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Logica Universalis

Aristotle's Prototype Rule-Based Underlying Logic

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Dedicated to William Corcoran, Ph.D., PE.
With love, gratitude, and admiration.

Abstract. This expository paper on Aristotle's prototype underlying logic is intended for a broad audience that includes non-specialists. It requires as background a discussion of Aristotle's demonstrative logic. Demonstrative logic or apodictics is the study of *demonstration* as opposed to persuasion. It is the subject of Aristotle's two-volume *Analytics*, as its first sentence says. Many of Aristotle's examples are geometrical. A typical geometrical demonstration requires a theorem that is to be demonstrated, known premises from which the theorem is to be deduced, and a deductive logic by which the steps of the deduction proceed. Every demonstration produces (or confirms) *knowledge* of (the truth of) its *conclusion* for every person who comprehends the demonstration. Aristotle presented a general *truth-and-consequence theory of demonstration* meant to apply to all demonstrations: a demonstration is an extended argumentation that begins with premises known to be *truths* and that involves a chain of reasoning showing by deductively evident steps that its conclusion is a *consequence* of its premises. In short, a demonstration is a *deduction* whose premises are known to be true. Aristotle's general theory of demonstration required a prior general theory of deduction presented in the *Prior Analytics*. His general *immediate-deduction-chaining theory* of deduction was meant to apply to all deductions: any deduction that is not immediately evident is an extended argumentation that involves a chaining of immediately evident steps that shows its final conclusion to follow logically from its premises. His deductions, both direct and indirect, were rule-based and not tautology-based. The idea of tautology-based deduction, which dominated modern logic in the early years of the 1900s, is nowhere to be found in *Analytics*. Rule-based (or "natural") deduction was rediscovered by modern logicians. To illustrate his general theory of deduction, Aristotle presented a prototype: an ingeniously simple and mathematically precise special case traditionally known as the *categorical syllogistic*. With reference only to propositions of the four so-called

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categorical forms, he painstakingly worked out exactly what those immediately evident deductive steps are and how they are chained to complete deductions. In his specialized prototype theory, Aristotle explained how to deduce from a given categorical premise set, no matter how large, any categorical conclusion implied by the given set. He did not extend this treatment to non-categorical deductions, thus setting a program for future logicians. The prototype, categorical syllogistic, was seen by Boole as a "first approximation" to a comprehensive logic. Today, however it appears more as the first of the dozens of logics already created and as the first exemplification of a family that continues to expand.

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1. Introduction

In the long history of this text even what is obvious has often been overlooked.

—Norman Kretzmann on a passage in the *Organon*, Buffalo, 1972.

This expository paper on Aristotle's prototype underlying logic is intended for a broad audience that includes non-specialists. It requires as background a discussion of Aristotle's demonstrative logic. Demonstrative logic presupposes the Socratic knowledge/belief distinction, between knowledge (beliefs that are known) and opinion (those that are not known). As said in the abstract, Aristotle's general theory of demonstration required the prior general theory of deduction presented in the *Prior Analytics*. His general *immediate-deduction-chaining theory* of deduction was meant to apply to all deductions. According to him, any deduction that is not immediately evident is an extended argumentation[1] that involves a chaining of immediately evident steps that shows its final conclusion to follow logically from its premises.

The task of demonstration is to make known what is not known: by extraction of information contained in what is known. Starting with what is known, demonstration produces knowledge of the unknown. It is a common and perhaps natural mistake to think that demonstration is limited to making evident what is not immediately evident. This mistake continues to be made: see von Plato [62, p. 5]. The premises of a demonstration must be known to be true, of course, but they do not need to be immediately evident. Indeed, as explained in Corcoran [11], for Aristotle the ultimate premises of demonstration were known by *epagoge* (traditionally translated 'induction') an arduous process of deriving knowledge from experience. For a more detailed treatment see Hintikka's excellent account in his 1980 article 'Aristotelian Induction', Hintikka [38].

[1] For an extended discussion of argumentation and argumentations, see Corcoran [14].

To illustrate his general theory of deduction, Aristotle presented a prototype: an ingeniously simple and mathematically precise special case traditionally known as the *categorical syllogistic*. With reference only to propositions of the four so-called categorical forms, he painstakingly worked out exactly what those immediately evident deductive steps are and how they are chained. In his specialized theory, Aristotle explained how to deduce from a given categorical premise set, no matter how large, any categorical conclusion implied by the given set. He did not extend this treatment to non-categorical deductions, thus setting a program for future logicians. The prototype, categorical syllogistic, was seen by Boole as a "first approximation" to a comprehensive logic. Today, however it is regarded as the first of the dozens of logics already created and as the first exemplification of a genus that continues to expand.

2. The Truth-and-Consequence Conception of Demonstration

Demonstrative logic or *apodictics* is the study of *demonstration* (conclusive or apodictic proof) as opposed to persuasion or even probable proof.[2] Demonstration produces knowledge. Probable proof produces *grounded* opinion. Persuasion merely produces *opinion*. Demonstrative logic thus presupposes the Socratic knowledge/belief distinction.[3] Every proposition that I know [to be true] I believe [to be true], but not conversely. I know that some, perhaps most, of my beliefs are not knowledge. Every demonstration produces *knowledge* of the truth of its *conclusion* for every person who comprehends it as a demonstration.[4] Strictly speaking, there is no way for *me* to demonstrate a conclusion *for* another person. There is no act that I can perform on another that produces the other's knowledge. People who share my knowledge of the premises must deduce the conclusion for themselves—although they might do so by autonomously following and reconfirming my chain of deduction.[5]

Demonstration makes it possible to gain new knowledge by use of previously gained knowledge. A demonstration reduces a problem to be solved to

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[2] As the words are being used here, demonstration and persuasion are fundamentally different activities. The task of demonstration is production of knowledge, which requires that the conclusion be true. The task of persuasion is production of belief to which the question of truth is irrelevant. Of course, when I demonstrate, I produce belief. Nevertheless, when I have demonstrated a proposition, it would be literally false to say that I persuaded myself of it. Such comments are made. Nevertheless, they are falsehoods or misleading and confusing half-truths when said without irony or playfulness.

[3] There is an extensive and growing literature on knowledge and belief. See Corcoran and Hamid [27].

[4] Aristotle seemed to think that demonstration is universal in the sense that a discourse that produces demonstrative knowledge for one rational person does the same for any other. He never asked what capacities and what experiences are necessary before a person can comprehend a given demonstration (Corcoran [14, pp. 22f]).

[5] Henri Poincaré (Newman [49, p. 2043]) said that he recreates the reasoning for himself in the course of following someone else's demonstration. He said that he often has the feeling that he "could have invented it".

problems already solved. See Corcoran [14, pp. 17 and 19], Spanish translation [16]. This point was central to the Peripatetic legacy. As noted by Ian Mueller [48, p. 1]: "Alexander [of Aphrodisias] is very concerned to defend Aristotelian logic as the tool (organon) of philosophy and science, a means for making unknown things known through known premises".

Demonstrative logic is not an exhaustive theory of scientific knowledge. For one thing, demonstration presupposes discovery; before we can begin to prove, we must have a conclusion, a hypothesis to try to prove. Apodictics presupposes *heuristics*, which has been called the logic of discovery. Demonstrative logic explains how a hypothesis is proved; it does not explain how it ever occurred to anyone to accept the hypothesis as something to be proved or disproved. If we accept Davenport's 1950 characterization [29, p. 9] that the object of a science is to discover and establish propositions about its subject matter, we can say that science involves heuristics (for discovering) and apodictics (for establishing). Besides the unknown conclusion, we also need known premises. Demonstrative logic does not explain how the premises are known to be true. Thus, apodictics also presupposes *epistemics*, sometimes called the logic of truth, which will be discussed briefly below.

Demonstrative logic is the subject of Aristotle's two-volume *Analytics*, as he said in the first sentence of the first volume, the *Prior Analytics* [1]—a point emphasized in 1989 both by Gasser [35] and by Smith [59, p. xiii]. Aristotle repeatedly referred to geometry for examples. However, shortly after having announced demonstration as his subject, Aristotle turned to *deduction*, the process of extracting information contained in given premises—regardless of whether those premises are known to be true or even whether they are true. After all, even false propositions imply logical consequences; we can determine that a premise is false by deducing from it a consequence we already know to be false. A deduction from unknown premises also produces knowledge—of the fact that its conclusion follows logically from (is a consequence of) its premises—not knowledge of the truth of its conclusion.[6]

In the beginning of Chapter 4 of Book A of *Prior Analytics* [1, 25b30], Aristotle wrote the following (1991 translation by Gasser [36, pp. 235f]):

> Deduction should be discussed before demonstration. Deduction is more general. Every demonstration is a deduction, but not every deduction is a demonstration.

Aristotle implied the same point elsewhere. In Chapter 10 of Book A of *Posterior Analytics*, [2, 76b23], he wrote the following (translation by Mure in McKeon [47, p. 29]):

[6] In some cases it is obvious that the conclusion follows from the premises, e.g., if the conclusion is one of the premises. However, in many cases a conclusion is temporarily *hidden*, i.e., cannot be seen to follow without a chaining of two or more deductive steps (Corcoran [25]). Moreover, as Gödel's work has taught, in many cases a conclusion that follows from given premises is permanently hidden: it cannot be deduced from those premises by a chain of deductive steps no matter how many steps are taken.

> All syllogism, and therefore a fortiori all demonstration, is addressed not to the spoken word but to the discourse within the soul.

Demonstrative logic is temporarily supplanted by deductive logic, the study of deduction in general. Since demonstration is one of many activities that use deduction, it is reasonable to study deduction before demonstration.

Although Aristotle referred to demonstrations[7] several times in *Prior Analytics*, he did not revisit demonstration per se until the *Posterior Analytics*, the second volume of the *Analytics*. Deductive logic is the subject of the first volume.

It has been said that one of Aristotle's greatest discoveries was that deduction is *cognitively neutral*: the same process of *deduction* used to draw a conclusion from premises known to be true is also used to draw conclusions from propositions whose truth or falsity is not known, or even from premises known to be false.[8] The same process of deduction used to extend our knowledge is also used to extend our opinion. Moreover, it is also used to determine consequences of propositions that are not believed and that might even be disbelieved or even known to be false.

Another of his important discoveries was that deduction is *topic neutral*: the same process of *deduction* used to draw a conclusion from geometrical premises is also used to draw conclusions from propositions about biology or any other field. His point, using the deduction/demonstration distinction, was that as far as the process is concerned, i.e., after the premises have been set forth, demonstration is a kind of deduction: demonstrating is deducing from premises known to be true.

Deduction is *content independent* in the sense that no knowledge of the subject matter *per se* is needed. It is not necessary to know the numbers, the subject matter of arithmetic, in order to deduce "No square number that is perfect is a prime number that is even" from "No prime number is square". Or more interestingly, it is not necessary to know the subject matter to deduce "Every number other than zero is the successor of a number" from "Every number has every property that belongs to zero and to the successor of every number it belongs to".

Moreover, Aristotle also discovered that deduction is *non-empirical* in the sense that external experience is irrelevant to the process of deducing a

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[7] As will be seen below, it is significant that all demonstrations mentioned in *Prior Analytics* are geometrical and that most of them involve indirect reasoning or *reductio ad absurdum*. Incidentally, although I assume in this paper that *Prior Analytics* precedes *Posterior Analytics*, my basic interpretation is entirely compatible with Solmsen's insightful view that Aristotle's theory of demonstration was largely worked out before he discovered the class of deductions and realized that it includes the demonstrations as a subclass (Ross [55, pp. 6–12, esp. 9]).

[8] Of course, demonstration is not cognitively neutral. The whole point of a demonstration is to produce knowledge of its conclusion. It is important to distinguish the processes of deduction and demonstration from their respective products, deductions and demonstrations. Although the process of deduction is cognitively neutral, it would be absurd to say that the individual deductions are cognitively neutral. How can deductions be cognitively neutral when demonstrations are not? After all, every demonstration is a deduction.

conclusion from premises. Diagrams, constructions, and other aids to imagining or manipulating subject matter are irrelevant hindrances to purely logical deduction. See *Prior Analytics* [1, 49b33–50a4] and Smith [59, p. 173].[9] In fact, in the course of a deduction, any shift of attention from the given premises to their subject matter risks the fallacy of *premise smuggling*—information not in the premises but intuitively evident from the subject matter might be tacitly assumed in an intermediate conclusion. This would be a *non sequitur*, vitiating the logical cogency of reasoning even if not engendering a material error.[10]

Aristotle did not explicitly mention the idea that deduction is information processing, but his style clearly suggests it. In fact, his style has seemed to some to suggest the even more abstract view that in deduction one attends only to the logical form of the argument ignoring the content entirely.[11]

For Aristotle, a demonstration begins with premises that are known to be true and shows by means of chaining of evident steps that its conclusion is a logical consequence of its premises. Thus, a demonstration is a step-by-step deduction whose premises are known to be true. For him, one of the main problems of logic (as opposed to, say, geometry) is to describe in detail the nature of the deductions and the nature of the individual deductive steps, the links in the chain of reasoning. Another problem is to say how the deductions are constructed. This might have been the, or one of the problems Aristotle promised to deal with when he wrote the following in the beginning of Chapter 4 of Book A of *Prior Analytics* [1, 25b26] (1989 translation by Smith [59, p. 4]).

> Let us now say through what premises, when,
> and how every deduction comes about.

As we show below Aristotle gave, in full detail, instructions by which each categorical deduction is constructed, but, as difficult and important this achievement was, it does not explain how every deduction "comes about". To do that it would be necessary to explain how humans came to possess the skill or operational knowledge of deduction. See Corcoran and Hamid [27] and also Corcoran [14] Spanish translation [16], where the problem is described in more detail.

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[9] Other writers, notably Kant and Peirce, have been interpreted as holding the nearly diametrically opposite view that every mathematical demonstration requires a diagram.

[10] Of course, this in no way rules out heuristic uses of diagrams. For example, a diagram, table, chart, or mechanical device might be heuristically useful in determination of which propositions it is promising to try to deduce from given premises or which avenues of deduction it is promising to pursue. However, according to this viewpoint, heuristic aids cannot substitute for apodictic deduction. This anti-diagram view of deduction dominates modern mainstream logic. In modern mathematical folklore, it is illustrated by the many and off-told jokes about mathematics professors who hide or erase blackboard illustrations they use as heuristic or mnemonic aids.

[11] This formalistic view of deduction is not one that I can subscribe to, nor is one that Aristotle ever entertained. See Corcoran [14]. The materialistic and formalistic views of deduction are opposite fallacies. They illustrate what Frango Nabrasa (per. comm.) called "Newton's Law of Fallacies": for every fallacy there is an equal and opposite fallacy—overzealous attempts to avoid one land the unwary student in the other.

Curiously, as Gasser and others have noted, Aristotle seems to have ignored a problem that deeply concerned later logicians, viz., the problem of devising a criterion for recognizing demonstrations. See Gasser [35].

Thus, at the very beginning of logic we find what has come to be known as *the truth-and-consequence conception of demonstration*: a demonstration is a discourse or extended argumentation that begins with premises known to be *truths* and that involves a chain of reasoning showing by evident steps that its conclusion is a *consequence* of its premises. The adjectival phrase 'truth-and-consequence' is elliptical for the more informative but awkward '*established*-truth-and-*deduced*-consequence'.

3. Demonstratives and Intuitives

Following the terminology of Peirce (b.1839–d.1914), a belief that is known to be true may be called a *cognition*. Cognitions that were obtained by demonstration are said to be *demonstrative* or *apodictic*. Cognitions that were not obtained by demonstration are said to be *intuitive*. In both cases, it is convenient to shorten the adjective/noun combination into a noun. Thus, we will speak of *demonstratives* instead of *demonstrative cognitions* and of *intuitives* (or *intuitions*) instead of *intuitive cognitions*. In his 1868 paper on cognitive faculties, Peirce has a long footnote on the history of the words 'intuition' and 'intuitive'. Shortly after introducing the noun, he wrote [52, pp. 11–12], "*Intuition* here will be nearly the same as 'premise not itself a conclusion'." For more of Peirce on intuition see [53].

Just as *individual* deductions are distinguished from the *general* process of deduction through which they are obtained, individual intuitions are distinguished from the general process of intuition through which they are obtained. Moreover, just as individual attempts to apply deduction are often arduous and often erroneous, individual attempts to apply intuition are often arduous and often erroneous. Not every intuition is "intuitively obvious" and not every belief thought to be an intuition actually is one, as Tarski said in [61], quoted from [39, pp. 110f and 117]. Intuitions may be said to be self-evident or immediate in any of several senses, but not in the sense of "trivial", "obvious", "easy", or "instant". The processes of deduction and intuition are equally fallible in the sense that there is no guarantee that attempts to apply them will always succeed.

Some writers subdivide intuitives into those that involve sense perception essentially and those known purely intellectually. However, other writers use different terminology for the two subclasses. They call intuitives known by senses 'inductions' and they restrict 'intuition' to intuitives known intellectually. For example, the ancient physician Galen (129–216 CE), wrote the following in his *Institutio Logica* [34, I.1], translated by Kieffer [42, p. 31].

> As human beings, we all know one kind of evident things through sense perception and another through ... intellectual intuition. These we know without demonstration. But things known neither by sense

> perception nor by intellectual intuition, we know through demonstration.

It is impossible to have informative demonstrative knowledge without intuitive knowledge.[12] This point was made by Plato, Aristotle, Galen, Leibniz, Pascal, and many others including Tarski [39, p. 117]. However, it is difficult for people to determine with certainty exactly which of their cognitions are intuitive and which are demonstrative. Peirce said in the 1868 paper that there is no evidence that we have the ability to determine, given an arbitrary cognition, whether it is intuitive or demonstrative [52, p. 12]. In his 1980 article "Aristotelian Induction", Hintikka [38] gave an excellent account of Aristotle's view of how intuitive cognitions are achieved. Hintikka's view agrees substantially with that of Beth [3, p. 34].[13]

[12] This passage refers to *informative* knowledge. It should not be taken to exclude the possibility of *uninformative* demonstrative knowledge not based on intuitive premises. For example, we have uninformative demonstrative knowledge of many tautologies, e.g., that every even number that is prime is a prime number that is even. Aristotle's syllogistic did not recognize tautologies and thus did not recognize the role of tautologies in deduction, which was one of Boole's revolutionary discoveries [20].

[13] It is important to understand how this terminology is to be used. For purpose of discussion, let us assume for the moment that once a person has a cognition, it is never lost, forgotten, or renounced. Let us further assume that people start out devoid of cognitions. As each cognition is achieved, it is established as an intuitive or as a demonstrative. For a given person, no cognition is both. However, I know of no reason for not thinking that perhaps some of one person's intuitive cognitions are among another person's demonstratives. A seasoned investigator can be expected to have a far greater number of intuitive cognitions than a neophyte.

In order to understand the truth-and-consequence conception of demonstration, it is useful to see how an "apparent" demonstration fails. Since a demonstration produces knowledge, there is no way for me to demonstrate something I already know. No argumentation whose conclusion is one of my present cognitions can ever become a demonstration for me. Let us exclude such cases. Any other argumentation that does not have the potential to become a demonstration for me in my present state of knowledge either has a premise that I do not know to be true or it has a chain of deduction that I cannot follow, that does not show me that the conclusion follows from its premises. The trouble is with the premise set or with the deduction—the data or the processing.

Now, if I have a demonstration that I wish to share with another, the situation is similar. The conclusion cannot be the other person's cognition. Moreover, the premises must all be the other person's cognitions. And finally, the other person must be able to follow the chain of deduction to its conclusion and through it come to know that the conclusion is a logical consequence of the premises.

None of the above should be taken to deny the remarkable facts of *deductive empathy*, without which teaching of logic would be impossible, and *demonstrative empathy*, without which teaching of mathematics would be impossible. Under demonstrative empathy, I include the ability to follow an argumentation whose premises and conclusion are known to me to be true and conclude that it would have demonstrated the conclusion if I had not already known it. As a practical matter, I must have demonstrative empathy in order to teach others the mathematics I know. Under deductive empathy, I include the ability to follow an argumentation whose premises are known by me to imply its conclusion and judge that it would have shown that the conclusion follows if I had not already known it. Further pursuit of this important topic would take us away from the immediate task.

4. Aristotle's General Theory of Deduction

Aristotle's general theory of deduction must be distinguished from the categorical syllogistic, the restricted prototype system he created to illustrate it. The latter will be sketched in the next section "Aristotle's Theory of Categorical Deductions". The expression 'immediate-deduction-chaining' can be used as an adjective to describe his general theory, which is based on two insights. The first is that in certain cases a conclusion can be seen to follow logically from given premises without recourse to any other propositions; these have been called *immediate* deductions[14] in the sense that no proposition *mediates* between the premises and conclusion in the process—*unmediated* would be a better word.[15] The second insight is that the deductions involving mediation are chainings of immediate deductions.[16]

Over and above the premises and conclusion, every deduction and thus every demonstration has a chain-of-reasoning that shows that the (final) conclusion follows logically from the premises—and thus that assertion of the premises is also virtual assertion of the conclusion.[17] The practice of clarifying the idea of an argument's validity by adding something to the effect "that assertion of the premises is also virtual assertion of the conclusion" was common in the 1800's, as exemplified by Whately's influential 1855 *Elements of Logic* [63, pp. 17, 40, 44 and passim].

An Aristotelian *direct* deduction based on three premises p1, p2, and p3, having the conclusion fc, and having a chain-of-reasoning with three intermediate conclusions ic1, ic2, and ic3, can be pictured as below. The question mark prefacing the conclusion merely indicates the conclusion to be deduced. It may be read, "Can we deduce?" or "To deduce". Here QED simply marks the end of a deduction much as a period marks the end of a sentence.[18]

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[14] Aristotle called an immediate deduction a *teleios syllogismos* or a complete syllogism, where by 'complete' he meant that nothing else is required to see that the conclusions follows. See Aristotle [1, 24b22], Boger [4, p. 188], and Smith [59, pp. 110 and 115].

[15] Remember 'immediate' does not mean "instantaneous": it has no temporal connotation. It takes time to grasp the argument and there may well be a time interval between the end of the grasping of the argument and the achieving knowledge of its validity. Cf. von Plato [62, p. 5].

[16] For an extended discussion of chaining and the epistemic problems it poses see (Corcoran [14, p. 33ff]). Aristotle and his commentators, both ancient and modern, are silent on this fundamental topic.

[17] In the case of immediate deductions, I must count a single link as a "degenerate" chain-of-reasoning. The act of deducing the conclusion from the premises is more than just the conclusion and premises. The conclusion follows without any act, but for me to deduce it, to see that it follows, requires an act.

[18] In a demonstration, it would be appropriate to take the QED marking the end of a deduction as an abbreviation of the traditional Latin *quod erat demonstrandum* (that which was to be demonstrated, or more properly, that which was required to be demonstrated), referring to the last intermediate conclusion. However, that would be inappropriate with deductions since a deduction is not necessarily a demonstration. Fortunately, those who prefer to take it as an abbreviation of Latin are free to use *quod erat deducendum* (that which was to be deduced, or more properly, that which was required to be deduced).

Direct Deduction Schema

p1
p2
p3
?fc
ic1
ic2
ic3
fc
QED

Note that in such an Aristotelian deduction the final conclusion fc occurs twice: once with a question mark as a goal to be achieved and once followed by QED as a conclusion that has been deduced—thus following the format common in Greek mathematical proofs, as previously noted by Robin Smith [59, p. 173]. Having a fully expressed goal at the outset is one of the important differences between a deduction and a calculation. Aristotle gives us a "formal system" but not a calculus. Also, note that intermediate conclusions are also used as intermediate premises. This picture represents only a direct deduction; a picture for indirect deduction is given below—after we consider a concrete example of a direct deduction.

Direct Deduction 1.

1. Every quadrangle is a polygon.
2. Every rectangle is a quadrangle.
3. Every square is a rectangle.
? Some square is a polygon.
4. Every square is a quadrangle. 3, 2
5. Every square is a polygon. 4, 1
6. Some polygon is a square. 5
7. Some square is a polygon. 6
QED

The example is from Aristotle's categorical syllogistic which is restricted to propositions in the four categorical subject-copula-predicate forms. In the below samples of categorical propositions, the *subject* is "square", the *predicate* "polygon", and the *copula* the rest.[19] Today, we would say that the copula is a logical or formal constant and that the subject and predicate are non-logical or contentful constants.

[19] In Greek as in English, in a categorical sentence such as 'Every square is a rectangle', the subject 'square' divides the copula 'Every ... is a'. Aristotle reworded his Greek in an artificial way so that the copula was entirely between the subject and predicate, which he called "terms" (using the Greek word for terminal, endpoint, end, limit, etc). He also moved the predicate to the front. For example, "Every square is a rectangle" would be worded "Rectangle belongs to every square". See the section "Colloquial and formalized languages" in Corcoran [19]. Also see [20].

Every square is a polygon.
No square is a polygon.
Some square is a polygon.
Some square is not a polygon.

It is worth emphasizing the difference between this subject-copula-predicate terminology and the subject-predicate terminology according to which a sentence is composed of subject and predicate. For example, in "Every square is a polygon", "Every square" would be the subject and "is a polygon" the predicate.

Since there are no "truth-functional constants", there is no way to form negations, double negations, or any other "truth-functional combinations" of categorical propositions.[20] Aristotle took what came to be called *the contradictory opposite* of a proposition to serve some of the purposes we are accustomed to assigning to the negation of the proposition. Using 'CO' to abbreviate 'contradictory opposite', we have the following pairings.

"Some square is not a polygon" is the CO of "Every square is a polygon", and vice versa.

"Some square is a polygon" is the CO of "No square is a polygon", and vice versa.

In every case, the contradictory opposite of a categorical proposition is logically equivalent to its negation. E.g., "Some square is not a polygon", which is the CO of "Every square is a polygon", is logically equivalent to "Not every square is a polygon", which is the negation of "Every square is a polygon". To understand inner working of syllogistic, we should remind ourselves that "Not every square is a polygon" is not a categorical sentence. Aristotle's concept of contradictory opposition enables him to implement indirect deduction without using negation.[21]

Today we have a law of double negation, that the negation of the negation of a proposition is distinct from but logically equivalent to the proposition. For Aristotle, however, every categorical proposition *is* the contradictory opposite of its own contradictory opposite. In his categorical syllogistic, there is no such thing as a double negation. His concept of contradictory opposition is entirely syntactic.

The picture for an *indirect* deduction, or *reductio-ad-impossibile*, resembles but is significantly different from that for a direct deduction. Indirect demonstrations are called *proofs by contradiction.* In such a deduction, after the premises have been assumed and the conclusion has been set as a goal, the contradictory opposite of the conclusion is assumed as an *auxiliary* premise. Then, a series of intermediate conclusions are deduced until one is reached

[20] A proposition that is a truth-functional combination of a set of propositions is composed of those in the set in such a way that its truth-value is determined by those of the propositions in the set. For example, "zero is even if one is odd" is a truth-functional combination of the two propositions "zero is even" and "one is odd", but "zero is even because one is odd" is a non-truth-functional combination. Aristotle did not make this distinction.

[21] To a modern logician this "work-around" seems very clever. But there is no independent evidence for thinking that Aristotle took any satisfaction in the device.

which oppositely contradicts a previous proposition. To represent a simple indirect demonstration, ∼fc (the contradictory opposite of the final conclusion) is added as a new assumption, the sign @ indicates auxiliary assumption, and the letter X indicates that the last intermediate conclusion ic3 "oppositely contradicts" one of the previous intermediate conclusions *or* one of the premises *or* even, in extremely rare cases, the auxiliary assumption. The rare case played an important role in the discovery of non-Euclidean geometry [13].

The sign @ can be read "Assume as an auxiliary assumption" or "Assume for purposes of reasoning". X can be read "A contradiction", or more literally "Which contradicts a previous proposition", where the relative pronoun refers to the last intermediate conclusion.[22]

Indirect Deduction Schema 1

p1
p2
p3
?fc
@∼fc
ic1
ic2
ic3
X
QED

Indirect Deduction 1.

1. Every quadrangle is a polygon.
2. Every rectangle is a quadrangle.
3. Every square is a rectangle.
? Some polygon is a square.
4. Assume: No polygon is a square.
5. No quadrangle is a square.1,4
6. No rectangle is a square. 2,5
7. Some rectangle is a square. 3

[22] In an indirect deduction, it would be inappropriate to take the QED marking the end of a deduction as an abbreviation of the traditional Latin *quod erat deducendum* (that which was to be deduced), referring to the last intermediate conclusion because the last intermediate conclusion is usually not the conclusion to be deduced. For a discussion of the unusual cases where it is, see Corcoran [13]. Euclid [31] avoided this awkwardness by repeating the final conclusion just after reaching his contradiction so that indeed the QED could always be taken as referring to the last intermediate conclusion. However, it would be less artificial to drop the idea of referring to the last intermediate conclusion by regarding QED as mere punctuation marking the end of a deduction.

In the example indirect deduction, the conclusion being deduced occurs only once where it is prefaced by the question mark; it never occurs as an intermediate conclusion. However, Aristotle's proof that every conclusion deducible directly from given premises can also be deduced indirectly probably depends on the possibility of having the stated conclusion occurring twice, the second time as an intermediate conclusion. See the diagram on page 115 in Corcoran [9].

8. Contradiction. 7, 6
QED

Like demonstration, deduction also makes it possible to gain new knowledge by use of previously gained knowledge. However, with deduction the reference is to knowledge that a conclusion *follows from* premises and not to knowledge of the *truth* of its conclusion. Again like demonstration, deduction reduces a problem to be solved to problems already solved. However, here the problem to be solved is "seeing" that the conclusion follows from the premises. The problems already solved are seeing that the conclusions of the rules of deductions follow from their respective premises. According to Aristotle, a "hidden" conclusion is seen to follow by means of chaining evidently valid arguments connecting that conclusion to premises. For the importance of the phenomenon of hiddenness as opposed to "evidentness" in logic see [25].

Aristotle's syllogisms were composed of premises, conclusions, and intermediate lines having truth-values such as in the above deductions even though there are no concrete examples in *Prior Analytics*. The concrete syllogisms were presented by using abstract syllogism schemas [21] such as the following.

Indirect Deduction Schema 2

1. Every Q is a P.
2. Every R is a Q.
3. Every S is an R.
? Some P is an S.
4. Assume: No P is an S.
5. No Q is an S. 1, 4
6. No R is an S. 2, 5
7. Some R is an S. 3
8. Contradiction. 7, 6
QED

5. Aristotle's Prototype Theory of Categorical Deductions

As an illustrative prototype special case of his general theory of deduction, Aristotle's theory of categorical deductions also had two types of deduction, direct and indirect. However, the categorical deductions used only categorical propositions and were constructed using only the eight "rules of deduction". Of the eight, seven are formal in the special sense that every two "applications" of the same rule are in the same logical form [9, p. 102]. For a more general treatment of logical form see [18] or [6, pp. 511–512]. The remaining rule amounts to the rule of repetition for categorical propositions. All eight are formal in the sense that every argument in the same form as an "application" of a given rule is an "application" of the same rule. Of the seven, three involve only one premise; four involve two premises. Those involving only one premise can be called *conversions*, since the terms in the premise occur in reverse order in the conclusion.[23] Following Boole's usage, those involving only two premises can be called *eliminations*, since one of the terms in the premises is "eliminated", i.e., does not occur in the conclusion.

Three Conversions

Every square is a rectangle	No circle is a rectangle	Some square is a rectangle
Some rectangle is a square	No rectangle is a circle	Some rectangle is a square

Two Universal Eliminations

Every rectangle is a polygon	No rectangle is a circle
Every square is a rectangle	Every square is a rectangle
Every square is a polygon	No square is a circle

Two Existential Eliminations

Every rectangle is a polygon	No rectangle is a circle
Some square is a rectangle	Some polygon is a rectangle
Some square is a polygon	Some polygon is not a circle

Aristotle collected what he regarded as evidently valid categorical arguments under the eight rules—although he did not refer to them as rules of deduction. Aristotle seemed to think that every other valid categorical argument's conclusion was hidden in the sense that it could not be seen to follow without chaining two or more of the evidently valid arguments. Moreover, he believed that any categorical conclusion that follows logically from a given set of categorical premises, no matter how many, was deducible from them by means of a deduction constructed using only his eight rules. In other words, he believed that every categorical conclusion hidden in categorical premises, once found, could be unveiled by applying his eight rules in a direct or indirect deduction. He had good reason for his belief and, as far as I know, he might have believed that he had demonstrative knowledge of it: a view argued forcefully by Smiley [58]. Aristotle's belief, but not Smiley's, has since been established using methods developed by modern mathematical logicians, See Corcoran's 1972 paper [7] and Smiley's 1973 paper [57].

It is important to notice that Aristotle's underlying deductive system was intended as an apodictic for showing that a given conclusion follows from given premises. It was not intended as a heuristic for finding the conclusions

[23] From Aristotle's point of view, the conclusions of the last two are [*outer*] *converses* of their respective premises in one modern sense of 'converse' [6, p. 189]. Moreover, the conclusions are logical equivalents of the premises. However, in the first case, the conclusion is neither a converse nor an equivalent of the premise. Furthermore, the first rule is rather artificial. From Aristotle's point of view, it is more evident that "Some square is a rectangle" follows from "Every square is a rectangle", avoiding the reversal of terms. Anyway, Aristotle's deduction of an existential conclusion from a universal premise has been mindlessly and unfairly criticized (Corcoran [9, pp. 104 and 126]; Smith [59, pp. xxv–xxvi]). It involves what has been called existential import (Corcoran [22]).

that follow from given premises. Deduction is a goal-directed activity not a goal-generating activity. Goal-generating activity has been called abduction, a word previously used for other activities. It would be better to coin a new word, perhaps *heuriscation* (from *heuristic*) or *telation* (from *telos* "goal", "purpose", "end", etc.) [25].[24]

As Robin Smith [59, p. 115] noted in 1989: "Aristotle's practice is almost always to state in advance the conclusion he is about to deduce". Aristotle's almost invariable practice of indicating the conclusion to be deduced after listing the premises to be used suggests that he intuitively recognized the goal-directedness of deduction. However, Aristotle never explicitly said that deduction is goal-directed. In fact, as far as I know no scholar writing before the present century explicitly made this point.

Accordingly, in the deductions below the question mark prefacing the conclusion is used to indicate the conclusion to be deduced. It may be read, "Can we deduce?" or "To deduce".

Certain features of Aristotle's rules are worth noticing. Each of the four forms of categorical proposition is exemplified by a conclusion of one of the four two-premise rules, giving them a kind of symmetry. In addition, in the seven rules just schematized, existential negative propositions such as "Some polygon is not a circle" are treated in a very special way. In the above schematization, there is only one occurrence of an existential negative, even though there are three occurrences of the existential affirmative. Moreover, although there are conversions for the other three, there is no conversion for the existential negative. Most strikingly, the existential negative does not occur as a premise. This means that no existential negative can be *used* as a premise in a direct deduction.

The existential negative's remarkably peculiar place in the syllogistic remains to be investigated. For example, even though the conversion of the universal negative into the existential is as obvious as the conversion of the universal affirmative into the existential it is not used in deductions.

$$\frac{\text{No circle is a rectangle.}}{\text{Some rectangle is not a circle.}}$$

Moreover, even though the existential negative is syllogistically deducible from the universal Aristotle never does a deduction.

1. No circle is a rectangle.
? Some rectangle is not a circle.
2. Assume: Every rectangle is a circle.
3. Some circle is a rectangle. 2
4. Contradiction. 3, 1
QED

[24] To be clear, in typical cases, deduction starts with a premise set and a conclusion not known to follow and not known to not follow. The reasoner first asks whether the conclusion follows. Second, the reasoner asks, if it follows, how we determine that it follows; and, if it does not follow, how we determine that it does not follow (Corcoran and Tracy [28]).

Recent work by Joray [41] followed up by Corcoran and Tracy [28] points to the importance of the missing fourth conversion.

6. Direct Versus Indirect Deductions

In Aristotle's general theory of deduction, direct and indirect deductions are equally important. As we will see below, both occur in the scientific and philosophical discourse that Aristotle took as his data. Thus, any theory that omitted one or the other would be recognized by its intended audience as inadequate if not artificial.

However, it is natural to ask the purely theoretical question whether it is necessary to have both direct and indirect deductions in Aristotle's special prototype theory, his categorical syllogistic. This question divides into two. First, is every conclusion deducible directly from given premises also deducible indirectly from the same premises? If so, direct deductions are not necessary. Second, is every conclusion deducible indirectly from given premises also deducible directly from the same premises? If so, indirect deductions are not necessary. By careful investigation of the details, it is easy to answer yes to the first question and no to the second.

To see that every direct deduction is replaceable by an indirect deduction having the same premises compare the following two easy deductions.

1. No circle is a rectangle.
2. Every square is a rectangle.
? No square is a circle.
3. No rectangle is a circle. 1
4. Every square is a rectangle. 2
5. No square is a circle. 3, 4
QED

1. No circle is a rectangle.
2. Every square is a rectangle.
? No square is a circle.
3. Assume: Some square is a circle.
4. No rectangle is a circle. 1
5. Every square is a rectangle. 2
6. No square is a circle. 4, 5
7. Contradiction. 6, 3
QED

The direct deduction on the left was transformed into the indirect deduction on the right by adding two lines. Between the statement of the conclusion goal and the first intermediate conclusion, I inserted the assumption of the contradictory opposite of the conclusion. Between the final conclusion and QED, I inserted "Contradiction". Thus, from a direct deduction I constructed an indirect deduction with the same conclusion and the same premises. It is evident that this can be done in every case, as Aristotle himself noted in *Prior Analytics* [1, 45a22–45b5]. Also see Corcoran [9, p. 115] and Smith [59, p. 154].

Now, let us turn to the second question: is every conclusion deducible indirectly also deducible directly so that indirect deductions are not necessary? Consider the following indirect deduction.

1. Every square is a rectangle.
2. Some polygon is not a rectangle.
? Some polygon is not a square.

3. Assume: Every polygon is a square.
4. Every polygon is a rectangle. 3, 1
5. X 4, 2
QED

It is obvious that neither premise is redundant; the conclusion does not follow from either one of the two alone. Thus, any deduction of the conclusion from them must use both of them. Notice that one of the premises is an existential negative. In this case, the existential negative was oppositely contradicted by the intermediate conclusion. In a direct deduction, one of the seven schematized rules would have to apply to the existential negative by itself or in combination with the other premise or with an intermediate conclusion. However, as we noted above, none of those rules apply to an existential negative premise. Therefore, no direct deduction is possible in this case.

The reasoning just used to show that this conclusion cannot be deduced from these premises by a direct deduction can be applied in general to show that no conclusion can be deduced directly from a set of premises containing an existential negative—unless of course the existential negative is redundant.

Thus, in Aristotle's categorical syllogistic, direct deductions are in a sense superfluous, whereas indirect deductions are indispensable.[25]

7. Geometric Background

It is difficult to understand the significance of Aristotle's logic without being aware of its historic context. Aristotle had rigorous training and deep interest in geometry, a subject that is replete with direct and indirect demonstrations and that is mentioned repeatedly in *Analytics*. He spent 20 years in Plato's Academy, whose entrance is said to have carried the motto: *Let no one unversed in geometry enter here*. The fact that axiomatic presentations of geometry were available to the Academy two generations before Euclid's has been noted often. Ross [54, p. 47] pointed out "there were already in Aristotle's time *Elements of Geometry*". Heath [37, Vol. I, pp. 116–117] wrote, "The geometrical textbook of the Academy was written by Theudius of Magnesia . . . [who] must be taken to be the immediate precursor of Euclid, and no doubt Euclid made full use of Theudius . . . and other available material". The central importance of mathematics in Aristotle's thought and particularly in his theory of demonstration has been widely accepted. See Beth [3, pp. 31–38].

This context makes it all the more remarkable that there is not one fully expressed or even well-sketched geometrical demonstration in *Prior Analytics* or even in *Analytics*. Today it is accepted that theory is developed from a basis of mastery of practice, that practice is improved by attention to theory, and that theory is tested by how well it explains what is found in practice and by how much it contributes to the understanding and improvement of practice. Judging by such standards it is difficult to praise *Analytics*. It is important

[25] Ironically perhaps, there are modern symbolic logic texts whose deductions are exclusively indirect (Jeffrey [40]).

to be clear about what *Analytics* isn't so that we can appreciate and benefit from what it is.

8. Aristotelian Paradigms

On page 24 of his influential 1962 masterpiece *The Structure of Scientific Revolutions* [44], Thomas Kuhn said that normal science "seems an attempt to force nature into the preformed and relatively inflexible box that the paradigm supplies". Continuing on the same page, he added two of the most revealing sentences of the book.

> No part of the aim of normal science is to call forth new sorts of phenomena; indeed those that will not fit the box are often not seen at all. Nor do scientists normally aim to invent new theories, and they are often intolerant of those invented by others.

The fact that he used words having pejorative connotations has not been lost on some scientists who regard Kuhn's book as unfairly derogatory and offensive [23]. He spoke of scientific revolutions as "paradigm shifts", which suggests unflattering comparison to figure-ground shifts in cognitive psychology, structure-ambiguity shifts in linguistics, and gestalt shifts in Gestalt psychology. In some cases, such as the Copernican Revolution, which is the subject of Kuhn's previous 1957 book [43], the comparison might seem somewhat justified.

If we replace Kuhn's words 'science', 'nature', and 'scientist' by 'logical theory', 'demonstrative practice', and 'logician', we would not be far off. The history of logic even to this day is replete with embarrassingly desperate attempts to force logical experience into inflexible paradigms. As late as Whately [63] was repeating as solid fact the absurdity that single propositions had no consequences, a view attributable to Aristotle.

Many of these attempts were based on partial understanding or misunderstanding of the relevant paradigm.[26] However, many were based on solid scholarship and insight. Many thinkers saw genuine inadequacies in the relevant paradigm, but failed to address them. However, many scholars mistakenly disputed well-founded aspects. Some were indeed pathetic in the wisdom of hindsight.[27] In contrast, a few were ingenious and will be remembered as

[26] One of the more ridiculous was to insist that singulars such as "Socrates is a Greek" was an ellipsis for a universal "Every Socrates is a Greek". This absurdity was designed to perpetuate the illusion that Aristotle's paradigm required that every proposition be categorical. The illusion was based on mistaking Aristotle's particular illustration of his general theory of deduction to be that general theory.

[27] Boole's attempt to force Aristotle's syllogistic into the equational logic mold could be seen to be pathetic once one sees that it led Boole into his "solutions fallacy" and blinded him to Aristotle's indirect deductions. Boole never even attempted to fit indirect deductions into his mold. Frege has a tangled explanation that indirect deductions are disguised direct deductions. Several decades later, Tarski [60, pp. 157–159] still told his readers that the expressions 'indirect proof' and 'proof by reductio ad absurdum' indicate direct proofs that use a certain logical law that he calls " the so-called LAW OF REDUCTIO AD ABSURDUM". On the same page, he pathetically alleged "Proofs of this kind may quite generally

solid contributions to logical wisdom, if not to mainstream logic. In the latter category, I put William of Ockham's brilliant attempt to account for empty terms in the framework of Aristotle's categorical logic [10].

It would be a serious mistake to think that by 'inflexible paradigms' I have in mind only those traceable to antiquity, although only ancient paradigms are relevant in this essay. For two millennia, logic was dominated by at least three paradigms apparently carrying Aristotle's *imprimatur*. Two of them are treated in this essay: the theory of categorical deductions and the truth-and-consequence theory of demonstration. A third important paradigm, Aristotle's logical methodology, including his method of establishing independence, is beyond the scope of this essay. It has been treated elsewhere [8,15,17]. Thus, nothing has been said in this essay about one of Aristotle's most lasting contributions: his method of counterarguments for establishing independence—that is, for producing knowledge that a conclusion does *not* follow from given premises.

Deductive logic has made immeasurable progress since Aristotle's theory of categorical deductions. More and more arguments have been subjected to the same kind of treatment that Aristotle gave to the categorical arguments. In some cases advances beyond the syllogistic were made by regarding syllogistic as a prototype rather than a paradigm.

In retrospect, the explosive increase in the field reported in the 1854 masterpiece [5] by Boole (b.1815–d.1864) merely served to ignite a chain reaction of further advances that continues even today. Aristotle's system did not recognize compound terms (as "triangle or square") or equations (as "$1+2=3$"). Boole's system recognizes both. Unlike other revolutionary logical innovators, Boole's greatness as a logician was recognized almost immediately. In 1865, hardly a decade after his 1854 *Laws of Thought* [8] and not even a year after his tragic death, Boole's logic was the subject of a Harvard University lecture "Boole's Calculus of Logic" by C. S. Peirce. Peirce opened his lecture [50] with the following prophetic words (Peirce [51, pp. 223–224]).

> Perhaps the most extraordinary view of logic which has ever been developed with success is that of the late Professor Boole. His book... *Laws of Thought*... is destined to mark a great epoch in logic; for it contains a conception which in point of fruitfulness will rival that of Aristotle's *Organon*.

Aristotle's special theory of categorical deductions recognized only four logical forms of propositions. It recognized only *dyadic* propositions involving exactly two [non-logical] terms. Today, infinitely many forms are accepted, with no limit to the number of terms occurring in a single proposition. In fact, as early as his famous 1885 paper "On the Algebra of Logic: A Contribution

Footnote 27 continued
be characterized as follows: we assume the theorem to be false, and derive from that certain consequences which compel us to reject the original assumption"—not a bad description of a genuine indirect deduction, but nothing like the direct deduction he had given as an example. Confusion continues in von Plato's 2017 book [62, p. 19].

to the Philosophy of Notation", Peirce [52, pp. 225–226] recognized in print simple propositions having more than two terms. Examples are the *triadic* proposition that the sign '7' denotes the number seven to the person Charles and the *tetradic* proposition that one is to two as three is to six. Peirce [53, pp. 407–408] revisited the topic in his 1907 manuscript "Pragmatism", where he presented his now well-known triadic analysis of propositions about giving such as "The person Abe gives the dog Rex to the person Ben".

Given Aristotle's interest in geometry and his historically important observations about the development of the theory of proportion (*analogia*), it is remarkable that in the *Organon* we find no discussion of tetradic propositions or proportionality arguments such as the following.

1 : 2 :: 3 : 6.	1 : 2 :: 3 : 6.	1 : 2 :: 3 : 6.
? 3 : 6 :: 1 : 2.	?1 : 3 :: 2: 6.	? 2 : 1 :: 6 : 3.

A significant amount of logical research was needed to expand the syllogistic to include the capacity to treat premise-conclusion arguments composed of conditionals whose antecedents and consequents are categorical propositions. The following is an easy example.

Direct Deduction 2.

1. If every rectangle is a quadrangle, then every quadrangle is a polygon.
2. If every square is a rectangle, then every rectangle is a quadrangle.
3. Every square is a rectangle.

? Some square is a polygon.

4. Every rectangle is a quadrangle. 2, 3
5. Every quadrangle is a polygon. 1, 3
6. Every square is a quadrangle. 3, 4
7. Every square is a polygon. 6, 5
8. Some polygon is a square. 7
9. Some square is a polygon. 8

QED

Aristotle's theory recognized only three patterns of immediate one-premise deductions and only four patterns of immediate two-premise deductions; today many more are accepted. In particular, he never discerned the fact pointed out by Peirce that to every deduction there is a proposition Peirce called a *leading principle* [52, p. 201] to the effect that its conclusion follows from its premises. It never occurred to Aristotle to include in his system such propositions as, for example, that given any two terms if one belongs to all of the other, then some of the latter belongs to some of the former.

The simple linear chain structures of Aristotle's deductions have been augmented by complex non-linear structures such as branching trees and nested[28] linear chains. Moreover, his categorical syllogistic has been subjected to severe criticism. Nevertheless, the basic idea of his demonstrative logic, the truth-and-consequence theory of demonstration, was fully accepted by Boole (Corcoran [21]). It has encountered little overt opposition in its over 2000-year history. It continues to enjoy wide acceptance in the contemporary logic community (Tarski [61]). Perhaps ironically, Peirce never expressed full acceptance and, in at least one place, he seems to say, contrary to Tarski and most modern logicians, that diagrams are essential not only in geometrical demonstrations [53, p. 303] but in all demonstration [53, p. 502].

9. Conclusion

Aristotle's *Analytics* contains two general theories, a theory of demonstration and a general theory of deduction, and one special theory of deduction, a prototype pointing beyond itself—in addition to other material including several metatheorems, several worked-out problems, scattered preliminary research notes that point to future developments, and a system of "reductions" often confused with or mixed in with deductions [12, 18].

The first two, described by him in broad terms in *Prior Analytics* and applied in *Posterior Analytics*, have had a steady, almost unchallenged, influence on the development of the deductive sciences and on theorizing about deductive sciences. His theory of demonstration was rooted in his knowledge of the demonstrative practice of mathematicians and others that he was familiar with. Moreover, his theory was intended to describe, clarify, and explain the processes and rules guiding that and future demonstrative practice. Tradition came to regard Aristotle's notion of demonstration as "*the notion of demonstration*". As Tarski [61, pp. 118–120] implies, formal proof in the modern sense results from a refinement and "formalization" of traditional Aristotelian demonstration.

The third, the special theory known as syllogistic, was succinctly described by Aristotle in meticulous detail in the first 6 chapters of *Prior Analytics*. Syllogistic has been subjected to intense, often misguided criticism. Quite properly, it has been almost totally eclipsed by modern logic.

Major commentators and historians of logic have failed to notice that a general theory of demonstration is to be found in *Analytics*. Łukasiewicz asserted that the *Analytics* did not reveal its purpose. As pointed out in my

[28] One nesting never considered by Aristotle but familiar in modern logic is the subdeduction. In the course of a deduction, an auxiliary subconclusion is set forth as a subgoal to be achieved on the way to achieving the initially chosen conclusion to be reached. Perhaps the simplest example is the case where the initial conclusion is a conjunction P&Q and the two subgoals are the two conjuncts P and Q. The syllogistic does not have conjunctions and, thus, has no need for such strategies. However, it is a kind of miracle that Aristotle's categorical deductions do not need indirect subdeductions, which are indispensable in modern systems. In the course of one *indirect* deduction, it might occur to the reasoner to set a subgoal and then to guess that one way of achieving the subgoal is indirect reasoning and so to begin an indirect subdeduction inside the initial indirect deduction (Corcoran 1974 [9, pp. 116–117]).

2015 lecture "The Aristotle Łukasiewicz omitted", he evidently skipped its first sentence (Corcoran [26]).

The failure to recognize Aristotle's truth-and-consequence theory of deduction did not begin with Łukasiewicz, of course. Sgarbi and Cosci [56] have collected essays documenting the reception of Aristotle's logic from the late middle ages though the renaissance to the beginnings of modern philosophy. This interesting collection—which will appeal to anyone interested in the history of syllogistic—documents a variety of interpretations but a unanimity of failures of several great philosophers to recognize Aristotle's demonstration theory.

Likewise, major commentators and historians of logic have failed to notice that a general theory of deduction, the immediate-deduction-chaining theory, is to be found in *Prior Analytics*. Without presenting a scintilla of evidence, Łukasiewicz [46, p. 44] said that Aristotle believed that "the categorical syllogistic is the only instrument of proof".[29] It has been widely observed that Aristotle's definition of deduction is much more general than required for the categorical syllogistic, but rarely do we find Aristotle credited with a general theory of deduction. For example, writing in the *Encyclopedia Britannica*, Lejewski [45, p. 58] noticed the wider definition. Instead of taking it as a clue to a wider theory, he criticized Aristotle's definition as being "far too general".

Finally, major commentators and historians of logic have even failed to notice that a special theory of deduction is to be found in *Prior Analytics*. In fact, Łukasiewicz did not even notice that there is *any* theory of deduction to be found anywhere in *Analytics*. He knew that every axiomatic or deductive science presupposes a theory of how deduction from its basic premises is to be conducted. Instead of recognizing that this was Aristotle's goal in *Prior Analytics*, he took Aristotle to be presenting an axiomatic science whose presupposed underlying theory of deduction was nowhere to be found in *Analytics*. More recently in 1980, writing in the prestigious *Encyclopedia Britannica* [30], Lejewski made the same mistake when he wrote [45, p. 59], "Aristotle was not aware that his syllogistic presupposes a more general logical theory, viz., the logic of propositions". According to the view presented here, Aristotle's categorical syllogistic includes a fully self-contained and gapless system of rules of deduction: it presupposes no other logic for its cogency.

Besides the general truth-and-consequence theory of demonstration, the general immediate-deduction-chaining theory of deduction, and the detailed special deduction theory called categorical syllogistic, Analytics contains much more, including observations that can be taken to be preliminary groundwork for another special theory that would supplement the categorical syllogistic. Cf. Corcoran and Tracy [28].

One of the extra-syllogistic observations that has received the lion's share of attention is the process or group of processes known as ecthesis or exposition.

[29] This is especially ironic because elsewhere in the same book Łukasiewicz said the *Prior Analytics* contains no instrument of proof at all. His view is that *Prior Analytics* presents an axiomatic theory whose underlying logic is presupposed but not presented.

In my opinion, the best place to begin study of ecthesis is Kevin Flannery's 1995 book [33]. There is more recent work on the subject [28,41]. In my opinion, further study of ecthesis is not likely to be rewarding and study of it by beginners will be found to be an exasperating distraction from the valuable lessons that can be learned from study of *Analytics*.

In several previous articles listed in the References, I give my textual basis, analysis, interpretation, and argumentation in support of the statements made above. My interpretation of Aristotle's general theory of demonstration agrees substantially with those of other logically oriented scholars such as Beth [3, pp. 31–51]. Moreover, there are excellent articles by John Austin, Michael Scanlan, Timothy Smiley, Robin Smith, and others criticizing the opponents of my approach to Aristotle's theory of categorical deduction and treating points that I have omitted [45,46].[30] This paper was intended not to contribute to the combined argument, which, though not perfect, still seems conclusive to me. Rather, my goal was to give an overview from the standpoint of demonstration.[31] This limited perspective brings out the genius and the lasting importance of Aristotle's masterpiece in a way that can instruct scholars new to this and related fields.

From a philosophical perspective the most important achievements of Aristotle's *Analytics* were his two general theories, the theory of demonstration and the general theory of deduction. In contrast, his special theory of deduction, the prototype syllogistic, is an aside that continues to obscure his lasting legacy. The arguments, spoken and unspoken, by modern logicians—that it is a distraction and that it should not be taught in college—carry weight. I have never taught syllogistic in any of the scores of logic courses I taught in my near half-century of teaching.

But from a historical perspective his two general theories recede into the shadows while the prototypical syllogistic stands out as a landmark. The richness of the prototypical syllogistic continues to unfold. The syllogistic was fully

[30] Timothy Smiley's 1973 work on the categorical syllogistic agrees in all essentials with mine. He independently discovered his main points about the same time that I discovered mine. It is an insignificant accident that my earliest publication on this subject predates his.

[31] In this paper I presented what I take to be the most basic and simplest of the theories of demonstration responsibly attributable to Aristotle. There are several passages, usually disputed and obscure, in which Aristotle seems to further elaborate his views of the nature of the ultimate premises of a demonstration, our knowledge of them, and what a deduction of a consequence from them shows. Here are some representative examples. He says that the ultimate premises must be "necessary" and known to be such, that it is impossible to demonstrate any of them using others as premises, and that the deductions must show that the facts referred to in the respective premises are the "causes" of those referred to in the respective conclusions. But how would one ever know that even one of these conditions is satisfied? For example, he never gives us a clue how to determine that one proposition cannot be deduced from any other true propositions.

These passages tend to deflect attention from the deep, clear, useful, and beautiful aspects of the *Analytics*. None of these ideas have yet played any role in modern understanding of demonstration. To have raised such murky and contentious issues would have made it difficult if not impossible to leave the reader with an appreciation of the clear and lasting contribution Aristotle made to our understanding of demonstration.

accepted by Boole who transformed it into Boolean logic, which encompassed incipient forms of class logic and propositional logic—two of the three branches of "symbolic logic"—the third being relation logic—before beginning unified into higher-order logic by Frege in 1879 [19].

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