



# Frege's Ontological Diagram Completed

David Makinson

**Abstract.** In a letter of 1891, Frege drew a diagram to illustrate his logical ontology. We observe that it omits features that play an important role in his thought on the matter, propose an extension of the diagram to include them, and compare with a diagram of the ontology of current first-order logic.

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## 1. Frege's Diagram

In a letter to Husserl dated 24 May 1891, Frege remarked

I hope to find some time soon to reply to your objections. All I should like to say about it now is that there seems to be a difference of opinion between us on how a concept word (common name) is related to objects. The following schema should make my view clear.

In Fig. 1 we reproduce Hans Kaal's translation of that schema.<sup>1</sup> Kaal leaves the German word *Bedeutung* untranslated. The round brackets and their contents are part of the original, as are the large brackets around the bottom item of the third column.

Frege did not intend this schema to explain the *nature* of the nodal items, only to display graphically the *interrelations* between them and, to that end, it is very helpful. But it also has some features that give pause.

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<sup>1</sup>*Letter to Husserl 24.5.1891*, extract translated by Hans Kaal in McGuinness et al. 1980 pp. 63–64, reproduced in Beaney 1997 pp. 149–150. In some other English translations of Frege's work, *Bedeutung* is translated as “reference” or as “meaning” (the latter perhaps rather misleadingly, cf. the discussion in Simons 1992 p. 758 note 15), while “proposition” and “concept word” in the top row of Fig. 1 are sometimes rendered as “sentence” and “predicate” respectively.

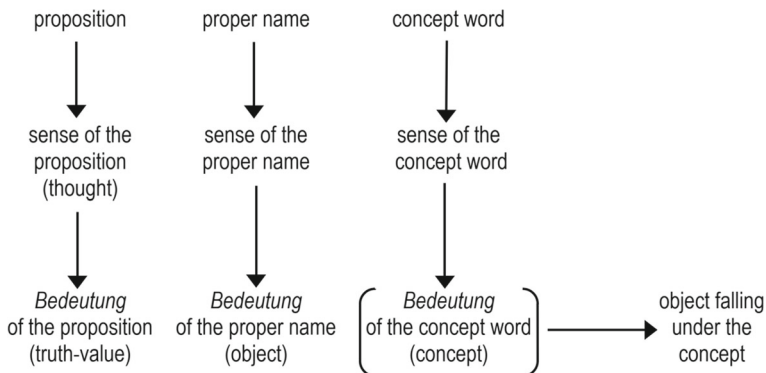


FIGURE 1. Frege's 1891 schema

- *It omits material, of three kinds:* (a) explanations of what the vertical arrows mean; (b) identification of horizontal connections between items in each of the three rows; (c) further nodes for items that play roles of varying importance in Frege's ontology.
- *It contains an oddity:* in the bottom row there is a node on the right for "object falling under the concept" even though the node "*Bedeutung* of the proper name (object)" already plays that role.
- *It prompts a question:* does Frege really want to say that the items in the bottom row emerge from the corresponding items in the middle row rather than immediately from those in the top row, as would seem more natural to some readers (and as the ontology is sometimes presented, e.g. in Burgess 2003 pp. 2–5)?

Regarding the last question, there are several texts where Frege repeats explicitly that references arise directly from senses, and only indirectly from linguistic items. For example, in a ms from the early 1890s:

Thus it is via a sense, and only via a sense, that a proper name is related to an object ... Logic must demand not only of proper names but of concept-words as well that the step from the word to the sense and from the sense to the *Bedeutung* be determinate beyond any doubt.<sup>2</sup>

Again in 1897:

Further, it is clear that we do not, properly speaking, ascribe truth to the series of sounds which constitute a sentence, but to its sense  
 ....<sup>3</sup>

<sup>2</sup> *Comments on Sense and Meaning*, ms of 1892–1895, translated in Beaney 1997 pp. 172–180 with the passage quoted on p. 180. Also in Hermes et al. 1979 pp. 118–125 with the passage quoted on pp. 124–125 and where "*Bedeutung*" is translated as "meaning".

<sup>3</sup> *Logic*, ms of 1897, translated in Beaney 1997 pp. 227–252 with the passage quoted on p. 229. Also in Hermes et al. 1979 pp. 126–151 with the passage quoted on p. 129.

Nevertheless, he sometimes writes more loosely. For example, the labels on the bottom row of Fig. 1 itself describe its nodes as the *Bedeutung* of the corresponding syntactic items in the top row. Other examples of this more relaxed way of writing can be found, for example, in mss of 1906 and 1914:

Proper names are meant to designate objects, and we call the object designated by a proper name its *Bedeutung*.<sup>4</sup>

If I say 'the thought that (16–2) is a multiple of 7 is true', I am treating *true* as a property of the thought, whereas it has emerged that the thought is the sense and the True the meaning of the sentence.

Of course treating truth as a property of sentences or of thoughts is in accord with linguistic usage.<sup>5</sup>

Thus, Frege has a formal doctrine and an informal practice on the connections between items in Fig. 1, and it would be desirable for an extended diagram to take both into account.

## 2. Frege's Diagram Completed

Figure 2 responds to the issues raised above. As well as eliminating the oddity, it introduces material from Frege's texts to fill gaps in Fig. 1 and reconciles the formal doctrine with its more casual formulation by representing the latter using compositions of links that are sanctioned by the former.

Like Frege's own diagram, Fig. 2 seeks to display what he sees as interrelations between items, without embarking on the task of clarifying their nature. For example, it says nothing about the ontological nature of the item "concept" which, apart from its negative properties of being non-extensional, non-syntactic and not a "sense", remains the subject of hermeneutic debate; nor does it say whether an "assertion" is a cognitive or social act or something else. Note also that Fig. 2, like Fig. 1, considers only the basic case of analysing a sentence attributing a one-place predicate to a single individual, as in the sentence "Frege is a logician", without truth-functional connectives, quantifiers, or other ingredients.

Figure 2 makes use of the following typographic conventions.

- Nodes are labeled in roman, underlining those that Frege regards as "unsaturated", while links between nodes are labeled in italics using a smaller font.
- Compositions of arrows are dotted, while their component arrows are unbroken.
- Arrows present in Frege's 1891 diagram are thick while those we have introduced are thin. Thus the 1891 figure (less its odd node, deleted)

<sup>4</sup>*Introduction to Logic*, ms of 1906, translated in Beaney 1997 pp. 293–298 with the passage quoted on p. 293. Also in Hermes et al. 1979 pp. 185–196 with the passage quoted on p. 191 and where "*Bedeutung*" is translated as "meaning".

<sup>5</sup>*Logic in Mathematics*, ms of 1914, translated in Hermes et al. 1979 pp. 203–250 with the passage quoted on p. 233.

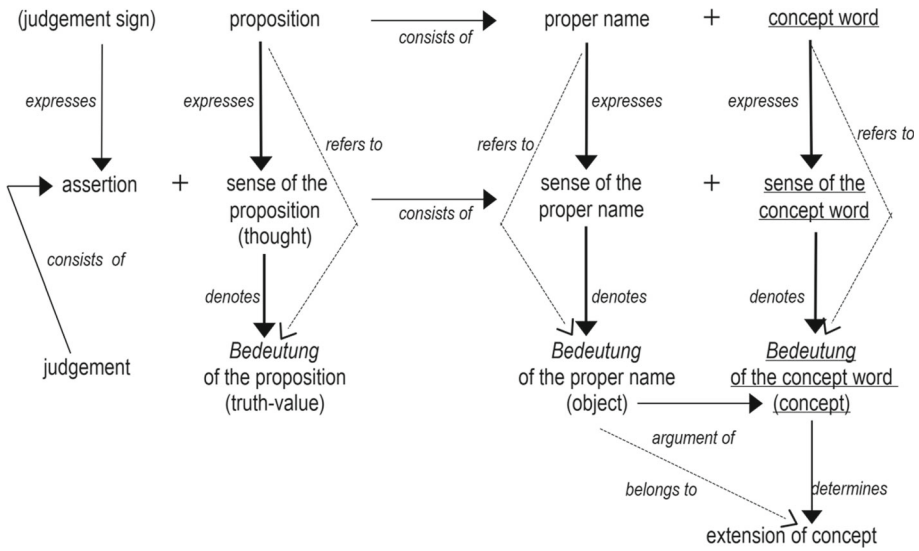


FIGURE 2. Frege’s schema completed

appears again as the part of Fig. 2 consisting of thick arrows and their end-nodes.

- The term “judgement sign” is bracketed because in natural language its work is done by context, intonation, or phrases like “I assure you that”. Nevertheless, it is made explicit in Frege’s *Begriffsschrift* (concept-script, ideography) as the well-known “judgement stroke” and so is included here.
- The plus sign is used for compounding–concatenation of syntactic items in the top row, and more abstract operations in the middle row. It is understood that the target of each of the three “consists of” arrows is the compound following its head, while the source is the single node next to its base. It is to emphasize this understanding that the arrow rising from “judgement” bends, to home in from the left: its target is the compound. Alternatively, one could enclose compound items in ovals with “consists of” arrows stopping at the perimeter and all other arrows entering or leaving from the interior. However, that would complicate the diagram for small advantage.
- The term “denotes” is used for direct links from middle to bottom rows, and “refers to” for indirect ones from top to bottom rows.
- For Frege, the extension of a concept is not just a set of objects, but a set of pairings. His term *Wertverlauf* is sometimes translated as “value range” or “course-of-values” (see e.g. Beaney 1997 page 135, note 2). The label “belongs to” is shorthand for “belongs / does not belong to”.

### 3. Current First-order Logic

Present-day logic drops many of the items in Frege's ontology, adds items, and subtly modifies some of the common part. In particular, it sharply separates sentences in natural language from their symbolic representations, with semantic analysis applied (at least, in the first instance) to the latter, as in Fig. 3.

In Fig. 3, “has as value” means “has as value, under a given valuation in a structure” and the label “element of” is shorthand for “is an element / is not an element of”. As in Fig. 2, the plus sign is for concatenation. The circled plus sign is for reverse concatenation; alternatively, in the middle row, one could reverse the order of constant and predicate, use plain concatenation, and cross the vertical arrows linking those items to ones above and below. That would give a slightly messier figure. One could also indicate that the leftmost item on the bottom row is determined by the ordered pair consisting of the other two items on that row; but that would perhaps be graphic overload.

### 4. Comparing the Diagrams

Why bother with such figures? Because they reveal at a glance the structure of the ontology, and help separate clearly questions about the nature of the nodal items from interrelations between them. In the more fanciful language of Thomas Le Myésier, writing in the 13th century about his liberal use of “horizontal trees”:

The reason I have made these figures, is that they are visible as a whole and at once. By looking at them, many things are called to the mind at the same time . . . The figure is caught in the imagination, then led to the memory and to the intellect and even further to the will. The soul that looks at the figure thus acquires knowledge and delights in it.<sup>6</sup>

In the present instance, several features of Frege's ontology stand out when we examine Fig. 2 and compare it with Figs. 1 and 3. They are perfectly well known, but it is convenient to have them visible in graphic form.

- Frege is less extensionalist than the casual reader, coming to his work after having learned some logic from today's textbooks, may assume. In particular:
  - The whole second row of Frege's “senses” in Figs. 1, 2 disappears in Fig. 3, as also does the non-extensional item “concept” in the third row.
  - The notion of the extension of a concept is absent from the original diagram in Frege's Fig. 1. It is introduced into Fig. 2 because it does appear in his work although, notoriously, he became dissatisfied with

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<sup>6</sup>Thomas Le Myésier *Breviculum*, cited by Even–Ezra 2021 with Latin original on p. 208 note 52 and an English translation on p. 47. Thomas was a disciple of Ramón Lull. We have taken the liberty of editing Even–Ezra's translation a little for fluidity.

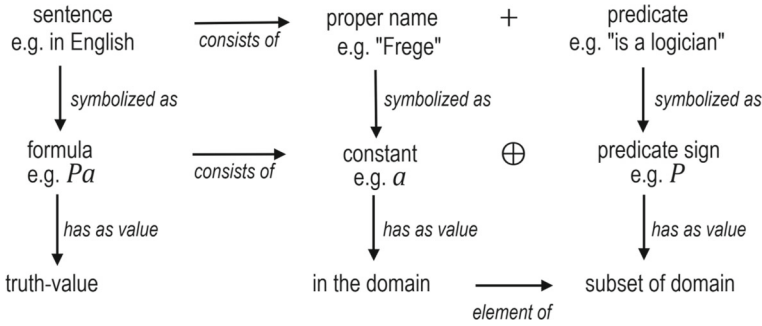


FIGURE 3. What present-day logic retains

- it.<sup>7</sup> But it takes centre stage in Fig. 3, as a subset of the domain that is the value, in a given structure, of a (one-place) predicate symbol.
- For Frege, the rightmost column of Fig. 2 is not ontologically homogeneous since the extension of a concept is “saturated” while the items above it are “unsaturated”, as marked graphically by our underlining.
  - Frege’s ontology gives a less prominent ontological role to his concept-script than one might have expected.
  - The concept-script does not enter into Figs. 1 and 2, whose only syntactic items are already-meaningful propositions of natural language and their components.
  - In contrast, in Fig. 3, valuation functions are defined on as-yet-uninterpreted symbolic expressions acting as surrogates for natural language ones.
  - On the other hand, it is striking that Figs. 1 and 3 share a common vertical structure: three descending paths, each containing three nodes. The items in the top row are syntactic and the bottom ones are, in their different ways, semantic; differences of content are thus greatest in the middle row.

<sup>7</sup>See e.g. Frege 1903 p. 253. Indeed, he came to think that the introduction of extensions of concepts is, at least in part, responsible for the contradiction in his attempted reconstruction of mathematics. For example, in a letter of 1912 to Jourdain he writes: “And now we know that when classes are introduced, a difficulty, (Russell contradiction) arises . . . Only with difficulty did I resolve to introduce classes (or extents of concepts) because the matter did not appear to me to be quite secure—and rightly so as it turned out.” This letter is translated in McGuinness 1980 p. 191 and is quoted in Hill 1995 p. 159.

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## References

- [1] Beaney, Michael (ed.): *The Frege Reader*. Blackwell, Oxford (1997)
- [2] Burgess, John P.: *Fixing Frege*. Princeton University Press, Princeton (2003)
- [3] Even-Ezra, Ayelet: *Lines of Thought: Branching Diagrams and the Medieval Mind*. University of Chicago Press, Chicago (2021)
- [4] Gottlob, Frege: 1903. *Grundgesetze der Arithmetik, Band II*. Jena. Reprinted by G. Olms, Hildesheim (1966)
- [5] Hermes, Hans, et al. (eds.): *Gottlob Frege: Posthumous Writings*. Blackwell, Oxford (1979)
- [6] Hill, Claire: Frege's letters. In: Hintikka, J.J.K. (ed.) *From Dedekind to Gödel*, pp. 97–118. Kluwer, Dordrecht (1995)
- [7] McGuinness, Brian (ed.): *Frege: Philosophical and Mathematical Correspondence*. Blackwell, Oxford (1980)
- [8] Simons, Peter: Why is there so little sense in *Grundgesetze*? *Mind* **101**, 753–66 (1992)

David Makinson

Ville d'Avray

France

e-mail: david.makinson@gmail.com

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