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To Peirce Hintikka's Thoughts

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Abstract. This paper compares Peirce's and Hintikka's logical philosophies and identifies a cross-section of similarities in their thoughts in the areas of action-first epistemology, pragmaticist meaning, philosophy of science, and philosophy of logic and mathematics.

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1. Introduction

This paper compares Peirce's and Hintikka's formal and logical philosophies, especially in terms of what I term their *action-first* (or *knowledge-last*) epistemologies. From this systematic perspective, I then identify a number of close similarities in their thoughts in the following areas of formal and logical philosophy.

1. Epistemology: Both Peirce and Hintikka developed epistemology as a sub-field of philosophy of science. They built upon a Socratic theory as the theory of inquiry, including methods of discovery and scientific reasoning. The result is a fallible epistemology which includes abductive moves in the

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model of inquiry grounded on interrogative moods of 'putting questions to Nature'.

2. Meaning: Both Peirce and Hintikka were proponents of subjunctive (pragmaticist) theory of meaning, which they explicated in *game-theoretic semantics*. Both rejected sense-data as the source of information and took justification of reasoning to be grounded on observational facts.
3. Philosophy of Science: Scientific discovery and hypothesis generation were the main interest of both Peirce and Hintikka, who emphasized the importance of creating the theory of the *economy of research*: Peirce explicitly in the *methodeutic* of scientific values (such as synechism as a non-epistemic combination of tychism and pragmatism) and in the cost-benefit analysis measuring the *uberty* of one's working hypotheses and research proposals, while Hintikka emphasized *strategic aspects of inquiry* and *the logic of question-answer structures* as essential factors of scientific reasoning and decision-making. Decisions to select or omit data in experimental work pertain to such economic and strategic considerations, as indeed do abductive hypothesis selection and uberty. All these are in both Peirce's and Hintikka's philosophies considered under a realist methodology.
4. Philosophy of Logic and Logic as a Theory of Inquiry: Both Peirce's and Hintikka's logical thought is characterized by algebraic and relational thinking, which shows in their emphasis on meta-theoretical ideas, centrality of epistemic modalities in logical analysis, and in taking syntax, semantics and pragmatics as a unity. Their respective philosophies of logic were guided by viewing logic as a model-building activity, not inferentialism, and taking possibilities as real (in Peirce's terms, scholastic realism). Moreover, we find the origins of epistemic logic, **KK**-thesis and the problem of cross-identification in quantified modalities dating back to Peirce's 1903–1906 writings on graphical logic.[1] Unlike Frege, both Peirce and Hintikka would take the development of the theory of quantification to follow actual mathematical practices and operations, and both made it clear how first-order and higher-order conceptualizations are to be distinguished from each other.

As to the differences, one could take Hintikka to be a realist with nominalist inclinations, whereas Peirce would be a realist in the scholastic (or Scotistic) sense. However, I conclude that Hintikka's version of nominalism that he took to result from quantification in independence-friendly (IF) logic might in fact be the only version of nominalism acceptable to Peirce, given (i) the "extreme scholastic realism" of the latter, (ii) the interpretation of possible worlds as "small worlds" in Hintikka's philosophy, which relies on the common ground of the players of the semantic game, and (iii) the fact that Peirce's theory of

[1] Hintikka had read the material published in Peirce's *Collected Papers* when he was conducting research for his *Knowledge and Belief: Introduction to the Logic of the Two Notions* in late 1950s and early 1960s.

quantification was, just as Hintikka's, aimed at capturing what is going on in actual mathematical practices.

2. Methodeutic and the Socratic Method as the Basis of the Theory of Knowledge

The closeness of these two thinkers can be uncovered by systematic, historical and textual considerations and methods. To begin with, let us compare the two quotations below:

> An experiment... is a question put to nature. Like any interrogation, it is based on supposition.

> The idea of thinking of scientific experiments as questions put to nature goes back to Bacon and Kant.

The previous passage is from Peirce (*Collected Papers* 5.168, 1903, hereon CP), that latter from Hintikka ([23, p. 222]). Placed in its proper context, what Peirce is telling here is that logic, which as an Aristotelian innovation has emerged as a special case of certain question-answer structures in which conclusions (as answers) are those that are necessitated by the premisses (as questions), has to take into account what the presuppositions of those questions are. Questions that follow the pattern of the scientific method are questions put to Nature, where researchers expect certain results to follow from those presuppositions. Questions ought to be designed so that Nature would reasonably be expected to produce a negative answer, or else an element of surprise would prevail. Either way, one of the underlying suppositions is the presence of the "external permanency" (CP 5.384) as the source of answers. This external permanency determines what the guesses, beliefs and conjectures of the clever interrogator are going to be. Here is what Peirce says in full:

> An experiment... is a question put to nature. Like any interrogatory, it is based on a supposition. If that supposition be correct, a certain sensible result is to be expected under certain circumstances which can be created, or at any rate are to be met with. The question is, Will this be the result? If Nature replies "No!" the experimenter has gained an important piece of knowledge. If Nature says "Yes," the experimenter's ideas remain just as they were, only somewhat more deeply engrained. If Nature says "Yes" to the questions, although they were so devised as to render that answer as surprising as possible, the experimenter will be confident that he is on the right track. (CP 5.168)

Indeed Peirce maintains that "there is a purely logical theory of how discovery must take place" (CP 2.107). Exploration of the structures of the logic of discovery was also one of Hintikka's major contributions in philosophy of science. One wants to find our what the "method of discovering methods" is and that "can only come from a theory of the method of discovery" (CP 2.108). The name Peirce gave to the theory of that method was *methodeutic* (he also called it Speculative Rhetoric). Peirce expected it to be "destined to grow into a colossal doctrine which may be expected to lead to most important philosophical conclusions" (CP 3.454). One of the expectations of methodeutic was to answer what the *theorematic* aspects of reasoning consist of. Peirce rightly admitted having "[in]sufficiently studied the methodeutic of theorematic reasoning" (CP 4.627, 1908). In Hintikka's work we see a continuation of what Peirce sketched as such methodeutic. An obvious such example is his "first real discovery", related to theorematic reasoning and which Hintikka interpreted in terms of the analysis of complex quantificational patterns. Generally, both favoured a broadly logical approach to the analysis of scientific method, placing deduction—rightly understood—at the core of that methodology [47]. Hintikka's 2007 book *Socratic Epistemology* may indeed be viewed as fulfilling the gap that Peirce lamented to exist in philosophy, which after all is supposed to be practiced in a scientific manner: "THE book on this subject remains to be written; and what I am chiefly concerned to do is to make the writing of it more possible" (CP 2.109).

There is an important lesson in Peirce's approach to the analysis of scientific method, namely that one must have a first-hand knowledge of what scientific thought is, and that over and above exact methods, this knowledge concerns what experienced scientists have at their disposal in the actual practice of scientific work.

> A study of logical methodeutic [is] illuminated by the light of a first-hand acquaintance with genuine scientific thought—the sort of thought whose tools literally comprise not merely Ideas of mathematical exactitude, but also the apparatus of the skilled manipulator, actually in use. (CP 6.488, 1908)

This was also Hintikka's desire for philosophers to take into their hearts. The worry is that contemporary philosophers of science are not that well trained or even interested in the substance of various sciences—and even when they are, they do not practice their philosophical work in such a wise that their philosophical proposals would become the kinds of research questions that would be conducive to the actual practices of scientists and their experimental work.

3. Pragmaticism as the Theory of Meaning

Peirce and Hintikka have laid a famous critique of both sense-data and intuitions in the methodology of philosophy.

> There are no such members of our world as sense-data. [7, p. 177]

Perception and cross-identification are primary, sense-data at most their hypostatization. Peirce expressed the idea in seeing our senses as "reasoning machines" (MS 831, MS 1101, NEM 2:1114). Also,

> The data from which inference sets out and upon which all reasoning depends are the perceptual facts. (CP 2.143)

To this criticism of raw or underived sense-data Peirce and Hintikka both added a criticism of the role of intuitions in philosophy:

> The truth of reasonings consist in that instinct's saying that they are true. Outside of a German treatise of logic, I have never met with so bald a fallacy as that. (CP 2.169)

Hintikka's criticism of the methodology of intuitions in contemporary philosophy appears, among others, posthumously in [26], but his anti-Cartesian thought it is present already in his early works.

In "Quine on Quantifying In" ([9], cf. [13]) Hintikka argued that the rather unmotivated idea of 'rigid designators' is not the same as the entities that are naturally to be considered as entities of cross-identification, namely individuating functions. Hintikka's criticism of Quine on his extreme narrowness in the conception of modality is manifold, but its upshot is an endorsement of the pragmatistic theory of meaning, which Quine of course did not have in virtually any sense of that Peircean term. In a somewhat less-known piece on comparing Quine and Peirce, Hintikka wrote:

> Postulation of certain meanings which the subject possesses surely has conceivable practical consequences. They can be brought out by asking what the speaker would have said and done had his past experiences been different. [10, p. 8]

Peirce's theory of meaning, pragmaticism, is indeed a method of subjunctively mapping our general principles of conduct that trace out conceivable practical consequences that the possession of a concept would elicit:

> The word pragmati[ci]sm was invented to express a certain maxim of logic... intended to furnish a method for the analysis of concepts. A concept is something having the mode of being of a general type which is, or may be made, the rational part of the purport of a word. ... The method prescribed in the maxim is to trace out in the imagination the conceivable practical consequences,—that is, the consequences for deliberate, self-controlled conduct,—of the affirmation or denial of the concept; and the assertion of the maxim is that herein lies the whole of the purport of the word, the entire concept. (CP 8.191)

The exact theory of pragmaticist theory is in both Peirce and Hintikka articulated in terms of a *game-theoretic semantics*: Peirce using games in the implicit sense in which general habits of action are the winning strategies [36,41,43]. Now Hintikka did happen to claim, in response to the question 'Who plays these semantic games?', that Peirce had taken the players to be human players: "Peirce presented his semantical games as games between two human players" [22, p. 538]. Yet in Peirce's game-theoretic semantics players are proper theoretical constructions: "In our make believe, two parties are feigned to be concerned in all scribing of graphs; the one called the Graphist, the other the interpreter" (MS 280, [43]).

> We devote a certain surface...to such a use that whatever proposition is expressed upon it shall be understood (in make-believe, you know, for we are only studying logic and not attaching any importance to the matter of the propositions which we take as examples), to have been asseverated by the graphist and to be implicitly believed by the interpreter. This surface we call the sheet of assertion. It is supposed, that is imagined in make-believe, to be the mirror of the state of mind of the interpreter...The Graphist is really Plastic Nature, or the Artifex of Nature. (MS 280, 1905)

Also [27] had erred in claiming Peirce not to have assigned appropriate logical roles to these actors in his semantic theory. The claim is refuted by almost every relevant passage from Peirce. In one of them, he tells that the semantic game is that between the Graphist as "the author of truth", whose task is to forbid falsity and permit truth, and the Interpreter, as the opponent of the Graphist whose task is to forbid truth and defend falsity:

> Now the graphist, as the author of truth (for we have seen that falsity is what he forbids and truth what he permits) and source of all the interpreter's knowledge must be recognized as being either Plastic Nature or the Artifex of Nature. The universe is simply that the collective whole of all things of which to the predication of whose existence the Graphist interposes no veto, or extends a positive permission.
>
> The reason why it is necessary to assume a Graphist as well as an interpreter is [that] logic cannot be successfully studied without perfectly clear ideas. Now the graphs and the sheet of assertion are represented as signs; but if they are signs, they must, according to the principles of pragmaticism, function as such. For it will be found to be a corollary from that principle that existence consists in action.
>
> We should come to the same conclusion that commonsense would have jumped to at the outset; namely, that the Graphist-mind and interpreter-mind *must have all the characters of personal intellects possessed of moral natures*. (MS 280, added emphasis)

The last sentence is also the key to why Peirce unhesitatingly took logic to be a normative science; a science which reposes on ethics, even esthetics. His answer has been analyzed in detail in [36,41], and so there is little need to go into those details here, but I will briefly return to the normativity of logic it in Sect. 5 below from another point of view, namely in relation to questions of identification, possibilities, and the common ground of the interlocutors.

4. Science: Its Values, the Economy of Research, and Forms of Reasoning

4.1. Values in Science

Peirce's valuable insight to inquiry has been that not all hypotheses concluded by abduction are worthy of further investigation. A hypothesis is to be submitted to an inductive trial provided that the hypothesis (i) is testable, which is ascertained by deduction, that it (ii) may explain some surprising fact(s), and that the hypothesis (iii) should follow the principles of the economy of research, in other words its formulation and refinement does not require a consumption of unreasonable amount of resources, time, energy and thought when compared to other conceivable rival hypotheses [48,50,51].

Peirce's economy of research proposes a cost-benefit analysis according to which "hypotheses that ought to be entertained becomes purely a question of economy" (CP 6.528). Given limited resources of money, time and energy, benefits are gained in choosing hypotheses to be tested that are prone to advance science in certain specific senses. The distinction of epistemic versus non-epistemic values evaporates. The real scientific values are those of *tychism*, *synechism* and *uberty*. It has recently turned out that principles of the economy of research guide the adoption of scientific values not only before the inductive testing (this is the usual acknowledgement in the literature) but also during the inductive phase of scientific inquiry [3].

This viewpoint leads to some of the least discussed aspects of both Peirce's and Hintikka's philosophy of science, namely the ethics of research as a question of the logic of science.

4.2. Omitting Data: An Example in the Economy of Research

Why do scientists sometimes behave as if they would intentionally be withholding information from public scrutiny and review? Hintikka [21] has analyzed the infamous Millikan oil-drop experiment and argued that the incomplete dataset presented as the outcome of those experiments was not a violation of ethical principles of science. On the contrary, the decision to omit certain data was part of the entirely appropriate research strategies. Those decisions are made by heeding to the principles of economy, which entitle us to conclude that some results need not to be included in the reporting of the experimental outcomes. At the moment when those decisions to exclude certain data are to be made, it suffices that the general course of events that Nature would yield is clear to our skilled manipulator. In Hintikka's words, "Never to omit data cannot be part of any realistic methodology" [23, p. 223]. It is the actual practices and the conduct of inquiry that determine what the decisions are that the experimenter is recommended to make in the light of the economy of research. Those practices follow the logic of science in which certain answers that Nature makes can be bracketed and thus can relatively safely be ignored, in so far as they do not fit the generalizable schema that is readily emerging from the patterns of the experiment and its interpretation under the given theoretical considerations.

4.3. Hypothesis Selection and the Principles of Economy

Peirce laid out similar views when he was mapping our what the logic of science would look like in practice. Facts what would make the proposed hypothesis objectively probable are strong recommendations to test the hypotheses in question. But hypotheses may appear likely because they conform to our experience and to the beliefs that already are in our possession, not because those would be the best one to be tested in certain other senses of the 'best'. Those actually to be tested have to be weighed against economic considerations, no matter how improbable or unlikely their initial probabilities are deemed to be. We would thus do well to value qualities that characterize hypotheses that have to be given up anyway. The following three come from Peirce's own shopping list:

> The third category of factors of economy, those arising from the relation of what is proposed to other projects, is especially important in abduction, because very rarely can we positively expect a given hypothesis to prove entirely satisfactory; and we must always consider what will happen when the hypothesis proposed breaks down. The qualities which these considerations induce us to value in a hypothesis are three, which I may entitle *Caution, Breadth, and Incomplexity.* (CP 7.220, 1901; EP 2: 108–9, added emphasis)

The quality of *caution* expresses the Socrates–Hintikka recommendation to break big questions into a series of small questions, which then can be reasonably interrogated and experimented with. Peirce found it effective to have a logarithmic strategy which prunes the search space by a clever formulation of a short series of *yes-no*-questions put to Nature. Hintikka [19] formulated the *yes–no*-theorem in the Tableaux system of his interrogative model of inquiry.

The quality of *breadth* is to favour generalisations of hypotheses that cover more ground than others and can accommodate different phenomena, thus saving scientists from repetitious and needless work. In Millikan's experiment, the skillful manipulator had already seen what such a generalization should look like and thus for economic reasons needed not include all the data that the experiment had produced in the calculations.

Incomplexity, which should not be confuse with outright simplicity of hypotheses, is to expedite inquiry by favouring hypotheses that are cheaper to test, in many senses of 'cheap'. Incomplexity thus can give us some solid stepping stones by which one might hope to uncover further and possibly more complicated structures.

The method of inquiry is thus *elenchus*, the Socratic method of questioning, in both Peirce's and Hintikka's philosophies of science. They repose on values that do not fall within the dichotomy of epistemic/non-epistemic in any obvious fashion, as they respect economic (Peirce) and strategic (Hintikka) considerations, both of which are two sides of the same, abductive coin.

5. Logic as a Calculus Versus Logic as a Universal System

5.1. Philosophy of Logic as Philosophy of Notation

Hintikka's famous distinction that unearthed certain Collingwoodean absolute presuppositions in one's fundamental thought is that of *logic as a universal means of expression* (including the ineffability of semantics thesis and a 'one-world' philosophy) versus *logic as a calculus*, as a re-interpretable method of reasoning, meaning and experimentation [16]. Peirce's tenet of logic as calculus included, in Hintikka's exposition, the emergence of the theory of modern logic as a development of methods of studying algebraic structures, iconicity of model-theoretic semantics, invention of various meta-logical methods and their importance in the theory of logic, application of various modalities in logical analysis, as well as taking syntax, semantics and pragmatics as a unity instead of separate areas of investigation [14].

Of particular note is the applicability of this distinction in the *philosophy of notation* [2]. A universalist notation would then be one in which the notational repertoire and its interpretations are supposed to be fixed, once and for all. For example the so-called theory of Euler Diagrams (or more appropriately speaking, Leibniz Diagrams) presupposes a universalist thesis according to which one would be led to claim that the validity of rules such as Barbara are to be seen as something like a 'free ride': that their justification and hence the relation between the premisses and conclusions would fall from the observational advantages that such diagrammatic notation has to offer over others. A calculist, in contrast, would hold the task of logic to be in analysis of inferences, and in the realization of the significations of the terms and notations used in those practices and operations. It would matter what the meanings of a certain piece of notation are, and those meanings do not fall from the physical appearance or analogue any more than they do from the characteristics of the notation alone as it appeals to the eye, the reason being that such interpretational tasks and aims may vary indefinitely.[2]

Frege is in Hintikka's application of the distinction an outspoken universalist, who believed in one true logic and in the fact that such logic suffices to characterize the realm of logical thought. One symptom of that belief is the Frege–Russell trichotomy, namely that the verb for *being* (Greek *estin*) is multiply ambiguous between various senses of 'is': the 'is' of existence, the 'is' of identity, the 'is' of predication, and the 'is' of subsumption, among others [17]. Hintikka has famously taken 'is' to be a simple concept in the Aristotelian fashion, in which the underlying logic of the behaviour of that verb is the same. Thus no separate sign, unlike in the Frege–Russell characterization, is needed to express various senses of this term. Phrased in terms of Peirce's philosophy of notation, 'is' is a *line of identity*, because in Peirce's theory of Beta graphs (and in its extensions, including some Gamma graphs, see below)

[2] Thus for instance the so-called Multiple-Readings argument that tries to distinguish diagrammatic from other kinds of notations is merely communicating an unarticulated expression of a universalist presupposition concerning logic [1].

it is only one sign, namely the line of identity, that suffices to capture all these different senses and uses of *estin*.

5.2. On the Origins of Modern Epistemic Logic

A much less-known connection between Peirce and Hintikka is the invention of epistemic logic and its application to logical analysis. Peirce developed an interpretation of modal operators on modal logic as *subjective possibilities*; thus they are epistemic concepts that are the duals of the knowledge operator. He proposed an interpretation of them in terms of a relation that obtains between various states of information (MS 467). He also proceeded to propose various systems of modal and epistemic logic, with different rules of transformations obtaining in them [28,37].

The following passages attest the first point well, namely that Peirce had an epistemic charaterization of modal notions:

> A modal proposition is a simple assertion, not about the universe of things, but about the universe of facts that one is in a state of information sufficient to know. (CP 4.520, cf. [4])

Moreover, in his 1906 sketches of the theory of *tinctured gamma graphs*, Peirce took the *verso* of the sheet of assertion to be "usually appropriated to imparting information about subjective possibilities or what may be true for aught we know" (CP 4.574).

In relation to systems of modal epistemic logic, Peirce argued that the **KK**-thesis is false: "There will be some peculiar and interesting little rules, owing to the fact that what one knows, one has the means of knowing that one knows–which is sometimes incorrectly stated in the form that whatever one knows, one knows that one knows, which is manifestly false" (CP 4.521, 1903; MS 467; [28]). Thus epistemic logics that are to analyse notions of knowledge would have to be, just as Hintikka argued, below **S5**.

5.3. The Tableaux Method and Its Multiple Discoveries

Peirce developed modal logics in his 1903 Lowell Lectures. The second lecture ends with a remark[3] that concerns certain logical properties of possibility, impossibility, necessity and contingency of the Alpha and Beta parts of his theory of existential graphs. Here Peirce presents a rule for Beta-possibility, which in modern terms is to show the existence of a model by the construction rules of the analytic tree (Tableau) method, first by reducing the question of the Beta-possibility to that of the question concerning Alpha-possibility by using a procedure that resembles quantifier elimination (that is, by rewriting certain Beta graphs that have been reduced to "adaptible pairs" of lines "without junctures"). The five operations that follow are meant to demonstrate the satisfiability of the problem thus reduced in the "universe of alpha possibility". These prescient observations then lead Peirce to begin the lecture three (drafted on October 5th, see R 462 and S-31) by investigating certain

[3] The ending that begins with the words "To fill my hour tonight...", see R S-32.

Beta-impossible statements. He invents a couple of examples that involve various types of modalities, and concludes that an impossibility involved in them consists of an impossibility for whatever exists, that is, whatever partakes of an individual existence. But there are, besides existence, relationships between things and their qualities and between things and laws, and it is those relationships, which Peirce terms "references" (see also R 490, [44] and below), that do not possess individuality in the same sense as the relations in the Beta graphs do. This perspective immediately leads to new kinds of logics that can analyze assertions that "deal with what can logically be asserted of meanings".

Where the Beta-impossibility and intractability really enter the scene is in testing the limits of the expressive power of the system to represent complicated statements of natural language. If those statements involve modal expressions, including expressions that are *esse in futuro*, and their meanings references to states of things that fail to obtain in the actual world, then their representation, as Peirce notices, have to take place in the Gamma part of the theory of logical graphs. However, even in the theory of Beta graphs one could try to represent meanings that are not obviously absurd or contradictory but are nevertheless peculiar. The question thus arises: How does one discover contradictions in complex Beta graphs?

Finding out contradictions in first-order formulas is indeed a non-trivial task. The semantic approach tells us that a statement is consistent if and only if it has a model. The tree method from Beth and Hintikka is routinely used in attempts to construct such a model, and a frustrated model-construction in which all branches of the tree lead to a contradiction and close shows that the statement in question inconsistent. The procedure necessitates some further details as well, such as how to apply the rule of existential instantiation, as it involves introduction of new constants, and which Peirce for that reason would take to constitute a *theorematic step* in the attempted model-construction process. Another is finding a right way to handle universal quantification, which being a nondeterministic procedure also involves theorematic steps on the development of strategies for the proofs that avoid construction of infinite branches that would never close.

Finding out the nature of the contradiction nevertheless leads Peirce in these lectures to propose a new rule to do such a consistency-check for the Beta graphs. He introduces some new terminology, such as that of an "adaptible pair" (R S-32). The definition of the adaptible pair is not clear, but the rule involves iterating the entire graph, followed by a conjoining of such adaptible pairs. He then suggests new individuals to be introduced, much like in existential instantiation. The rule strives to expand the graph to the point in which one could literally see that it is impossible, that is, that the expanded graph involves a contradiction. Peirce concludes that the contradiction in his example is of the same general type as what the pseudograph would do in graphs that express paradoxical statements. Though the definition and his presentation and explanation of the new consistency-checking rule is far from complete, the idea of the proposal, namely that in trying to discover hidden contradictions in complicated Beta graphs there is a rule that can be systematically followed, is quite remarkably on the right track.

An earlier and considerably more incomplete attempt from October 2 (Lecture III(b), R 457) confirms that Peirce is after something like a strategic rule that is to aid the search for a tableaux-like model-construction: if it is impossible that an application of transformation rules to a proposition ever leads to a contradiction, how to ascertain oneself of that impossibility? Peirce says that a wrong or a bad application of a rule "may lead to a result which can obviously never lead to the cancelling of the sheet when the original graph otherwise treated might have done so" (R 457). A wrong application of the universal rule may indeed lead to an infinite branch that will never close. Peirce's remedy is to modify rules of transformation such that only reversible ones are admissible, and then let only those to be applied so that one is always entitled to backtrack in a branch of the tree to a previous node in which to try out some alternative proof strategy.

I wanted to explain this unknown historical development in some detail here since it is obviously congenial to Hintikka's later work in logical methods and analysis. Just as Hintikka would emphasize over a century later [43], the nature of logic has a lot to do with the presence of such model-construction processes. Indeed Peirce has proposed an application of the tree method already in the 1885 paper "On the Algebra of Logic: A Contribution to the Philosophy of Notation". Beth and Hintikka would systematize the Gentzen method so as to involve the labelling of trees with semantic values. In fact the sequent calculus is also a Peircean innovation, who in his earlier, 1880 paper "On the Algebra of Logic" would take the calculus to be about the deductive consequence relation. This is clearly shown in the rules for the copula and its properties [30]. His calculus is a Boolean algebra and obviously agrees with classical propositional logic. Peirce achieved a sequent calculus for his logic immediately in the 1880 paper.[4]

5.4. "References": The Problem of Cross-Identification and the Method of Gamma Graphs

In the above we saw Peirce mentioning an important class of relationships which he had termed the *references*. His motivation in much of the Lowell Lectures, though he did not actually manage to deliver those parts to the audience, was indeed to understand the nature of reasoning that involves propositions with intensional notions. He develops the Gamma systems, including a treatment of references, as logical representations that could deal with qualities

[4] "My system of 'Existential Graphs"', Peirce explains in hindsight, "puts in a clearer light a truth first virtually enunciated by my student (afterward professor) O. H. Mitchell". Mitchell showed that deductive reasoning "can always be reached by adding to the stated antecedents and subtracting from stated consequents, being understood that if an antecedent be itself a conditional proposition, its antecedent is of the nature of a consequent" (MS 905, 1908). These two operations that adequately characterize logically necessary reasoning are much exploited in modern systems of proof sequents. He admitted that he had not fully grasped the import of Mitchell's work when his 1885 paper was written up in 1883.

and laws. I mention his innovation here simply because it marks an important precursor to the philosophical question of identification that emerged in the wake of the development of quantified modal logics in the 1950 and 1960s. Hintikka's contributions have been vital in highlighting the importance of the philosophical and conceptual nature of the problems involved in those logics.

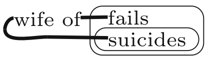
Peirce had returned to the question of references in his 1906 National Academy of Sciences address. One quarter into the presentation, he recounts the audience the following points:

> In all my attempts to classify relations, I have invariably recognized, as one great class of relations, the class of *references*, as I have called them, where one correlate is an existent, and another is a mere possibility; yet whenever I have undertaken to develop the logic of relations, I have always left these references out of account, notwithstanding their manifest importance, simply because the algebras or other forms of diagrammatization which I employed did not seem to afford me any means of representing them. I need hardly say that the moment I discovered in the *verso* of the sheet of Existential Graphs a representation of a universe of possibility, I perceived that a *reference* would be represented by a graph which should cross a cut, thus subduing a vast field of thought to the governance and control of exact logic. (MS 490; CP 4.579; [44]).

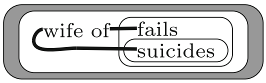
Peirce's 1906 Academy talk has a good claim of making him one of the founders of modern philosophical logic. In tackling these two puzzles he comes to establish the philosophical significance of multi-modal logics, quantification and identity in modal contexts, the idea of world-lines (references), as well as what is now known as 'Peirce's Puzzle', one version of which he mentions in his paper "Prolegomena to an Apology for Pragmaticism" (1906). Moreover, Peirce's system of conventions for the graphical logic is a novel approach for the meaning of logical constants, including logical connectives and quantifiers, and obviously relies neither on the presuppositions of proof-theoretic semantics or on the model-theoretic, permutational one.

What is known as Peirce's Puzzle is the question of the meaning of indefinites in the context of conditional sentences, for which Hintikka [25] proposed a truth-value-gap approach in order to resolve the puzzle. Various examples of Peirce's Puzzle, which Peirce himself proposed to analyze in terms of quantified modalities and thus in terms of concepts that involve references, come from his unpublished *Logic Notebook*. We can provide here a sample of them:

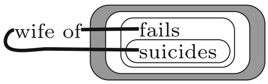
[MS 339, *Logic Notebook*, 1908 September 7 [320r].



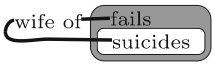
There is a married woman and should her husband fail she will commit suicide (under the actual circumstances). [But it is not said that his failure will have any connexion with her suicide.]



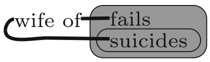
Under all circumstances there would be a married woman who, should her husband fail, would commit suicide.



There is a married woman and under all circumstances, *the fact would be*, that if her husband fails she will commit suicide.



There is a married woman; and if her husband *might* fail she will suicide.



There is a married woman and should her husband fail she might commit suicide.

Peirce's solution to Peirce's Puzzle is thus also an example of how conceptual questions of cross-identification enter the logical analysis. He explicitly considered that problem in his quantified epistemic logic.

Below is another example of a formula in which the problem of quantifying into—namely the problem of line passing into the tinctured (argent) area of modality—has to be explicitly addressed:

> I will now, in the briefest manner possible illustrate some of the features of it which might have escaped notice at first view. Figure 1 is to be understood as lying on the argent of the recto. It asserts that there is a man who is at once loved by an existing woman and loves an existing woman.

Fig. 1

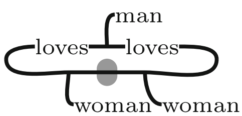
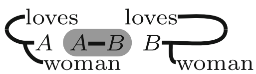


Fig. 2

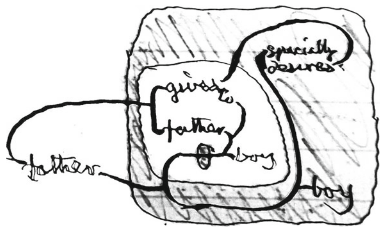


> Now in order to understand the ligature passing through the azure enclosure we put instead of that part of the graph its equivalent in Fig. 2. This reads the woman who loves is by name A; the woman who is loved is by name B. And it is subjectively impossible (that is, is contrary to what is known by the Graphist) that A should be B. In other words the woman who loves and the woman who is loved (whom the graph does not assert to be otherwise known to the

> Graphist) are known by the Graphist not to be the same person. Therefore, Fig. 1 asserts that there is a man who is loved by one woman and loves a woman known by the Graphist to be another. If the area of the cut had been in argent instead of azure the meaning would have been the same, since we have to take the Graphist's word for it that he does know them not to be the same, and he would give his word to the effect that they were not the same if the area of the cut had been argent.

Fifty years later, the issue of cross-identification became intensively studied by many philosophical logicians, including Hintikka and his students. Esa Saarinen [49] came close to Peirce's idea of cross-identification in emphasizing *continuity principles* involved in the processes of identification and re-identification. Notationally, those principles can be captured by the continuity of Peirce's line of identity, which in cases such as above are in fact now also lines of *identification*. They differ from identity lines only in the respect that they are lines (and thus graph-replicas and not ligatures) that can cross cuts.

This special behaviour of the line comes especially clear from one of Peirce's own examples which is reproduced below. In it, the two instances of the boy may not be, as far as it is known (namely that it is subjectively possible), the same [44]:

> I had better give one more illustration of the interpretation of graphs:
>
> 
>
>
> *There is a father and if any boy of his specially desires anything the father will give that thing to some other boy of his.*

Another common ground between Peirce and Hintikka in their interpretations of the interplay of quantifiers and modalities is the assumption of the presence of the *common ground*. Various possible states of information are, as we can detect in both Peirce's and Hintikka's theories, to be considered as real possibilities: they are possibilities that *can become actual* (MS 280; [40]). Thus some of their parts or stages have something in common in order to enable cross-identification. Peirce's theoretical agents of the Graphist and the Interpreter are at this moment introduced as constructions whose communication of the meanings of concepts of logic is possible, by virtue of the fact that they share the common ground which logical discourse is taken to be about [37]. Reaching that common ground is, in turn, possible as soon as the elements in the universe of discourse are sufficiently continuous. Such continuity has moreover to be represented in a continuous fashion; it may well be the graphical method that enables one to do that in a completely satisfactory manner.

5.5. Semantics of Questions

The semantics of questions is an important subfield of the theory of inquiry and hence epistemology. Both Peirce and Hintikka proposed the method of a logical analysis of questions in a closely related fashion. Peirce took interrogatives to be specific kinds of imperatives (that is, requests for information) in his theory of signs—also known as semeiotic, to be conceived as a general theory of logic. A closely related approach is found in Hintikka's development of the theory of the semantics of questions.

An example that comes from Peirce suffices to prove the point. In 1893 he studied the nature of assertions, writing that "An assertion has its *modality*, or measure of assurance, and a question generally involves as part of it an assertion of emphatically low modality. In addition to that, it is intended to stimulate the hearer to make an answer" (CP 4.57). For example, the question

> Does this road lead to the city?

can according to Peirce be replaced with

> "This road leads, perhaps, to the city. I wish to know what you think about it." (CP 4.57, 1893)

Here Peirce analyses the meaning of an interrogative as a request for information. He is seen to apply the same strategy as in his late theory of abductive inference, in which the conclusion of an abductive inference is likewise an invitation by Nature to proceed pursuing a systematic investigation of that suggestion. The specific linguistic form of these conclusions is a co-hortative mood [28].

The analogues do not end here. How close Hintikka in his theory of the semantics of questions comes to Peirce's example above is shown by the fact that the meaning of questions in not only to request certain information, but to request that a certain information *would become known* to the questioner. Moreover, since epistemology for both Peirce and Hintikka is what I have termed action-first epistemology, and as it is subservient to the method of inquiry, such knowledge is possible only if it is derived from a certain *class of habitual action*, namely from the response to a *request to make the knowledge known* to the questioner. This shows at once its imperative (and by implication, a co-hortative) character as a request that can be reformulated as a directive "Bring it about that I would know p". This is exactly how Peirce proposes the analysis of the meaning of questions to proceed in the above quotation and elsewhere, and it is exactly what his late and admittedly incomplete theory of inquiry as a methodeutical enterprise was also intended to articulate: that abductive conclusions are requests to Nature to act in a certain way in certain kinds of circumstances, so that the skillful manipulators (the experimenters) would become in a position to gain important pieces of information as part of their *yes–no*-interrogation strategies.

6. Nominalism as a Difference?

Though Peirce's and Hintikka's logical thoughts share surprising similarities, there are also some marked differences. Peirce's *synechism* is not, sight unseen, compatible with a kind of nominalism that we find in Hintikka's logical philosophy, as the latter takes elements in the universe of discourse to be spatiotemporally continuous individuals, and quantifiers in the proposed reduction of all second-order quantification to the first-order level [5,24] to range over them. Peirce's notion of continuity is the *true continuum*, which cannot be conceived as a point-set structure or to be modelled by standard set-theoretic constructions. However, Hintikka's critique of axiomatic set theory is in fact on the same track; the alternative he has offered is a kind of partial set theory, with indefiniteness of the concepts that talk about its elements, and including an admission of the existence of the greatest element [20]. Hintikka has also argued that his revisionist approach in the philosophy of set theory would serve as a better model for quantificational logics that stem from the 'independence-friendly' patterns of quantification.

But the reductive argument that Hintikka has offered in defending IF logic as a first-order theory in the foundations of mathematics seems to be a crystalline expression of nominalist doctrines. However, the kind of nominalism that Hintikka took to be a derivative of quantificational IF logic might in fact be the only version of nominalism in town that could have been acceptable to Peirce. For Peirce took pragmaticism to conform to "extreme scholastic realism", while Hintikka interpreted possible worlds as "small worlds". Their domains have to have at most some proper subset in common and thus also contain possibilities that are real, in the Scotus–Peirce sense of the real, in which modes of identification that trace those common parts and their points of divergence are of perennial importance. Moreover, Peirce's theory of quantification aims at capturing what is going on in actual mathematical practices, and to analyze those practices. Thus the kind of nominalism that results from reducing higher-order notions (including the Skolem functions) to first-order concepts would have been acceptable to Peirce as long as it would derive its rationale from the analysis of the meaning of quantifiers in the light of their role in the actual practices of mathematical reasoning, as it is only those meanings that would yield an improved understanding of the substance of mathematical reasoning.

This rationale stands in sharp contrast to the behaviour of quantifiers in the Frege–Russell logic, much criticized by Hintikka. Peirce's lifelong goal was to understand the practices of mathematical reasoning and to do so he developed a broad theory of logic that could analyze the nature of that reasoning. Peirce's logic of quantifiers took note both of the signs of generality that express important logical concepts as well as the complex interplay of such signs that express mathematical concepts some of which cannot be expressed by ordinary first-order quantification but which are expressible in Hintikka's IF logic (MS 430; [44]). As Peirce developed his logic quite independently of the kinds of formal considerations that led Frege to his doctrines, he did not fell victim to the emerging conceptions of logic that came to define the better part of the 20th-century logic. Rather, Peirce could be seen as a pioneer of precisely how to overcome the limitations of logical formalism when the purpose is to represent actual and unrestricted patterns of dependence and independence of quantified variables in the mathematical theory building, free from the straightjackets of the Frege–Russell types of formalism.

One should also mention as a separate difference the rarity of finding in Hintikka's works any conceptualizations of the theory of evolution or evolutionary biology to explain human epistemic success. Peirce's inquiry-driven epistemology, on the contrary, is largely defined by attempts to reinterpret the Darwinian theory of evolution, especially in the light of what much later became known and accepted as the inheritance of learned characteristics, also known as the Baldwin Effect ([42], attributed to James M. Baldwin, Peirce's colleague and an editor of his many contributions). Philosophy of biology indeed is one of the few areas of philosophy that Hintikka did not write much about.

7. Conclusion

Numerous other points of contact, both systematic and historically acquired, could be exposed between Hintikka and the philosopher he might have admired and respected above all others [12,14]. When Peirce wrote that philosophy is "The Queen Bee of the Sciences" (MS 280, [37]), what he meant was not only a recognition of philosophy as a science among or even above other sciences, but also an injunction of the maxims of the ethics of terminology and the ethics of notation into the development of logical methods that are to be applied to the purposes of serious philosophizing. These maxims presuppose the logic-as-a-calculus viewpoint which implies that language is inept for expressing thought in full. Thus proper nomenclatures and logical notations need to be developed to carry out that work. Since philosophy should stick to the vernacular as far as possible, but also since exact words are hard or impossible to find yet we should be historically and etymologically faithful to our meanings, it is crucial that new terms, new conceptions and new notations are justified when no other concept is found that could serve the purpose equally well or better. Thinking in terms of ordinary language will in the end be a flawed enterprise anyhow. In the conceptual and logical analysis of philosophical ideas that are often vague and imprecise, one should use technical terms and design well-thought-out notational, graphical and whatnot representational improvements tailored to serve the scientific and logical ends of the analysis. When Hintikka took a drop of logic in inquiry to square it with ordinary language, what he had in mind was precisely such considerations. But that would be just another manifestation of the depth and the values of what I have here termed to be the action-first-and-knowledge-last epistemologies that we find common in Peirce's and Hintikka's logical philosophies.

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