Influences of spin on the properties of a weak-coupled magnetopolaron in quantum dot*

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Considering the influences of the spin on the ground state energy, the properties of a weak-coupled magnetopolaron in quantum dots are studied by using a linear combination operator and unitary transformation method. The numerical calculation results for CaP crystals have been given as examples.

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The properties of weak- and strong-coupled magnetic polaron in a parabolic quantum dot were researched by mean of the linear combination operator method of Wang[1,2]. The impact of weak magnetic field on spin transport and spin relaxation properties of a system with Rashba spin-orbit interaction was investigated in the diffusion approximation[3,4].

The induced potential and the self-energy of an interface magnetopolaron were studied by Wei using the Green-function method[5]. Li[6] et al calculated the ground state energy of a magnetopolaron in polar crystals at different spin states. However, the influence of spin on the properties of magnetopolaron in quantum dots had not been studied by using a linear combination operator method. In this paper, considering the influence of spin, the magnetopolaron properties are researched by using the linear combination operator method.

We consider that in the system the electrons are much more confined in one direction (taken as the z direction) than in other two directions, x and y. Therefore, only the electrons moving on the x-y plane need to be considered. We assume that the confinement potential in a QD is parabolic,

$$V(\rho)=\frac{1}{2}m^*\omega_0^2\rho^2, \tag{1}$$

where m^* is the band mass of electron, ρ is the coordinate vector of a two-dimensional system, and ω_0 is the confinement strength of a quantum dot. We take the magnetic field B as B=(0,0,B), and the Hamiltonian of the electron-phonons system is given by

$$H=\frac{p_z^2}{2m^*}+\frac{1}{2m^*}(p_x-\frac{\beta^2}{4}y)^2+\frac{1}{2m^*}(p_y+\frac{\beta^2}{4}x)^2+\sum_w\hbar\omega_L a_w^+a_w+V(\rho)+\left(\frac{e\hbar}{m_e c}\right)SB+\sum_w\left(V_w^*e^{-iw.r}a_w^++h.c\right), \tag{2a}$$

$$V_w^*=\frac{i}{\omega}(\frac{2\pi e^2\hbar\omega_L}{\varepsilon V})^{1/2} \quad \text{and} \quad \frac{1}{\varepsilon}=\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0},$$

$$\beta^2=\frac{2eB}{c}, \quad \omega_c=\frac{eB}{m^*c}, \tag{2b}$$

$$\omega_e=\frac{eB}{m_e c}, \text{ and } \alpha_L=\frac{m^*e^2}{\varepsilon\hbar^2}\sqrt{\frac{\hbar}{2m^*\omega_L}}, \tag{2c}$$

where S is the spin operator of an electron, m_e is the mass of the free electrons, ω_c and ω_e are the cyclotron resonance frequency of the electron and the free electron in magnetic field , respectively. $a_w^+(a_w)$ is the creation(annihilation) operator of the bulk LO-phonons with a wave vector $\boldsymbol{w}$. and $r=(\rho, z)$ is the coordinate of the electron. After an unitary transformation of Eq. (2a), the following equations can be obtained,

$$U_1=\exp(-i\sum_w a_w^+a_w wr), \tag{3a}$$

$$U_2=\exp[\sum_w(a_w^+f_w-a_wf_w^*)]. \tag{3b}$$

The linear combination operators of the coordinate and momentum of electron can be obtained,

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$$p_j = (\frac{m^*\hbar\lambda}{2})^{1/2}(b_j + b_j^+), \quad (4a)$$

$$\rho_j = i(\frac{\hbar}{2m^*\lambda})^{1/2}(b_j - b_j^+), \quad (4b)$$

where $j = (x,y)$, f_w and λ are the variational parameters, and the transformed Hamiltonian can be written as

$$H^` = U_2^{-1}U_1^{-1}HU_1U_2 . \quad (5)$$

The following ground state wave function is used in the calculations,

$$|\psi\rangle = |\phi(z)\rangle |0\rangle_a |0\rangle_b , \quad (6)$$

where $|\phi(z)\rangle$ is the moving electron wave function in z direction. Since the electrons are much more strongly confined in z direction than in other two directions and confined in a infinite slit narrow layer, so $\langle\phi(z)|\phi(z)\rangle = \delta(z)$, where $|0\rangle_a$ is the unperturbed zero phonon state, and $|0\rangle_b$ is the vacuum state of b operator. The expectation value of Eq.(5) with respect to $|\Psi\rangle$ can be expressed as

$$E_0 = \langle\psi|F(\lambda, f_q)|\psi\rangle . \quad (7)$$

Using the variational method, we have

$$F(\lambda, f_q) = \frac{\hbar\lambda}{2} + \frac{\hbar\omega_c}{8}\frac{1}{\lambda} + m_s\hbar\omega_e - \alpha_L\hbar\omega_L . \quad (8)$$

For the convenience, we choose the units in the usual polaron units $(\hbar = 2m^* = \omega_L = 1)$ and define the quantum dot confinement length as $l_0 = \sqrt{\frac{\hbar}{m^*\omega_0}}$, then the ground state energy of a weak-coupled magnetopolaron can be written as

$$E_0 = \frac{\lambda}{2} + \frac{\omega_c}{8\lambda} + \frac{2}{l_0^4} + m_s\omega_e - \alpha_L . \quad (9)$$

By using variational method for λ and take λ in formula (9), the absolute value of the ratios between the spin energy of electron and the total energy of the magnetopolaron, self energy, Landau ground state energy, and the coupling energies can be written as p_1,p_2,p_3, and p_4, respectively.

In this section, to show more obviously the influences of the spin on the properties of a weak-coupled magnetopolaron in quantum dot, we perform a numerical calculation for GaP [6] crystal, of which the experiment numerical data are taken as follows:

$$\varepsilon_\infty = 8.46,\ \varepsilon_0 = 10.28,\ \alpha_L = 0.201,\ \frac{m^*}{m_e} = 0.338,$$

$$\omega_L = 7.591\times10^{13}s^{-1},\ \text{and}\ \hbar\omega_L = 49.97\text{meV}.$$

The results are shown in Figs.1-4. Fig.1 shows the relations between the magnetopolaron ground state energy E_0 and the spin-up(spin-down)splitting energy with the quantum dot confinement length l_0 at different external magnetic fields.

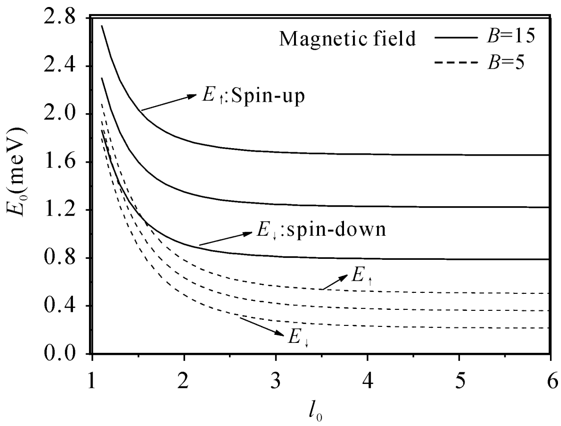

Fig.1 The relations between the magnetic polaron ground state energy E_0 and the confinement length l_0 at different magnetic fields.

From the figure, one can see that due to the influences of spin, the ground state energy splits into two energies $E_\uparrow$, $E_\downarrow$, and the ground state energy E_0 decreases with the increasing of quantum dot confinement length l_0.This is because the ground state energy and the square of quantum dot confinement length l_0 are in inverse ratio relation, that means when the quantum dot confinement length increases, the quantum dot confinement strength ω_0 becomes small, so the ground state energy decreases. The change of the ground state energy E_0 with the quantum dot confinement length l_0 is not obvious when confinement length l_0>2. We find that the spin-up (spin-down) splitting energy $E_\uparrow$ ($E_\downarrow$) changes large when the magnetic field B increases, and the difference of spin-up (spin-down) splitting energy and the ground state energy is very obvious with the change of the confinement length l_0 at a confinement length l_0>1.3. This is because the electron cyclotron resonance frequency ω_c and the free electron cyclotron resonance frequency ω_e in quantum dots are linear dependent on B.The change of ω_c is not notable when l_0 becomes large, but the change of ω_e is much larger with the magnetic field. As a result, the influences of the spin on the ground state energy are very obvious.

Fig.2 shows the relation between the magnetic polaron ground state energy or the spin-up (spin-down) splitting energy and *B,* with different quantum dot confinement length l_0. From the figure, one can see that the ground state energy E_0 increases with the increasing of B, and at the same magnetic field B, the ground state energy E_0 decreases with the increasing of the confinement length l_0. This is because that there is a confining potential to the motion of the electrons. When l_0 decreases, the thermal motion energy of the electron and the interaction between the electron and phonons, which take photons as medium, will be enhanced because the range of the particles motion becomes small. As a result, the magnetic polaron ground state energy changes large. We also find that the difference of the spin-up (spin-down) splitting energy and the ground state energy E_0 increases with the increasing of the magnetic field B, and the influences of spin is obvious. The conclusion is in accord with Ref. [7].

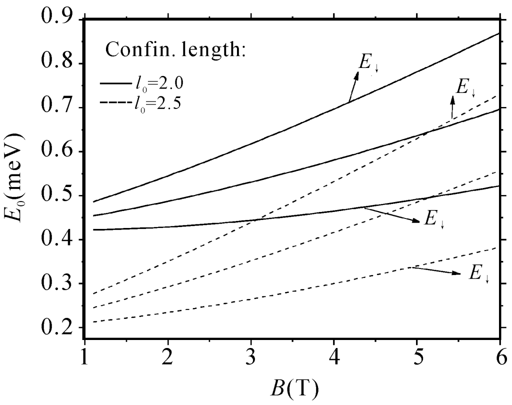

Fig.2 The relations between magnetic polaron ground state energy E_0 and the magnetic field *B*.

Fig.3 shows the relational curves between the absolute value p_1, p_2, p_3 and p_4 of the rations of the spin energy and the ground state energy, self energy, Landau energy, and the coupling-energy as a function of the quantum dot confinement length l_0 at different magnetic fields. From the figure, we can see that p_1 and p_2 increase with the increasing of the confinement length l_0. This is because that there is a confining potential in quantum dot, which confines the motion of the electrons, and the confinement potential ω_0 decreases when the confinement length l_0 increases, and as a result, the magnetic polaron ground state energy and self energy decrease, so the p_1 and p_2 increase. The absolute value p_3 of the ratio between the spin energy and the Landau energy decreases with the increasing of the confinement length, which is because p_3 only relies on the frequency λ. The vibration frequency λ decreases when the confinement length l_0 increases, so p_3 decreases with the increasing of confinement length l_0. In other words, the spin energy is less than the Landau energy all the time, of which the conclusion is in accord with the Ref. [7]. At the same time, we also find that p_1 decreases with the increasing of *B,* when $l_0 < 3.5$, but p_1 increases with the increasing of *B,* when $l_0 > 3.5$. So our conclusions are that the spin energy of electrons only depends on *B* and crystal properties, which may be great or less than the ground state energy. The p_2 decreases with the increasing of the magnetic field *B* when $l_0 < 7$, but the range of decreasing is much smaller, which shows the spin energy may be greater than the self energy with the increasing of *B*. The p_3 and p_4 decrease with the increasing of *B* when l_0 is a fixed value, and the range of p_3 decreasing becomes large with the l_0 increasing, but the p_4 does not change with the increasing of the confinement length l_0, which only depends on *B* and crystal properties.

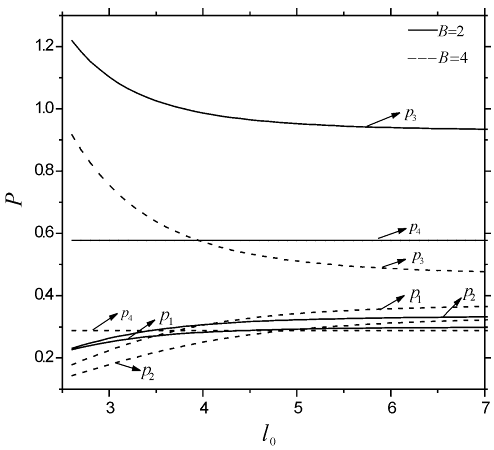

Fig.3 The relations between the *p* and the confinement length l_0 at different magnetic fields

Fig.4 shows the relations between p_1, p_2, p_3, p_4 and the magnetic field *B*. From the curves, one can see that p_1,p_2 ,p_3 and p_4 increase with the increasing of *B* respectively if l_0=3, which indicates the influences of spin-up (spin-down) splitting energy on the ground state energy are more obvious when l_0 is a fixed value. At the same time, we can find that p_1 increases with the increasing of l_0 when the magnetic field *B* is a constant. Moreover, the range of increasing will decrease with the increasing of *B*. This is because the confinement strength ω_0 decreases when l_0 increases, and the ground state energy becomes small, so p_1 increases with the increasing of l_0.While the magnetic field is increasing, the ground state energy is also increasing, and the influences of spin on the ground state energy is not obvious. p_2 increases with the increasing of the confinement length l_0 when *B* is a fixed value, but the range of increasing will decrease with the increasing of *B*. p_3 decreases with the increasing of the confinement length l_0 when *B* is a constant, moreover the range of decreasing will decrease with the increasing of B. The conclusion is in accord with the conclusions of Fig.3, but p_4 is not related to the confinement length l_0, because it only depends on the magnetic field B and the crystal properties.

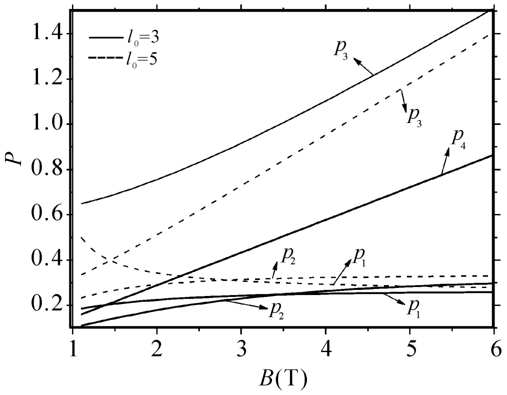

Fig.4 The relations between *p* and the magnetic field *B* at different quantum dot confinement lengths.

In conclusion, to understand the influences of spin, the properties of the magnetopolaron have been researched in a parabolic QD using a linear combination operator method in the electron-LO-phonon weak-coupled region. The results indicate that the ground state energy decreases with the increasing of the confinement length, in which the influences of spin on the ground state energy can not be ignored when l_0>1.3.

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