# Teleportation of an arbitrary unknown *N*-qubit entangled state under the controlling of *M* controllers*

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A new quantum protocol to teleport an arbitrary unknown $N$-qubit entangled state from a sender to a fixed receiver under $M$ controllers($M < N$) is proposed. The quantum resources required are $M$ non-maximally entangled Greenberger-Horne-Zeilinger (GHZ) state and $N$-$M$ non-maximally entangled Einstein-Podolsky-Rosen (EPR) pairs. The sender performs $N$ generalized Bell-state measurements on the $2N$ particles. Controllers take $M$ single-particle measurement along x-axis, and the receiver needs to introduce one auxiliary two-level particle to extract quantum information probabilistically with the fidelity unit if controllers cooperate with it.

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The principle of quantum mechanics supplies many interesting applications in the field of information in the last decades, such as quantum computation[1-2], quantum cryptography[3-4] and quantum teleportation[5-6]. The quantum teleportation process allows telaporting an unknown state from a sender Alice to a remote receiver Bob, with the help of quantum resources such as entangled states of Einstein-Podolsky-Rosen (EPR) pairs[7], Greenberger-Horne-Zeilinger (GHZ) pairs[8], W-state[9-11] and classical communication. It can be said that any tasks in quantum information processing and distributed quantum computing can be performed by using the enough quantum resource and the classical communication, although the classical communication is relatively cheap, the generation and distribution of entangled states (i. e., establishment of quantum channels) are expensive. Therefore the common strategy aims to consume as less as possible the quantum resource for a concrete task. In addition, most of the teleportation schemes, exploited the maximally entangled states as the quantum channels[12-18]. But in real situation the sender and the receiver may not share maximally entangled state. So in practical quantum information processing, we must consider the above two aspects.

As known to all, quantum secret sharing (QSS) [19-22]can be also looked upon as the so-called controlled teleportation (CT): Alice teleports her quantum state to a Bob, the receiver, under control of the remaining Bobs, the controllers. In the protocols mentioned above, a receiver and a controller may be exchanged: any Bob can serve as either of the receiver or a controller. The role of the controllers is that they are given the right to decide whether and when the task should be processed, while the role of the receiver is to further handle the state after obtaining it. So, as far as the CT is concerned, it is more reasonable to assign beforehand (rather than to choose at random) who are controllers and who is the receiver. The appropriate assignment depends on concrete factors such as available equipments and specialized functions of the partners.

This work concerns with probably CT of the most general $N$-qubit entangled state. For convenience, we name the receiver Bob and the $M$ controllers Charlie$_1$, Charlie$_2$,⋯ Charlie$_M$ . Our protocol works for $M < N$ and consumes $M$ GHZ trios plus ($N$ -$M$) EPR pairs. So this scheme is not symmetry, namely the receiver and a controller can't exchange. Then this CT scenario can't be looked as QSS. From another point, in this controlled teleportation, the numbers of GHZ state are $M$ and not $N$($M < N$), so this protocol is very economical.

In Ref.[23], Man and Xia have proposed a very economical and feasible way to teleport an arbitrary unknown $N$-qubit entangled state from a sender to a fixed receiver under control of $M$ ($M < N$) controllers. In that scheme, the quantum resources required are $M$ maximally entangled GHZ trios plus and ($N$ -$M$) maximally entangled EPR pairs. In this paper, we expend Ref. [23] by using $M$ non-maximally entangled GHZ trios plus and ($N$ -$M$) non-maximally entangled EPR pairs. In this way, when the receiver Bob introduces an auxiliary particle A, the successful probability will be raised.

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Now let us depict our protocol. For simplicity we first let $N = 2$, $M = 1$. The state of Alice's two qubits of 1 and 2 which Alice wishes to teleport to Bob under control of Charlie will be of the form,

$$|\chi\rangle_{12} = \alpha|00\rangle_{12} + \beta|10\rangle_{12} + \gamma|01\rangle_{12} + \delta|11\rangle_{12}, \tag{1}$$

where nothing is known about the coefficients of $\alpha$, $\beta$, $\gamma$ and $\delta$ except that they satisfy the normalization $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Our protocol requires that Alice, Bob and Charlie need sharing a GHZ state,

$$|G\rangle_{A_1B_1C_1} = \mu'|000\rangle_{A_1B_1C_1} + v'|111\rangle_{A_1B_1C_1}, \tag{2}$$

of which qubit $A_1$ ($B_1$ and $C_1$ ) is held by Alice (Bob and Charlie), and the coefficients $\mu'$ and $v'$ are known by Bob. They satisfy the normalization $|\mu'|^2 + |v'|^2 = 1$. In addition, Alice and Bob share an EPR pair,

$$|B\rangle_{A_2B_2} = \mu|00\rangle_{A_2B_2} + v|11\rangle_{A_2B_2}, \tag{3}$$

where the coefficients $\mu$ and $v$ are known by Bob too. They satisfy the normalization $|\mu|^2 + |v|^2 = 1$. Qubit $A_2$ is with Alice, and qubit $B_2$ is with Bob. Before going into any detail, we introduce some notations. The two eigenstates of a qubit in the $z$-basis $\{|0\rangle, |1\rangle\}$ are related to those in the $x$-basis $\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$,

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{4}$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{5}$$

Without losing the generalization, we define another complete orthogonal set of four states, called generalized Bell states, which for a system of two arbitrary qubits $X$ and $Y$ are given by

$$|B_{00}\rangle_{XY} = a|00\rangle_{XY} + b|11\rangle_{XY}, \tag{6}$$

$$|B_{01}\rangle_{XY} = c|01\rangle_{XY} + d|10\rangle_{XY}, \tag{7}$$

$$|B_{10}\rangle_{XY} = d|01\rangle_{XY} - c|10\rangle_{XY}, \tag{8}$$

$$|B_{11}\rangle_{XY} = b|00\rangle_{XY} - a|11\rangle_{XY}, \tag{9}$$

where $|a|^2+|b|^2 = |c|^2+|d|^2 = 1$ and ${}_{xy}\langle B_{ii'} || B_{jj'}\rangle_{xy} = \delta_{ii'}\,\delta_{jj'}$ . We assume that Alice is able to perform a generalized-Bell-state measurement (GBM), which is meant as a projective measurement onto one of the four generalized Bell states given by eqs. (6)-(9). The coefficients of $a, b, c$, d are known by Bob too. The combined state of the total system composed of the secret state $|\chi\rangle_{12}$ and the quantum channels $|G\rangle_{A_1B_1C_1}$ and $|B\rangle_{A_2B_2}$ can be represented as,

$$|T\rangle_{12A_1B_1C_1A_2B_2} = |\chi\rangle_{12}|G\rangle_{A_1B_1C_1}|B\rangle_{A_2B_2} =$$
$$\frac{1}{\sqrt{2}}\sum_{i,j,k,l,m=0}^{1}|B_{ij}\rangle_{1A_2}|B_{kl}\rangle_{2A_1}|m\rangle_{C_1}|\phi_{ijklm}\rangle_{B_2B_1}, \tag{10}$$

where

$$|\phi_{ijkl}\rangle_{B_2B_1} = \xi_{ijkl}|00\rangle_{B_2B_1} + \zeta_{ijkl}|01\rangle_{B_2B_1} + \sigma_{ijkl}|10\rangle_{B_2B_1} + \tau_{ijkl}|11\rangle_{B_2B_1}.$$

It is now clear from eq. (10) that if Alice makes GBMs on qubit-pairs $(1, A_2)$ and $(2, A_1)$ with outcomes $\{i, j, k, l\}$ corresponding to find $|B_{ij}\rangle_{1A_2}$, $|B_{kl}\rangle_{2A_1}$, and Charlie measures qubit $C_1$ in the $x$-basis with an outcome $\{m\}$ corresponding to find $|\tilde{m}\rangle$ , then the state of Bob's qubits $B_2$ and $B_1$ is projected onto $|\phi_{ijkl}\rangle$. If both Alice and Charlie communicate with Bob via reliable classical channels about their measurement outcomes, then Bob is able to transform $|\phi_{ijkl}\rangle$ to Alice's original state $|\chi\rangle$ in eq. (1).

Firstly, Bob performs on qubits $B_2$ and $B_1$ single-qubit unitary operations $U_1$ which are composed of Pauli operators, $\sigma_x$, $\sigma_y$, and $\sigma_z$. Secondly, Bob introduces an auxiliary particle A with its initial state $|0\rangle_A$, and makes another unitary transformation $U_2$ for particles B2, B1, and A. Finally, Bob measures the state of auxiliary particle A. If the result $|0\rangle_A$ is measured, quantum teleportation is successful. Otherwise, teleportation fails.

Although non-maximally entangled quantum channel is more practical than maximally entangled ones, the success probability is always less than unity, so the scheme becomes probabilistic rather than deterministic. Tab.1 shows the explicit expressions of measurement results of *ijklm* and unitary transforms of $U_1$ and $U_2$.

**Tab.1 The expressions of measurement results of *ijklm* and unitary transforms of $U_1$ and $U_2$**

| case | *ijklm* | $U_1$ | $U_2$ |
|---|---|---|---|
| 1 | 00000 | $(I \otimes I)_{B_2B_1}$ | (1) |
| 2 | 00001 | $(I \otimes \sigma_z)_{B_2B_1}$ | |
| 3 | 00110 | $(I \otimes \sigma_z)_{B_2B_1}$ | (2) |
| 4 | 00111 | $(I \otimes I)_{B_2B_1}$ | |
| 5 | 11000 | $(\sigma_z \otimes I)_{B_2B_1}$ | (3) |
| 6 | 11001 | $(\sigma_z \otimes \sigma_z)_{B_2B_1}$ | |
| 7 | 11110 | $(\sigma_z \otimes \sigma_z)_{B_2B_1}$ | (4) |
| 8 | 11111 | $(\sigma_z \otimes I)_{B_2B_1}$ | |
| 9 | 00010 | $(\sigma_z \otimes i\sigma_y)_{B_2B_1}$ | (5) |
| 10 | 00011 | $(I \otimes i\sigma_y)_{B_2B_1}$ | |
| 11 | 00100 | $(I \otimes i\sigma_y)_{B_2B_1}$ | (6) |
| 12 | 00101 | $(I \otimes \sigma_x)_{B_2B_1}$ | |

| | | | |
|---|---|---|---|
| 13 | 11010 | $(\sigma_z \otimes \sigma_x)_{B_2B_1}$ | (7) |
| 14 | 11011 | $(\sigma_z \otimes i\sigma_y)_{B_2B_1}$ | |
| 15 | 11100 | $(\sigma_z \otimes i\sigma_y)_{B_2B_1}$ | (8) |
| 16 | 11101 | $(\sigma_z \otimes \sigma_x)_{B_2B_1}$ | |
| 17 | 01110 | $(\sigma_x \otimes \sigma_z)_{B_2B_1}$ | (9) |
| 18 | 01111 | $(\sigma_x \otimes I)_{B_2B_1}$ | |
| 19 | 10000 | $(i\sigma_y \times I)_{B_2B_1}$ | (10) |
| 20 | 10001 | $(i\sigma_y \times \sigma_z)_{B_2B_1}$ | |
| 21 | 10110 | $(i\sigma_y \otimes \sigma_z)_{B_2B_1}$ | (11) |
| 22 | 10111 | $(i\sigma_y \otimes I)_{B_2B_1}$ | |
| 23 | 01000 | $(\sigma_x \otimes I)_{B_2B_1}$ | (12) |
| 24 | 01001 | $(\sigma_x \otimes \sigma_z)_{B_2B_1}$ | |
| 25 | 01010 | $(\sigma_x \otimes \sigma_x)_{B_2B_1}$ | (13) |
| 26 | 01011 | $(\sigma_x \otimes i\sigma_y)_{B_2B_1}$ | |
| 27 | 01100 | $(\sigma_x \otimes i\sigma_y)_{B_2B_1}$ | (14) |
| 28 | 01101 | $(\sigma_x \times i\sigma_y)_{B_2B_1}$ | |
| 29 | 10010 | $(i\sigma_y \otimes \sigma_x)_{B_2B_1}$ | (15) |
| 30 | 10011 | $(i\sigma_y \otimes i\sigma_y)_{B_2B_1}$ | |
| 31 | 10100 | $(i\sigma_y \otimes i\sigma_y)_{B_2B_1}$ | (16) |
| 32 | 10101 | $(i\sigma_y \otimes \sigma_x)_{B_2B_1}$ | |

To reconstruct the original state under the basis $\{|000\rangle_{B2B1A}, |100\rangle_{B2B1A}, |010\rangle_{B2B1A}, |110\rangle_{B2B1A}, |001\rangle_{B2B1A}, |101\rangle_{B2B1A}, |011\rangle_{B2B1A}, |111\rangle_{B2B1A}\}$, the unitary transformation $U_2$ may be taken as the following form,

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{11}$$

where $A_1$ and $A_2$ are $4 \times 4$ matrices which can be written as

$$A_1 = \mathrm{Diag}(a_1, a_2, a_3, a_4) \text{ , and}$$
$$A_2 = \mathrm{Diag}(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \sqrt{1-a_4^2}) , \tag{12}$$

where $a_i$ ($i$ = 1,2,3,4 and $|a_i| \leqslant 1$). Its value depends on the measurement result of *ijklm*.

Tab.2 shows the explicit values of $a_1$, $a_2$, $a_3$, and $a_4$.

**Tab.2 Explicit values of $a_1$, $a_2$, $a_3$, and $a_4$. of $U_2$**

| $U_2$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| (1) | 1 | $\mu a/\nu b$ | $\mu' a/\nu' b$ | $\mu\mu' a^2/\nu\nu' b^2$ |
| (2) | 1 | $\mu a/\nu b$ | $\mu' b/\nu' a$ | $\mu\mu'/\nu\nu'$ |
| (3) | 1 | $\mu b/\nu a$ | $\mu' b/\nu' a$ | $\mu\mu'/\nu\nu'$ |
| (4) | 1 | $\mu b/\nu a$ | $\mu' b/\nu' a$ | $\mu\mu' b^2/\nu\nu' a^2$ |
| (5) | 1 | $\mu a/\nu b$ | $\nu' c/\mu' d$ | $\mu\nu' ac/\nu\mu' bd$ |
| (6) | 1 | $\mu a/\nu b$ | $\nu' d/\mu' c$ | $\mu\nu' ad/\nu\mu' bc$ |
| (7) | 1 | $\mu b/\nu a$ | $\nu' c/\mu' d$ | $\mu\nu' bc/\nu\mu' ad$ |
| (8) | 1 | $\mu b/\nu a$ | $\nu' d/\mu' c$ | $\mu\nu' bd/\nu\mu' ac$ |
| (9) | 1 | $\nu c/\mu d$ | $\nu\mu' b/\mu\nu' a$ | $\nu\mu' cb/\mu\nu' da$ |
| (10) | 1 | $\nu d/\mu c$ | $\mu' a/\nu' b$ | $\nu\mu' da/\mu\nu' cb$ |
| (11) | 1 | $\mu c/\nu d$ | $\mu' b/\nu' a$ | $\nu\mu' db/\mu\nu' ac$ |
| (12) | 1 | $\nu c/\mu d$ | $\mu' a/\nu' b$ | $\nu\mu' ca/\mu\nu' db$ |
| (13) | 1 | $\nu c/\mu d$ | $\nu' c/\mu' d$ | $\nu\nu' c^2/\mu\mu' d^2$ |
| (14) | 1 | $\nu c/\mu d$ | $\nu' d/\mu' c$ | $\nu'\nu/\mu'\mu$ |
| (15) | 1 | $\nu d/\mu c$ | $\nu' c/\mu' d$ | $\nu'\nu/\mu'\mu$ |
| (16) | 1 | $\nu d/\mu c$ | $\nu' d/\mu' c$ | $\nu'\nu d^2/\mu\mu' c^2$ |

For example, if $i = 0$, $j = 1$, $k = 1$, $l = 0$, and $m = 0$, and making the unitary transformation $U_1$ ($\sigma_x$ and $i\sigma_y$) in Tab.1 for $B_2$ and $B_1$, respectively, Bob introduces an auxiliary particle A with initial state $|0\rangle_A$, the state of particles $B_2$ and $B_1$ will become,

$$|\phi_{0110}\rangle_{B_2B_1}|0\rangle_A = \nu\nu' cd\alpha|000\rangle_{B_2B_1A} + \mu\nu' d^2\beta|100\rangle_{B_2B_1A} +$$
$$\nu\mu' c^2\gamma|010\rangle_{B_2B_1A} + \mu\mu' dc\delta|110\rangle_{B_2B_1A}, \tag{12}$$

So we choose $U_2(14)$ in Tab.2 as,

$a_1 = 1$, $a_2 = \dfrac{\nu c}{\mu d}$, $a_3 = \dfrac{\nu' d}{\mu' c}$, and $a_4 = \dfrac{\nu'\nu}{\mu'\mu}$ as shown in Tab.2, then

$$U_2(14)\ |\phi_{0110}\rangle_{B_2B_1}|0\rangle_A = \nu'\nu\ cd(\alpha\ |00\rangle_{B_2B_1} + \beta\ |10\rangle_{B_2B_1} +$$
$$\gamma|01\rangle_{B_2B_1} + \delta\ |11\rangle_{B_2B_1})|0\rangle_A + (\nu' d\sqrt{\mu^2 d^2 - \nu^2 c^2}\beta|10\rangle_{B_2B_1} +$$
$$\nu c\sqrt{\mu'^2 c^2 - \nu'^2 d^2}\gamma|01\rangle B_2B_1 + dc\sqrt{\mu'^2\mu^2 - \nu'^2\nu^2}\delta|11\rangle_{B_2B_1})|1\rangle_A \tag{13}$$

Finally, Bob measures the state of auxiliary particle A. If the result $|0\rangle_A$ is measured, quantum teleportation is successfully realized with the probability of 16 in Tab.2, Otherwise, teleportation fails. It is obvious when

$$a = b = \frac{1}{\sqrt{2}},\ c = d = \frac{1}{\sqrt{2}},\ \mu = \nu = \frac{1}{\sqrt{2}},\ \mu' = \nu' = \frac{1}{\sqrt{2}} ,$$

the auxiliary particle A is not needed.

Next we generalize the case in the previous section with $N = 2$ and $M = 1$ to an arbitrary $N \geqslant 2$ and $M < N$. The $N$-qubit state to be teleported is the form,

$$|S\rangle_{1,2\cdots N} = \sum_{a_1,a_2\cdots a_N=0}^{1} \alpha_{a_1,a_2\cdots a_N}|a_1, a_2\cdots a_N\rangle_{1,2\cdots N} , \tag{14}$$

where $\sum_{a_1,a_2\cdots a_N=0}^{1}|\alpha_{a_1,a_2\cdots a_N}|^2 = 1$, and $\alpha_{a_1,a_2\cdots a_N} \neq \prod_{i=1}^{N}\alpha_{a_i}$.

To realize this general task, the partners need sharing $M$ GHZ trios and $N$ - $M$ EPR pairs,

$$|G\rangle_{A_iB_iC_i} = \mu'|000\rangle_{A_iB_iC_i} + \nu'|111\rangle_{A_iB_iC_i} , \tag{15}$$

where $i = 1,2\cdots\cdots M$ , $|\mu'|^2 + |\nu'|^2 = 1$ , and

$$|B\rangle_{A_jB_j} = \mu|00\rangle_{A_jB_j} + \nu|11\rangle_{A_jB_j} , \tag{16}$$

where $j = M+1, M+2, \cdots N$, $|\mu|^2 + |\nu|^2 = 1$. For each GHZ trio $|G\rangle_{A_iB_iC_i}$, qubit $A_i$ is held by Alice, and qubit $B_i$ and $C_i$ are held by Bob and Charlie, respectively. For each EPR pair $|B\rangle_{A_jB_j}$, qubit $A_j$ is kept by Alice, and Bob keeps particle $B_j$. The total system state composed of Alice's secret $|S\rangle_{1,2\cdots N}$ and the states of the quantum channels of $|G\rangle_{A_iB_iC_i}$ and $|B\rangle_{A_jB_j}$ are

$$|T\rangle = |S\rangle_{1,2\cdots N} \otimes \prod_{i=1}^{M} |G\rangle_{A_iB_iC_i} \otimes \prod_{j=M+1}^{N} |B\rangle_{A_jB_j} =$$

$$(\frac{1}{\sqrt{2}})^M \left|B_{k_1l_1}\right\rangle_{1A_1} \left|B_{k_2l_2}\right\rangle_{2A_2} \cdots \left|B_{k_Nl_N}\right\rangle_{NA_N} |\widetilde{m}_1\rangle_{C_1} |\widetilde{m}_2\rangle_{C_2} \cdots |\widetilde{m}_M\rangle_{C_M} \times (\left|\varphi_{k_1l_1\ldots k_Nl_Nm_1\ldots m_M}\right\rangle_{B_1B_2\ldots B_N}) , \quad (17)$$

where

$$\left|\varphi_{k_1l_1\ldots k_Nl_Nm_1\ldots m_M}\right\rangle_{B_1B_2\ldots B_N} = \sum_{i_1,i_2\ldots i_N=0}^{1} X_{i_1\ldots i_N}^{k_1l_1\ldots k_Nl_Nm_1\ldots m_M} \left|i_1i_2\ldots i_N\right\rangle_{B_1B_2\ldots B_N}.$$

The coefficients $X_{i_1i_2\ldots i_N}^{1A_1\ldots NA_Nm_1\ldots m_M}$ are determined by the measurement results of both Alice and Charlie. The process is as follows,

1) Alice has the unknown state $|S\rangle_{1,2\ldots N}$, $M$ non-maximally entangled GHZ states $|G\rangle_{A_iB_iC_i}$ in the eq. (15) with $i$=1,2,… $M$ and $N$-$M$ non-maximally entangled EPR pairs of $|B\rangle_{A_jB_j}$ in the eq. (16) with $j = M+1,\ldots N$. Alice sends qubits $B_1, B_2\ldots B_M$ and $B_{M+1}, \ldots B_N$ to Bob, and $C_1, C_2\ldots C_M$ to Charlie, respectively.

2) Alice makes $N$ GBMs on the qubit pairs of $\{1, A_1\}$, $\{2, A_2\}$, … and $\{N, A_N\}$ with the outcomes $\{k_1, l_1\}$, $\{k_2,l_2\}$, …and $\{k_N,l_N\}$, if she finds $\left|B_{k_1l_1}\right\rangle_{1A_1}$, $\left|B_{k_2l_2}\right\rangle_{2A_2}$,... and $\left|B_{k_Nl_N}\right\rangle_{NA_N}$, respectively.

3) Charlie makes a single particle measurement along $X$-axis on particles $C_1, C_2, \ldots C_M$, respectively. The value is represented by $m_i (i = 1, 2, \ldots M) = 0$ or 1.

4) According to the result of Alice and Charlie, Bob needs single-qubit operation on $B_1, B_2, \ldots B_M$ with $I, \sigma_x, i\sigma_y$ or $\sigma_z$ composed by pauli matrices. Then Bob introduces an auxiliary particle $A$ with its initial state $|0\rangle_A$ and makes another unitary transformation $U_2(N)$ on particles of $B_1, B_2 \ldots B_N$ and $A$. $U_2(N)$ may be taken the form of the following $2^{N+1} \times 2^{N+1}$ matrices,

$$U_2(N) = \begin{pmatrix} A_1(N) & A_2(N) \\ A_2(N) & -A_1(N) \end{pmatrix} , \quad (18)$$

where $A_1(N)$ and $A_2(N)$ are $2^N \times 2^N$ matrices and may be written as,

$$A_1(N) = \mathrm{Diag}(a_1, a_2, a_3, \cdots, a_{2^N}), \text{and}$$

$$A_2(N) = \mathrm{Diag}\left(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \cdots, \sqrt{1-a_{2^N}^2}\right),$$

where the values of $a_1, a_2, a_3, \ldots, a_2N$ are related to the coefficients of channel $\mu$, $\nu$, $\mu'$, $\nu'$ and generalied Bell states *a, b,c* and *d*.

Finally, Bob measures the auxiliary particle A. If the result is $|0\rangle_A$, the scheme is successful; otherwise, it fails.

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