



Perspectives on the Logical Study of Language

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Abstract. Published originally as “Loogisen kielentutkimuksen näköaloja”, *Ajatus* 19, (1956), pp. 81–96, the following piece by JAAKKO HINTIKKA is the first essay he published in his mother tongue of Finnish. It is seen to provide both a state-of-the-art review of current topics emerging in the philosophy of language in the mid-1950, as well as outlines of HINTIKKA’s own evaluation of major theses of that era, in particular those of QUINE’s and WITTTGENSTEIN’s concerning language use. HINTIKKA evaluates contributions that the logical study of language use can make to the solving of philosophical questions. Published in 1956 in *Ajatus: The Yearbook of the Philosophical Society of Finland*, HINTIKKA’s essay served to introduce these topical issues to the Finnish-speaking audience soon after the original works had appeared in print. It also puts the theses into the perspectives of HINTIKKA’s own fledgling logical philosophy. One can see the germs of game-theoretical semantics in his comments on WITTTGENSTEIN’s notion of a language-game, and his remarks on general habits of action anticipate the importance of strategic rules in HINTIKKA’s mature, inquiry-led and ‘action-first’ epistemology. This is manifest in HINTIKKA’s characteristic approach to the methodology of philosophy of science and mathematics as an attempt to understand the nature of scientific and mathematical practices and operations through the logical analysis of their central concepts. Topics indicated below thus became the hallmarks of HINTIKKA’s next sixty years of work, which saw the development of a range of logical methods by which language and its use can be studied, and in particular in such a way that progress can be made in solving problems of genuinely philosophical nature that one encounters across various fields of science. This piece was very briefly summarized in English in 1963 by ARTO SALOMAA (*The Journal of Symbolic Logic* 28(2), 1963, p. 165), and it is translated from the original Finnish by JUKKA NIKULAINEN and AHTI-VEIKKO PIETARINEN. [A.-V. P., J. N.]

Mathematics Subject Classification. Primary 03A10, Secondary 01A70.

Updated version of the original publication from *Ajatus* 19, (1956), pp. 81–96. Reprinted with permission.

Keywords. Hintikka, Carnap, Quine, Wittgenstein.

1.

A large part of the best philosophical thinking of our century¹ stems from prewar Cambridge and the brilliant cluster of luminaries including BERTRAND RUSSELL, G. E. MOORE, J. M. KEYNES, A. N. WHITEHEAD and LUDWIG WITTGENSTEIN, not to mention many other minor talents who worked there. In his “Two Memoirs” [5], Lord KEYNES has instructively described the discussions and ways of thinking of that circle that have, directly or indirectly, left a permanent mark on the philosophical discourse of our times. KEYNES recounts that the most typical line was: “What *exactly* do you mean?” The implicit presupposition was that, once it had become clear what each and everyone actually meant with their words and questions, philosophical problems would then resolve themselves.

This way of thinking portrays the entire mode of philosophizing today. The logical study of language has received a great deal of attention in the contemporary philosophical world. In it, one attempts to solve philosophical problems by asking what the words and concepts contained in them actually mean. Logical empiricism already arrived at the conclusion that a general theory of signs and meaning forms the basis of philosophy. In the Anglo-Saxon world, the proponents of various shades of ‘analytical philosophy’ take philosophizing to mean dissecting the meaning and the use of words. Both of these trends owe much to LUDWIG WITTGENSTEIN, who in his early work *Tractatus Logico-Philosophicus* generalized his thought to propositions according to which all (real) philosophy is the critique of language and that most traditional philosophical problems are based on the misunderstanding of the logic of our language [9, 4.0031, 4.003].

2.

When taken to such extremes, stressing the linguistic nature of philosophical questions may appear to be a rather dubious move. It seems to deprive philosophy of its great value as a general ‘science of world-views’ that unites different walks of life. It also makes most of the history of philosophy a mere collection of more or less rudimentary mistakes. It comes as no surprise that the logical study of language and philosophers adhering to it have been disparaged, sometimes harshly. Indeed the springboard for such critique has often been the need to emphasise the value and ‘significance’ of philosophy against real or perceived attacks.

This is no place to dwell upon the nature of philosophy or the right subject matter of philosophical inquiry. Let us merely notice that the criticism

¹Translators’ note: The 20th-century.

towards the logical study of language is quite too often based on a superficial understanding of what it has to offer. One neglects the background and the real meaning of logical questions; one fails to see where the philosophical ‘critique of language’ could lead to when taken far enough. Logicians themselves are not exculpated from these misunderstandings. It is easy to point out the narrowness of projects that many logically-minded philosophers have, and it is easy to criticize the contributions in the philosophy of language in general. It does not denigrate CARNAP to say that in his work many central philosophical problems are neither solved nor even attended to. But at the same time it has to be said that philosophical inquiry on language in general and its logical side in particular has reached the point in which most interesting perspectives are opened up and questions are constantly deepening. One of the most famous philosophical works of the 1930s was Carnap’s masterpiece on the logical syntax of language, which solely explored the formal structure of language. It did not suffice to solve the philosophical problems of language; other aspects than the formal structure of language had to be taken into account as well. Thus in his ‘semantical’ works of the 1940s, CARNAP examined the possibilities of interpreting formal logical systems, the relationships of the signs of logical systems to the reality which they represent. At this moment a central question is the *active use of language* as the touchstone of philosophical problems. Next, I will attempt a brief sketch of some problems with this enterprise.

3.

A good starting point is provided by two recently published substantial works in which the logical problems of language are dealt with in a profound manner. These are WITTGENSTEIN’s posthumous *Philosophical Investigations* [10] and essays by the American QUINE written ‘from the point of view of a logician’ [7]. In many ways these books are at odds with each other. QUINE grounds even his philosophical positions in the results of logical research, whereas in WITTGENSTEIN’s book you would be hard-pressed to find logical formulas at all. QUINE does not even mention WITTGENSTEIN nor does WITTGENSTEIN’s ageless work refer to QUINE or to the existence of most other modern philosophers. It is remarkable that in such different works there are important confluences in the issues that they tackle.

The novelty of the issues QUINE tackles is not immediately evident. His thoughts are seamlessly connected to the earlier logico-philosophical discussion. This is particularly so with regards to the critique he directs at the concept of analyticity and which has a central role in his thinking; it is the result of those discussions which, alongside QUINE, such great minds as CARNAP, CHURCH, GOODMAN, TARSKI and WHITE have taken part.

In philosophical vocabulary, sentences whose truth is determined solely by virtue of the meaning of their words and not at all by the facts of the matter, are called *analytical*. This definition is not particularly illuminating, however, because the question of the meaning of words is one of the most difficult problems in the philosophical theory of language. Thus the concept

of analyticity has been investigated by using such notions as definition, substitution, necessity and the semantical rule. We need not go through QUINE's judgment on each of these attempts [7, pp. 20–37]. It suffices to get at the basic nature of his critique. QUINE does not deny that one can, especially in simplified systems of grammar constructed by logicians, give precise and adequate rules for a sentence to be analytical. He doubts not so much the possibility than usefulness of such attempts. He claims that defining analyticity by semantical rules does not help us *understand* the nature of analyticity. It only reduces the problematic concept of 'analytical' to the equally elusive concept of 'a semantical rule'. "We understand what expressions the rules attribute analyticity to, but we do not understand what the rules attribute to those expressions" [7, p. 33], QUINE says.

The way in which QUINE thus frames the issue differs completely from the way on which e.g. CARNAP grounds his *Logische Syntax der Sprache*, perhaps even more radically than QUINE himself realizes. This becomes clear when one considers the extent to which QUINE's arguments can be applied. In exactly the same way as QUINE criticizes the possibilities of the earlier formal logical approaches to help us understand the concept of analyticity, one may criticize the formal approaches that try to help us understand other and more fundamental logical concepts. Examples are concepts such as 'every' and 'some', which incidentally speaking do have a pivotal meaning in QUINE's logic. Logical laws governing these concepts have exhaustively been presented in the formal system called predicate calculus. But if one applies QUINE's arguments literally to predicate calculus, one concludes that even with all its rigorous rules, it cannot help us understand the concepts of 'every' and 'some'. For the rules of predicate calculus are formal rules, and as rules invariably fail to answer why exactly these rules were chosen from the set of rules that formally speaking are equally justified.

Semantical rules offer us no more help. The way in which the truth of sentences containing the words 'every' and/or 'some' is defined in the semantical systems of CARNAP and TARSKI is in many ways an illustrative case in point. In them the statement 'every x ' is formalized e.g. as ' (x) ' and 'some x ' as ' $(\exists x)$ '. These strings of symbols are jointly called quantifiers. Simplifying a little, a typical formulation of CARNAP's truth-definition for a sentence containing quantifiers is as follows:

- \mathfrak{S}_k has the form $(i_j)(\mathfrak{A}_i)$ and every value of i_j [...] has the property determined by \mathfrak{A}_i .
- \mathfrak{S}_k has the form $(\exists i_j)(\mathfrak{A}_i)$ and at least one value of i_j has the property determined by \mathfrak{A}_i [1, p. 42].²

In this definition itself, words 'every' and 'some' are used. It can be comprehended only by someone who understands what these words mean. It cannot be used to define them on the whole but only in the restricted formal language in which quantifiers are present.

²Translators' note: This quotation replaces the simplified version that occurs in the original text and which lacks the symbols.

For reasons not to be delved into here, QUINE does not apply his criticism to predicate calculus or to TARSKI's theory of truth, but only to the concept of analyticity. However, the implications of his arguments outlined above do indicate even clearer than the original reasoning what is essential in them. QUINE would be the last one to take what was elaborated above as a criticism towards predicate calculus or TARSKI's theory of truth. These branches of logic get the job done, namely to find and articulate those laws of logic that concepts like 'every' and 'some', or 'true', obey. What the arguments above show is that *with respect to some other issues*, predicate calculus as well as TARSKI's theory of truth cannot bring about full clarity. QUINE's criticism of the concept of analyticity can be understood in the same way. It does not show that we could not come up with exact logical laws that the concept 'analytical' obeys. The point is that such laws according to QUINE do not help us really understand the concept of analyticity.

4.

What does Quine then mean with his keyword *understanding*? What is the issue he is concerned with? QUINE's works offer no direct answer though they contain some curious hints. When talking about synonymity he suggests that it be clarified "in terms of (linguistic) behavior". When discussing CARNAP's attempt at clarifying the concept of analyticity by artificially simplified languages and the rules that govern their structure QUINE writes:

Appeal to hypothetical languages of an artificially simple kind could conceivably be useful in clarifying analyticity, if the mental or behavioral or cultural factors relevant to analyticity—whatever they may be—were somehow sketched into the simplified model [7, p. 36].

And in his essay "The Problem of Meaning in Linguistics" [7, pp. 47–64] Quine addresses those problems that await a grammarian when exploring a new and altogether foreign language, with the aid only of the speech of its users and other action and behaviour.

The use of language, the habits and customs of the users, are according to QUINE thus the best steps for understanding language. This idea can be interpreted in various ways. It might seem that QUINE would like to subsume the logical study of language under such forms of investigation, or in the very least to be dependent on them, which CARNAP calls [1, pp. 9–10] pragmatic: investigations that are focussed on the physiological, psychological, sociological or ethnographical prerequisites of language use. Such an idea can nonetheless be thwarted by the same criticism that already FREGE and HUSSERL once levied at the psychologizing of logic. And this interpretation is not the only possible one. Quite independently of what QUINE's own position may be, one could say that the gist of the matter lies elsewhere. I think CARNAP was wrong when he considered all such research to concern pragmatics in which one must refer to the language user and not only to the structure of the language and the reality it describes [1, pp. 9–10]. Just as well as one can study the structure of

a language irrespective of a certain accent or a certain typeface, it is equally possible to study the use of language purely from the logical point of view, irrespective of the psychological or sociological conditions of private speakers. And it is this kind of research that best answers that cluster of questions that has become the focal point of a logical philosophy of language: How and for what reason is language so profoundly and dramatically successful instrument in the reciprocation among people and in the interaction between a human being and nature that surrounds him? The latter question is at the forefront when mathematical symbolic systems are at issue. Here the prominent American logician LEON HENKIN has clearly laid out the nature of the problem. Understanding mathematics as a system of manipulating symbols, he says, “would still leave unexplained how the symbolism of mathematics functions as a language useful in the interactions between the users of the symbolism and their physical environment” [2, p.27]. The entire interest in the use of language that recently has so vehemently manifested itself becomes sensible if it is born out of such a deeper understanding. One no longer searches for logical laws for their own sake. In many instances logical inquiry has indeed thoroughly uncovered them. Interest has shifted or is beginning to shift to the interpretational problems facing these laws. Why just these logical laws ‘happen’ to be valid among many other and from the formal point of view equally possible laws? What is the significance of these laws as part of the fabric of human knowledge and human life? Are they purely conventional? How do they relate to the laws of nature? It is natural that in examining these kinds of questions, investigating the *use* of language as an integral part of human agency becomes essential. Investigating the language use at once delivers new methods for examining those philosophical problems that are intertwined with linguistic factors, which research that concerned only the formal structure of language was never able to do. Understood in this way, the mission of the logical study of language may not cease with the search for the formal laws of language nor even with the study of the static ‘meaning relations’ between language and the reality it represents.

5.

This view on the tasks and methods of a logical study of language is very close to what WITTGENSTEIN is doing in practice. The first thing he in his *Philosophical Investigations* points out is how little the speaking about the meaning of individual words helps us in understanding the functioning of language [10, pp. 1–16]. Rather than talking about the general concept of the ‘meaning of a word’, it is according to WITTGENSTEIN far more appropriate to say that the meaning of a word is its *use* within the context of a complete language. His most important philosophical method is to study exactly these kinds of fragments of the use of a complete language; he strives to “study the phenomena of language in primitive kinds of application in which one can command a clear view of the aim and functioning of the words” [10, p. 4 (§ 5)]. WITTGENSTEIN has as his central methodological concept the idiosyncratic concept of a

language-game (*Sprachspiel*). In modern philosophy, language has often been compared to a game, but usually the purpose has been to throw light on the formal character of the rules of language. It has been said that a logically perfect language is like a game of chess: certain characters are operated with using precise rules without having to think about their meaning or interpretation. WITTGENSTEIN gives this parable an entirely new twist. He states that formal rules do not turn games into games proper. One can move chesspieces about the board following the rules of the game even when the game itself is not being played; this can happen for example when analyzing games that have already been played. A game is only characterized by typical circumstances that when they prevail we say that a game is ‘really being played’. In a similar vein, what makes a language is not its formal rules but the use peculiar to it. This can be purely recreational, like the plays of the children when they are learning to speak. But it can also refer to the normal colloquial use or some derivative of such use. Or we may, say, imagine an invented or artificially simplified habit of using a language which, for the purposes of illustration, could emerge among a primitive tribe. “It is easy to imagine a language consisting only of orders and reports in battle. Or a language consisting only of questions and expressions for answering yes and no. And innumerable others” [10, p. 8 (§ 19)]. All of these kinds of uses WITTGENSTEIN calls language-games. The essential point is that language shows up intertwined with certain human habits of action. “Here the term ‘*language-game*’ is meant”, WITTGENSTEIN says, “to bring into prominence the fact that the *speaking* of language is part of an activity, or of a form of life” [10, p. 11 (§ 23)].

According to WITTGENSTEIN, philosophical problems are born when some word is used outside of its original context of action. One can try to solve or rather dispel philosophical problems by pointing out that the words that are the source of the problems have no use outside of their original ‘language-games’.

When philosophers use a word—‘knowledge’, ‘being’, ‘object’, ‘I’, ‘proposition’, ‘name’—and try to grasp the *essence* of the thing, one must always ask oneself: is the word ever actually used in this way in the language-game which is its original home. [10, p. 48 (§ 116)]

What WITTGENSTEIN himself tries to do is to “. . . bring words back from their metaphysical to their everyday use” [10, p. 48 (§ 116)].

6.

When WITTGENSTEIN and QUINE stress the significance of the use of language as the touchstone of its philosophical and logical problems, the remarkable thing in their stance is not its novelty. Rather it can be said that they come close to the view on the logical study of language that a very different stripe of linguists have often emphasized. “Eine Sprache vorstellen heißt, sich eine

Lebensform vorstellen" [10, p. 8 (§ 19)],³ says WITTGENSTEIN, and precisely in the spirit of this maxim have such philologists as SAPIR, BLOOMFIELD and WHORF highlighted the interconnectedness of linguistic phenomena to the habits and customs—indeed, ultimately to the entire culture—of its users. Philosophers like CASSIRER and anthropologists like MALINOWSKI have made the same point with their own respective nuances. A proof in philosophers' interest in problems of language use is provided by works such as CHARLES MORRIS' book "Signs, Language and Behaviour" [6].

Perhaps the most interesting in both QUINE but especially in WITTGENSTEIN is the manner in which they tackle those basic difficulties which behaviouristic linguistics leads to. The use of language is always something that is temporally and spatially bounded. Logical laws of language must, in contrast, be conceived as *general* laws, whose validity and applicability is not restricted to any singular cases of use. What is the relationship between these two views?

This problem has different facets depending on one's perspectives. QUINE examines problems of an hypothetized linguist when he attempts to study an alien language of some foreign tribe solely by observing the everyday life of its speakers. The task our linguist faces is to determine when a sequence of phonemes that occurs among the language of our imaginary 'primitive speakers', or can be formulated by means of that language, forms a meaningful sentence. According to QUINE, our linguist must refer at least to four increasingly larger classes of sequences of phonemes. QUINE calls them *H*, *I*, *J* and *K* [7, pp. 53–54]. *H* is the class of all those utterances that the linguist has been witnessing and has deemed appropriate given the reactions they have elicited. *I* is the class of all those spoken sentences which a competent observer sometimes—in the past, present or future—observes and deems appropriate given the doings of the language users. *J* is the class of all those sequences of phonemes, which our primitive speakers sometimes happen to utter and which are sensible to them, irrespective of whether they are within or without observation of an expert linguist. Finally, *K* is the class of all those sequences of phonemes which *would, if uttered*, be sensible to the imaginary native speakers. It is precisely this class *K* which is of interest to a logician or a linguist who is occupied with finding out what the general laws of language are. But at the same time it is this class *K* that is the most troublesome one. It surpasses the limits of empirical research, because its very definition includes the conditional statement 'would—if.' So what does this statement cover, QUINE asks. "What is the rationale behind that infinite additional membership of *K*, over and above the finite part *J*? This vast supplementary force of 'could', in the present instance and elsewhere, is perhaps a vestige of Indo-European myth, fossilized in the subjunctive mood" [7, pp. 54]. QUINE thus has an inkling of the question harbouring some deep waters but he provides no satisfactory answer to the problem he himself has evoked.

³Translators' note: "And to imagine a language means to imagine a form of life."

Even the form in which QUINE frames the problem is not the final or the most illuminative one. Apparently the talk of the infinite class K originates when our imaginary grammarian observed, or thought to have observed, certain laws, certain general rules that the native speakers' language conforms to. It is with them that he defines the class K . Insofar as the talk about the class of all possible meaningful sentences has any epistemic value, it reduces to the talk about those laws which observed meaningful sentences conform to.

And what possibilities does our linguist have in order to establish these laws? Let us continue QUINE's thought-experiment by imagining that the linguist now lives among the very tribe that he is investigating. The linguist follows the discussions of the language users also when the native speakers discuss their own language, what they say about their own language, how they correct each others' grammatical errors, resolve linguistic issues or misunderstandings between them and how they teach their own children or strangers to speak their language. Our linguist may himself learn to use the language he investigates, and it could step by step become as familiar to him as his mother tongue. There is no fixed boundary between the problems of a native and a foreign language. From the logical point of view, there is not even any principled difference. The linguist must often concede having misunderstood some law of the alien language; but this may well happen with his native language as well. He might have to doubt whether he really has uncovered the right rules of a foreign tongue; but in principle he could be met with the same doubts with respect to his mother tongue. Furthermore, the methods of eradicating these doubts are exactly the same in both cases. Any explanation that native speakers can give to each other, and any explanation that the native speaker can give to himself is in principle also within the reach of the linguist.

Or are they? Is it not also true that linguistic phenomena are fundamentally entwined to the psyche of the speaker? Is it not the case that, although a stranger can in practice learn an alien language, his or her psychical composition remains unchanged? He understands the new language slightly differently from the native speakers; he relates slightly different mental imagery to its words than the native speakers do; he interprets its sentences slightly differently from the people in his surroundings. The phenomenal aspect of language is different to him than to those of the others. Is it not true that this difference also influences how linguistic laws are understood?

This question need not be addressed here. What is essential is that the question does not take use beyond a private language argument. The question whether our linguist understands the language in the same way as the native speakers do is precisely the same question as whether two English speakers understand English in the same way. We are interested in the problems of language in general, not in any problem of a private language; therefore unfamiliarity or acquaintance with the language are altogether inessential tangential issues.

7.

We have arrived at the perspectives that WITTGENSTEIN applied. He thought that the problems stem from the familiarity of linguistic phenomena rather than their peculiarity. We overlook the most important aspects of linguistic phenomena precisely because of their simplicity and their quotidian nature. This is even more so because the problems related to the generality of the laws of language manifest themselves outside of the context of language. The problem is ultimately not the linguistic character of laws but their generality. According to WITTGENSTEIN, there is no fundamental difference in learning the laws of language to learning other rules of action. There is no method of learning or definition that would specifically be about linguistic phenomena. Sometimes ostensive definitions, in which the word 'book', for example, is defined by pointing at a certain book and which in such a demonstrative fashion makes it clear to the hearer what the words means, have been thought to be such 'boulevards to language'. WITTGENSTEIN quickly shows, however, that ostensive definitions belong to a very definite language-game outside of which they have no use. Crucially, espousing this language-game cannot itself take place with ostensive definitions. It is learnt by practice, not by demonstration.

One of WITTGENSTEIN's most central problems was this *problem of general rules of action*. He kept coming back to it late in his life; he grabbed onto it with all his characteristic intensity. Much of his *Philosophical Investigations* is devoted to this problem [10, pp. 53–88].

Again it does not behoove us here to review or assess WITTGENSTEIN's arguments in their entirety. The crux of the matter nonetheless is the tension between two different classes of criteria, both of which concern the meaning of correctly obeying a general law. Let us for the moment set aside the traditional baggage of these terms and call them the extensional and intensional criteria.⁴ It is clear that the application of a rule must ultimately be the criterion of whether one has understood it correctly. If one applies the rule wrongly or cannot apply it at all, we do not say that the rule was understood. Such criteria we could call *extensional*. On the other hand, it is equally clear that when we say that someone has understood a rule, we do not mean only the empirical fact that he in all circumstances thus far encountered correctly applied the rule. What we are after is that we are striving to state that he has acquired the ability to proceed applying the rule correctly *ad infinitum*, which is to say that the correct application of the law is not accidental or fortuitous but results from a real cause, which leads to the right result also in those cases that we have not witnessed. The application of a law must in other words have a basis, e.g. some rule, clause or definition expressible and understandable by finite means. Criteria of that sort could be termed *intensional*. Both extensional and intensional criteria leave room for an 'area of vagueness', a 'latitude' of interpretation. Each application is singular and finite in nature; nothing logically follows from it with respect to other applications. What happens in a

⁴These terms are not WITTGENSTEIN's.

potentially infinite set of other applications invariably remains open. On the other hand, each clause, each picture and each formula can in principle be interpreted in more than one way. A private rule becomes unequivocal only as a part of an entire language-game; each language-game readily assumes a set of other rules to be understood and mastered. That mathematical formulae are unambiguous results from them occurring as part of vast mathematical practice.

8.

This kind of a situation is liable to typify all human knowledge. Interestingly, a similar situation arises in certain theories of modern logic which WITTGENSTEIN does not refer to and which he was unlikely to have been acquainted with. I have come to a conclusion [4] which can be said to reduce many important fundamental problems in mathematics and logic to problems concerning the concept of *arbitrary sets*. The concept of a 'set' is then to be taken fully 'extensionally', without any reference to possible ways of definition and without any guarantees of even a possibility of a definition. How has the discourse on such sets arisen then? Apparently in such a wise that resembles determination of a general law through its applications: a set is conceived as a result of an infinite number of construction steps or choices of action, the collection of which then determines it. The problem is that no one can ever carry out all these steps or choices. The talk of arbitrary sets even necessarily leads us to the talk of uncountable sets that cannot even be approximated by finite operations. The sets we cope with in practice are usually 'effectively defined'. These effective methods of definition may be seen to correspond to the aforementioned 'intensional' criteria. The tension between two species of criteria corresponds to the question of how and to what extent can arbitrary sets be defined by effective methods or by means derived from effective methods. In logic, talk about sets is always tantamount to talk about properties defining those sets, which in turn reduces to talk about functions. We are thus essentially dealing with classification of arbitrary functions. As far as functions on integers are concerned, this question is addressed in the branch of mathematical logic variously known as the theory of recursive functions, the theory of computable functions, and recursive number theory, but most aptly known as the theory of recursive *and non-recursive* functions. From the point of view of the history of ideas, its invention and development during the last two decades shows an interesting parallel to WITTGENSTEIN's investigations on the character of general rule-following. Significantly, it was precisely the theory of recursive functions that gave birth to several of the most important recent logical discoveries, such as CHURCH's impossibility result concerning decision method for predicate calculus, which contains as a conclusion GÖDEL's famous incompleteness theorem. It is no coincidence that the classification of functions pursued by the theory of recursive functions is connected to some of the deepest mathematical and logical problems. Above all, such is the continuum hypothesis, which for so long has defied and still does defy the efforts of logicians and mathematicians.

Notably, already in the 1920s the great HILBERT tried to grapple the continuum hypothesis with the classification of functions [3]. It is no secret that the desire to fulfill HILBERT's grand idea drives the efforts of many a clever logician within the theory of recursive and non-recursive functions.

9.

We are thus dealing with extremely deep and wide-ranging matters. Perhaps the Spenglerian [8, ch. 1] idea of mathematics as the allegory and the purest expression of the Faustian spirit is not too far fetched. According to SPENGLER, the essential feature of this mathematics is the idea of the *general morphology of mathematical operations*. This very same idea is at bottom the point of that classification of functions which the recursive theory of functions aims at. The study of recursive functions is a much more articulate expression of the general tendency mentioned by SPENGLER than his own examples. The continuum hypothesis is a far superior Faustian counterpart to the classical problems in the Greek mathematics than SPENGLER's own examples of convergence in infinite series or transformations of elliptic and algebraic integrals into multiple-periodic functions [8, p. 64]. Here we are not dealing with fundamental constraints of our ways of thinking in the manner in which squaring the circle fundamentally constrained the Greek mathematics. In the theory of recursive functions and in the continuum hypothesis we are dealing with fundamental difficulties related to the general conception of a 'mathematical operation'. As we saw above, fundamental questions on the nature of mathematical laws and mathematical sets is connected to questions on the nature of general laws of conduct on the whole. Taking SPENGLER's bold allegory further we may state that the interest of logical and philosophical inquiry in these problems portends a sort of self-criticism and soul-searching of the Western *Geist*. If you will pardon the adage, this self-criticism clad in the logical form of the Faustian spirit has a major symbolic significance, and the results following from its examination can have a massive indirect relevance when evaluating cultural phenomena of altogether different sort.

10.

Hopefully these in many ways deficient and perhaps imprecise indications suffice to establish that the modern problems of the logical study of language and the perspectives they offer have a great and not least a philosophical significance, in the most exacting sense of the word.

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Received: July 26, 2018.

Accepted: July 27, 2018.