between the *MSE* of the change of the Brillouin gain coefficient and α for different n_0 values.

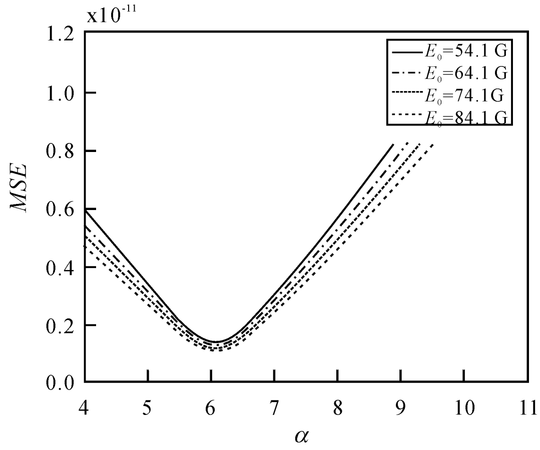

Fig.5 Relation between the *MSE* of the Brillouin gain coefficient and α for different E_0 values.

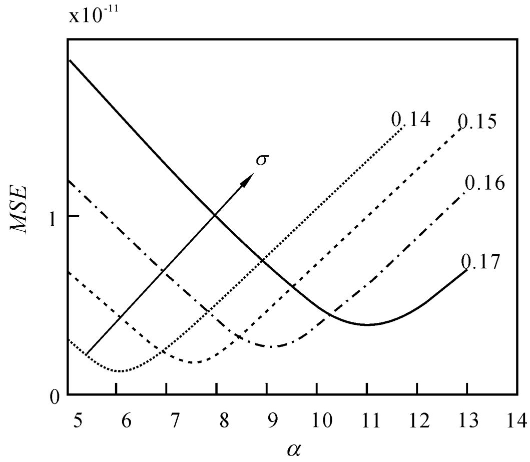

Fig.6 Relation between the *MSE* of the Brillouin gain coefficient and α for different σ Values.

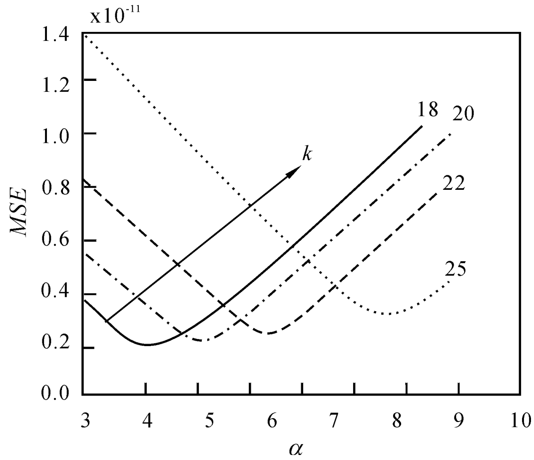

Fig.7 Relation between the *MSE* of the Brillouin gain coefficient and α for different *k* Values.

In conclusion, the relation between the change of the Brillouin gain coefficient and the strain is not always linear We can always find that a curve is more like a beeline, which is convenient for the testing of the temperature and the strain by measuring the change of the Brillouin power and can make the result more precise. Parameters α, σ, and k are the main influence factors in the above simulation. So the theoretical analysis will give us a help for devising optical-fiber sensors and selecting the more appropriate optical fibers.

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