Study of a displacement sensor based on transmission varied-line-space phase grating

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A displacement sensor based on transmission varied-line-space (VLS) phase gratings is proposed. The relationship between the properties of the sensor and the parameters of VLS is discussed. Compared with the displacement sensor manufactured by the reflective VLS grating, this type of sensor contains a grating with simpler structure and high diffraction efficiency. It also has good stability with the change of temperature.

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The fiber bragg grating has been widely used in temperature and pressure sensors [1-4]. A fiber optic displacement sensor manufactured by reflective VLS grating was first proposed by W. B. Spillman Jr in 1989 [5]. Further studies on the sensor of this type have been continued ever since [6-8]. As the reflective VLS grating, the key element of the sensor, is difficult to manufacture, lots of research on the design and manufacture of VLS grating have been also done [9-13]. In this paper, a displacement sensor based on transmission VLS phase gratings is proposed. The resolution of the numerical control photolithography equipment in Hitachi Company of Japanese can reach 0.02 nm[14], while the photolithography technology of excimer laser etching and UV laser engraving can reach the resolution of sub-micrometer. The precision of grating's groove depth is better than 0.1 μm.

The structure of a transmission VLS phase grating with rectangular grooves is shown in Fig.1. The refractive indices of the dielectric material and air are noted by n_1 and n_2, respectively. w is the thickness of the grating, and d_n, a_n and b_n are the line space, the material width and the slit width of the n th period of the grating, respectively. The Cartesian coordinates are used. The grating plane is within the o-xy plane and the line spaces d_n crease along the positive direction of x axis. A monochromatic plane wave traveling in z coordinate is incident on the grating normally. θ is the diffraction angle.

The change of d_n can be described with a polynomial of m orders:

$$d = d(x) = \sum_{k=0}^{m} a_k x^k \tag{1}$$

When m=1, the simplest case, the changing of d_n becomes:

$$d_n = d_0 - Gx_{n-1} \qquad n=1,2,3,\cdots \tag{2}$$

where, d_0 is the initial line space, G is a changing coefficient and x_n is the sum of the previous n line spaces:

$$x_n = \sum_{k=0}^{n} d_k \tag{3}$$

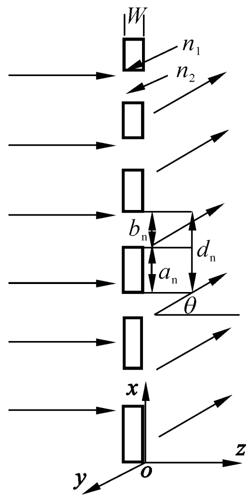

Fig.1 The structure of transmission VLS phase grating.

Using the mathematical induction method, the change of d_n described in Eq. (2) can also be expressed as:

$$d_n = d_0 (1-G)^n \tag{4}$$

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It can be seen that the line space d_n varies as a geometric series. When d_n changes slowly, the diffraction properties of the VLS grating can be qualitatively analyzed by the following grating equation:

$$d\sin\theta = j\lambda_a \qquad j=0, \pm 1, \pm 2, \cdots \tag{5}$$

where j is the diffraction order and λ is the wavelength of incidence. When white light normally incidents on the grating, the diffraction intensities of different wavelengths are not the same in a given direction. Light with the wavelength decided by Eq.(5) has maximum intensity. Putting the diffraction light in this direction into a spectrometer, we will get a spectrum. For a VLS grating, d in Eq.(5) can be treated approximately as the initial line space of the illuminating area. When the grating shifts along x axis, the corresponding initial line space d will change synchronously. Therefore, the peak wavelength in the spectrum will also change. And subsequently the displacement of the grating can be calculated through measuring the change of the peak wavelength in the spectrum.

Actually, the diffraction intensity of the VLS grating can be resolved based on the diffraction theory. Suppose a monochromatic plane wave with wavelength of λ and amplitude of E_0 normally incidents on the grating. According to the scalar diffraction theory, in the direction of θ, the complex amplitude of the diffraction wave caused by an element dx on the position x of the grating is:

$$\mathrm{d}E = [E_0 \mathrm{d}x \mathrm{e}^{\mathrm{i}\frac{2\pi \cdot n(x)\cdot w}{\lambda}}]\mathrm{e}^{\mathrm{i}\frac{2\pi \cdot x\sin\theta}{\lambda}} \tag{6}$$

where, $n(x)$ is the refractive index of the grating on point x. By integrating Eq.(6), the complex amplitude of the wave diffracted by the nth period of the grating can be obtained:

$$E_n = E_0[\alpha_n \mathrm{e}^{i(2\pi wn1/\lambda)}\mathrm{e}^{i[\frac{2\pi(x_{n-1}+a_n)}{\lambda}\sin\theta]} + \beta_n \mathrm{e}^{i(2\pi wn_2/\lambda)} \cdot \mathrm{e}^{i[\frac{2\pi(x_n-b_n)}{\lambda}\sin\theta]}] \tag{7}$$

where

$$\alpha_n = a_n \frac{\sin(\frac{\pi a_n \sin\theta}{\lambda})}{\frac{\pi a_n \sin\theta}{\lambda}} \quad \beta_n = b_n \frac{\sin(\frac{\pi b_n \sin\theta}{\lambda})}{\frac{\pi b_n \sin\theta}{\lambda}} \tag{8}$$

Thus, the complex amplitude of diffracted wave caused by the anterior N periods of the grating is:

$$E(\theta) = \sum_{n=0}^{N-1} E_n = E_0 \sum_{n=0}^{N-1}[\alpha_n \mathrm{e}^{i(\frac{2\pi wn_1}{\lambda})} \cdot \mathrm{e}^{i[\frac{2\pi(x_{n-1}+a_n)}{\lambda}\sin\theta]} + \beta_n \mathrm{e}^{\mathrm{i}(\frac{2\pi wn_2}{\lambda})} \cdot \mathrm{e}^{i[\frac{2\pi(x_n-b_n)}{\lambda}\sin\theta]}] \tag{9}$$

The related diffraction efficiency is:

$$\eta = [E(\theta)\cdot E(\theta)^*]/E_0^2 = \left|\sum_{n=0}^{N-1}[\alpha_n \mathrm{e}^{\mathrm{i}(\frac{2\pi wn_1}{\lambda})} \cdot \mathrm{e}^{\mathrm{i}[\frac{2\pi(x_{n-1}+a_n)}{\lambda}\sin\theta]} + \beta_n \mathrm{e}^{\mathrm{i}(\frac{2\pi wn_2}{\lambda})} \cdot \mathrm{e}^{\mathrm{i}[\frac{2\pi(x_n-b_n)}{\lambda}\sin\theta]}]\right|^2 \tag{10}$$

It shows that the diffraction efficiency is related to the diffraction direction θ, the incident wavelength λ, the initial line space d_0 and the changing way of the line space (a_n,b_n). When the grating is illuminated by white light, the peak wavelength at a given diffraction direction is related to the position of the grating.

Fig.2 gives a calculated example, from which the displacement of the grating can be deduced from the increment of the peak wavelength.

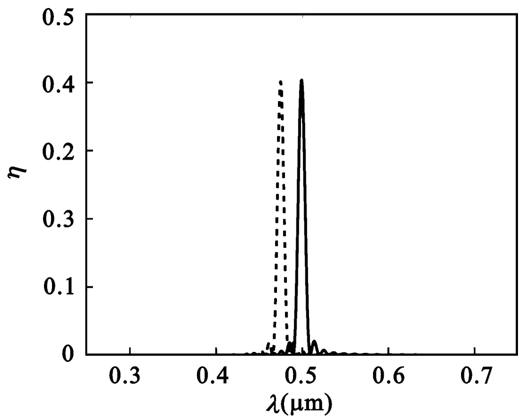


Fig.2 The spectrum of diffraction light in a given direction before and after the movement of VLS grating, respectively. Real line: before movement; Dashed line: after movement.

$$w(n_1 - n_2) = (k + \frac{1}{2})\lambda \ , \quad k=0, \ 1, \ 2\cdots \tag{11}$$

The energy of the zero diffraction order can be zero, and most diffraction energy will concentrate on the first diffraction order. So, the properties of the sensor can be improved by using phase grating.

According to the diffraction efficiency of VLS grating given by Eq. (10), we can analyze the influence of the parameters of VLS on the sensitivity and dynamic range of the sensor. The line space of the grating changes in geometric series as shown in Eq.(4), and the spectrum of the incident wave is in gaussian function,

$$E(\lambda) = \exp[-\frac{(\lambda - \lambda_0)^2}{|\Delta\lambda|^2}] \tag{12}$$

where, λ_0 is the central wavelength, λ_0=0.5 μm, and Δ=0.25 μm. Since that the input/ output properties of the sensor will not be affected by the move direction of the grating, we can assume that the VLS moves to the positive direction of x axis.

Supposing a given grating with a_n/b_n=1, w = 0.7 μm, G = 0.000 1. The values of d_0 are 3 μm, 5 μm and 10 μm, respectively. And the diffraction direction θ is along the first diffraction order of the incident wavelength of 0.7 μm. Using Eq.(4), we can obtain curves for the properties of the displacement sensor. The calculating results are shown in Fig.3. Fig.3(a) gives the input/output curves. The lateral axis denotes the displacement of the grating, δx, while the longitudinal one denotes the displacement of the peak wavelength in the output spectrum, $\delta\lambda$. In Fig.3(b), the relationship between δx and the peak diffraction efficiency is displayed.

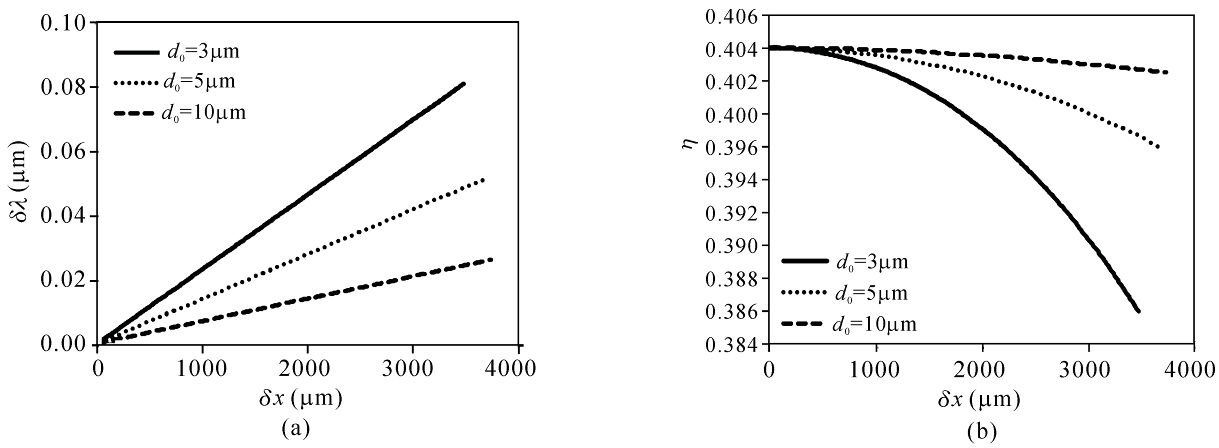


Fig.3 The influence of d_0 on the properties of the sensor, w=0.7 μm, G=0.000 1. (a) The input/output curves; (b) The relationship between the displacement of the gratings and the peak diffraction efficiency.

Fig.3(a) suggests that the grating with the line space changing in this way is ideal for sensor.This is consistent with the conclusion of the reflective grating sensor [9]. When d_0 is smaller, the slope of the curve becomes larger, which stands for a higher sensitivity of the sensor. Curves in Fig.3 (b) imply that for a given value of d_0, when the δx increases, the peak diffraction efficiency decreases. And with a smaller d_0, the diffraction efficiency decreases more quickly, which shows a narrower dynamic range of the sensor.

For a grating with the initial line space d_0=5 μm, a_n/b_n =1, w=0.7 μm, we choose G to be 0.000 1, 0.000 2 and 0.000 3, respectively. The calculated input/output curves are given in Fig.4(a). Fig.4(b) shows the relationship between the displacement of the grating, δx and the peak diffraction efficiency.

The thickness of grating and the diffraction direction may influence the diffraction efficiency of the waves with different wavelengths. For a given thickness and given diffraction direction, only the wave which satisfies both Eqs.(5) and (11) can reach the highest diffraction efficiency. If the grating moves towards the direction that the line space decreases, the wavelength of peak efficiency will decrease according to Eg. (5) and the thickness of the grating will no longer satisfy Eg. (11). Then the diffraction efficiency will fall down. The larger the displacement of the grating is, the lower the diffraction efficiency will be. As a result, the measuring range will be restricted.

In practical application, the thickness of grating and the diffraction direction are chosen according to the actual measuring requirements. If the measured movement is in a fixed direction, supposing it is in the direction of line space decreasing, the peak wavelength in the output spectrum will surely move towards short wavelength. In order to enlarge the dynamic range of the sensor, the thickness w of grating and the diffraction direction θ can be determined by Eq. (5) and Eg. (11) with a wavelength in the red end of input spectrum. If the direction of measured movement is uncertain, the values of w and θ should be calculated by Eq. (5) and Eq. (11) with the central wavelength of the incident spectrum. Taking a grating with d_0=5.0 μm, a_n/b_n=1, and G=0.000 1 as

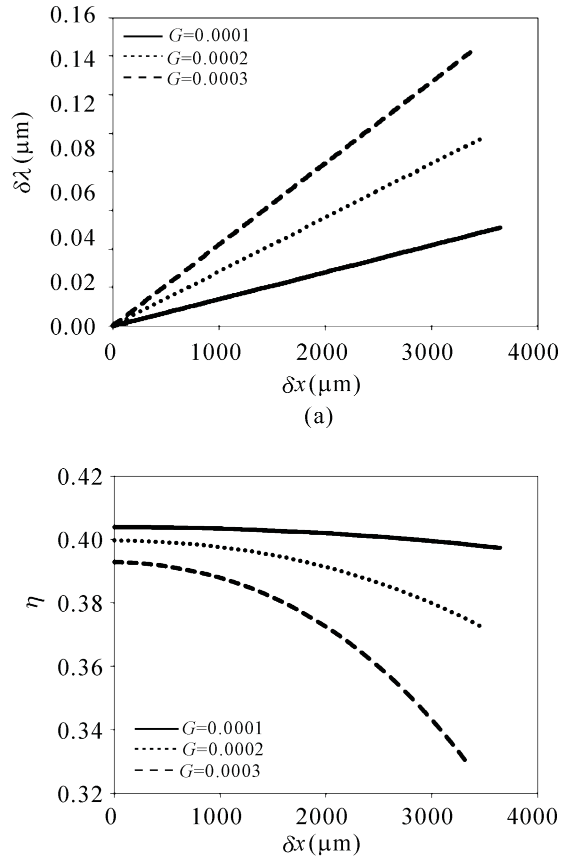


Fig.4 The influence of *G* on the properties of the sensor, *w*=0.7mm, d_0=5 μm. (a) The input/output curves (b) The relationship between the displacement of the gratings and the peak diffraction efficiency.

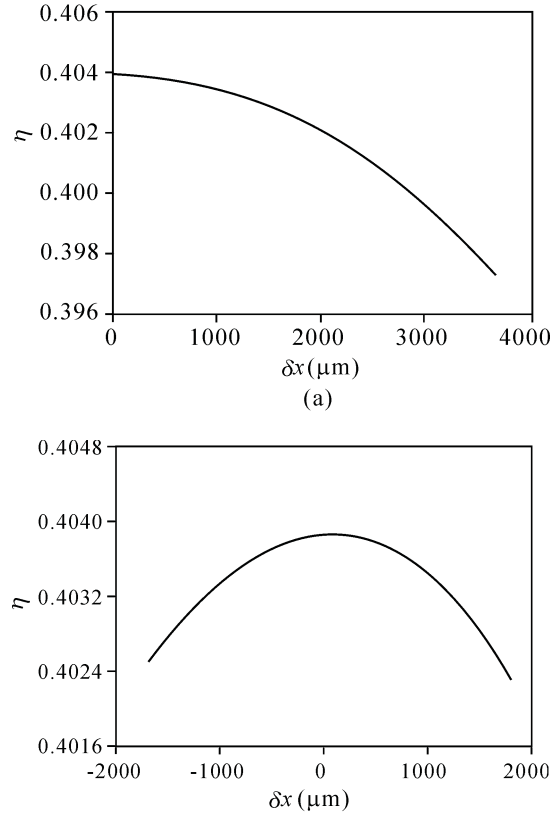


Fig.5 Influence of the grating's thickness and the diffraction direction on the peak diffraction efficiency. (a) Grating moves in a single direction. *w*=0.7 μm, *θ* corresponds to the first diffraction order of 0.7 μm wavelength. (b) Grating moves in both positive and negative directions. *w*=0.7 μm, and *θ* corresponds to the first diffraction order of 0.5 μm wavelength.

an the calculated results for both cases mentioned above are shown in Fig.5. The lateral axis denotes the displacement of the gratings, while the longitudinal one denotes the peak diffraction efficiency in the output spectrum.

Above all, when the initial line space of the grating d_0 is smaller or the line space changing coefficient *G* is higher, the sensitivity of the sensor becomes higher and the dynamic range will be narrower. It appears that the values of d_0, *G*, *w*, and *θ* should be chosen appropriately to meet the measuring requirement in the practical application.

The measuring precision of the sensor is determined by its input/output property and the measuring precision of the spectrometer. Nowadays, the measuring precision of the spectrometer can reach 0.001 nm [15], as a smallest displacement of wavelength $\delta\lambda=10^{-3}$ nm. According to the curves in Fig.3 (a) and Fig.4(a), we can get the measuring precision of the sensor in each case, which is shown in Tab.1. It is hopeful that the measuring precision of the sensor can be improved further by decreasing d_0 or increasing *G* as long as the fabrication technology of grating is allowed.

Tab.1 The measuring precision of the sensor with different values of d_0 and *G*.

	G=0.0001			*G*=0.0002	*G*=0.0003
	d_0=10 μm	d_0=5 μm	d_0=3 μm	d_0=5 μm	d_0=5 μm
The measuring precision (nm)	144	72	43	36	24

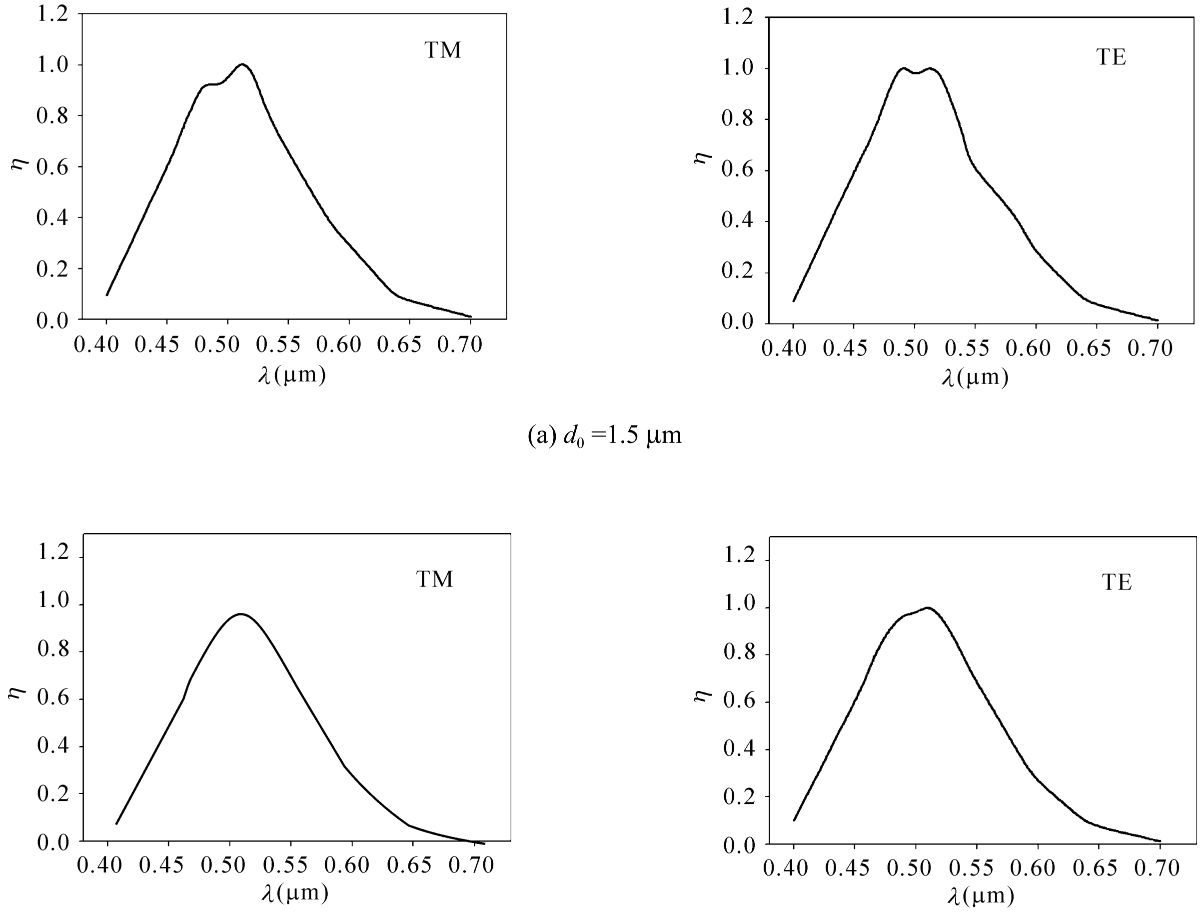
Actually, either the line space of the grating and the changing coefficient can decrease infinitely. They are restricted by the dynamic range as mentioned above. Besides, when d_0 is close to the order of wavelength, the polarization dependence of the diffraction field will affect the properties of the sensor. Diffraction efficiencies for different polarizations are different, which should be analyzed separately.

In Fig.1, suppose the polarizing direction of TE (TM) wave to be perpendicular (parallel) to the paper surface. When TE or TM wave incidents on the grating, the electromagnetic field around the grating can be calculated by the FDTD algorithm and the complex amplitude distribution on the rear plane of the grating can be obtained. Then, according to the Fourier optics, one can make the Fourier transform of this complex amplitude distribution to get the distribution of the Fraunhofer diffraction field.

Let d_0 be 1.5 μm, G be 0.0001, and θ_1 be the direction of the first diffraction order of the central wavelength of incident light. The diffraction efficiencies of all wavelengths in the direction of θ_1 were calculated for TM and TE polarizations, respectively. The normalized efficiencies are displayed in Fig.6. The horizontal axis denotes the wavelength, while the vertical one denotes the diffraction efficiency.

As shown in Fig.6 (a), the shapes of the output spectra for both TE and TM waves are distorted so that the peak wavelength is hard to read.

It can be seen from Fig.6 (b), that when d_0=3.0 μm, the peak diffraction efficiencies of TE wave are still not obvious, while the shape of output curve for TM wave becomes smooth, and the peak wavelength can be obtained. So the value of d_0 of the VLS phase grating for displacement sensor can not be decreased infinitely. Besides, TM polarization is the better choice in practical application.



(a) d_0 =1.5 μm

(b) d_0 =3.0 μm

Fig. 6 Diffraction efficiencies of TM and TE waves with different initial line spaces.

The displacement sensor has linear input/output property if the line space of VLS phase grating changes as a geometric series.It is concluded that the sensitivity of the sensor will become higher and the dynamitic range will be narrower as decreasing the value of d_0 or increasing the value of G. Appropriate values of d_0 and G should be chosen depending on the measuring requirement in the practical application. Considering the polarization properties of diffraction and the measuring precision limitation of the sensor, the TM polarization is priority when the line space is near to the order of wavelength.

References

[1] LIU Qin-peng, QIAO Xue-guang, and JIA Zhen-an, J of Optoelectronic·Laser, **17** (2006), 1440.(in Chinese)
[2] YU Zhi-hui,YU Chong-xiu, and WANG Xu, J of Optoele ctronic·Laser, **17** (2006), 54. (in Chinese)
[3] LIU Qin-peng, QIAO Xue-guang, and JIA Zhen an, J of Optoelectronic·Laser, **18** (2007), 147. (in Chinese)
[4] ZHU Hai, LI Zhi-zhong, and SUN Rong-shan, J. of Optoelec tronic·Laser, **18** (2007); 536. (in Chinese)
[5] Spillman W.B. Jr. Applied optics, **28** (1989), 3550.
[6] Spillman W.B. Jr. SPIE, **2467** (1995)187.
[7] Laclair Jr.R., D. Spillman and W. B. Long, SPIE, **2840** (1996), 137.
[8] LI Bing-shi, and WU Zhong. SensorWorld, **11**(2004) 32. (in Chinese)
[9] WU Zhong, and LI Bing-shi. SensorWorld, **9** (2005),23. (in Chinese)
[10] LI Bing-shi, and WU Zhong. SensorWorld, **5** (2004), 19. (in Chinese)
[11] WANG Wei, and YANG Hou-min. Acta Optica Sinica, **19** (1999),1158. (in Chinese)
[12] SHI Lun, and HAO De-fu. Optics and Precision Engineering, **9** (2001), 285. (in Chinese)
[13] SHI Lun, Microfiabricat io n Technology, **4** (2002), 32. (in Chinese)
[14] Harada T., and Kita T. Appl.Opt. , **19** (1980), 3987.
[15] HUANG Yong-lin, TONG Zheng-rong, and XIANG Yang, Chinese Journal of Lasers, **29** (2002),1015. (in Chinese)
[16] Taflove A., Hagness A. C. Computational Electrodynamics: The Finite-Difference Time-domain Method [M]. 2nd ed. Artech House, 2000.