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Leibniz's Ontological Proof of the Existence of God and the Problem of »Impossible Objects«

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Abstract. The core idea of the ontological proof is to show that the concept of existence is somehow contained in the concept of God, and that therefore God's existence can be logically derived—without any further assumptions about the external world—from the very idea, or definition, of God. Now, G.W. Leibniz has argued repeatedly that the traditional versions of the ontological proof are not fully conclusive, because they rest on the tacit assumption that the concept of God is *possible*, i.e. free from contradiction. A *complete* proof will rather have to consist of two parts. First, a proof of premise

(1) God is possible.

Second, a demonstration of the "remarkable proposition"

(2) If God is possible, then God exists.

The present contribution investigates an interesting paper in which Leibniz tries to prove proposition (2). It will be argued that the underlying idea of God as a necessary being has to be interpreted with the help of a distinguished predicate letter 'E' (denoting the concept of existence) as follows:

(3) $g =_{\mathrm{df}} \iota x \Box E(x)$.

Proposition (2) which Leibniz considered as "the best fruit of the entire logic" can then be formalized as follows:

(4) $\Diamond E(\iota x \Box E(x)) \rightarrow E(\iota x \Box E(x))$.

At first sight, Leibniz's proof appears to be formally correct; but a closer examination reveals an *ambiguity* in his use of the modal notions. According to (4), the *possibility* of the necessary being has to be understood in the sense of something which possibly *exists*. However, in other places of his proof, Leibniz interprets the assumption that the necessary being is *impossible* in the diverging sense of something which *involves a contradiction*. Furthermore, Leibniz believes that an »impossible thing«, y, is such that contradictory propositions like $\mathrm{F}(y)$ and $\neg F(y)$ might both be true of y. It will be argued that the latter assumption is incompatible with

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Leibniz's general views about logic and that the crucial proof is better reinterpreted as dealing with the necessity, possibility, and impossibility of *concepts* rather than of *objects*. In this case, the counterpart of (2) turns out to be a *theorem* of Leibniz's second order logic of concepts; but in order to obtain a full demonstration of the existence of God, the counterpart of (1), i.e. the self-consistency of the concept of a necessary being, remains to be proven.

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1. Leibniz's General Critique of the Ontological Proof

In several papers dating from 1676 onwards, Leibniz explained why he considered the traditional proof of the existence of God (as invented by St. Anselm and modified by Descartes and Spinoza) as *insufficient*. Thus in the "Meditations on knowledge, truth, and ideas" of 1684 (which contains an extensive discussion of the basic principles of Descartes' theory of knowledge), Leibniz analyzes the "old argument for the existence of God" as follows:

> The argument goes like this: Whatever follows from the idea or definition of a thing can be predicated of the thing. God is by definition the most perfect being, or the being nothing greater than which can be thought. Now, the idea of the most perfect being includes ideas of all perfections, and amongst these perfections is *existence*. So existence follows from the idea of God. Therefore [...] God exists.
>
> But this argument shows only that *if God is possible* then it follows that he exists. For we can't safely draw conclusions from definitions unless we know first that they are *real* definitions, that is, that they don't include any contradictions. If a definition does harbour a contradiction, we can infer contradictory conclusions from it, which is absurd.[1]

Hence, according to Leibniz, the traditional proof establishes the truth of the *conditional* statement 'If God is possible, then God exists'. But since the possibility, i.e. the self-consistency, of an arbitrary concept C may not generally be taken for granted, a *complete* demonstration requires in addition a proof of the *antecedent* 'God is possible'.[2]

In this connection two different conceptions of God have to be distinguished: (A) God as the *most perfect being* ("ens perfectissimum"), and (B)

[1] Cf. GP 4, p. 424; the English translation has been adopted from Bennett [7]. Cf. also GP 4, p. 359: "As I have argued elsewhere it must generally be observed that nothing certain can be inferred from a definition about the defined entity as long as it has not been secured that the definition expresses something possible. For if it implies a hidden contradiction, it may happen that something absurd will be derived from it."

[2] Cf. also the summary in Nolan [13]: "[...] Descartes' version of the ontological argument is incomplete. It shows merely that if God's existence is possible or non-contradictory, then God exists. But it fails to demonstrate the antecedent of this conditional."

God as the *necessary being* ("ens necessarium"). Accordingly, a complete proof of the existence of God will either consist of the two propositions

(1_A) The most perfect being is possible
(2_A) If the most perfect being is possible, then it exists

or of the two propositions

(1_B) The necessary being is possible
(2_B) If the necessary being is possible, then it exists.

Leibniz used to illustrate the necessity of the requirement of self-consistency of a concept by means of the example of »the fastest motion« which, allegedly, "entails an absurdity":

> Suppose there is a wheel turning with the fastest motion. Anyone can see that if a spoke of the wheel came to poke out beyond the rim, the end of it would then be moving faster than a nail on the rim of the wheel. So the nail's motion is not the fastest, which is contrary to the hypothesis.[3]

Unfortunately, this famous example is quite inapt to illustrate the point in question, because—according to modern physics—the concept of the fastest motion doesn't contain a contradiction at all; it rather forms a cornerstone of Einstein's theory of relativity. However, in other papers Leibniz put forward more convincing examples of (implicitly) contradictory concepts such as »the greatest number« or »the greatest figure«.[4] Moreover, Leibniz pointed out that without the requirement of the self-consistency of the definition, the basic idea of the ontological proof might be misused to show not only the existence of a most perfect and necessary *God*, but similarly also the existence of a »most perfect *man*« or the existence of a »necessary *beast*«:

> For example, let an entity A be defined as the absolutely necessary beast. Then one can argue that A has to exist as follows: Whatever is absolutely necessary will exist (by an indubitable axiom); now A is absolutely necessary (by definition), therefore A exists. But this is absurd, and one has to object that this definition or idea is impossible [...].[5]

The variant of the ontological proof which makes use of the conception of the most perfect being has been investigated above all in the paper "Quod Ens Perfectissimum existit" which Leibniz composed in 1676 after having visited Spinoza in The Hague.[6] Leibniz's ideas about the »most perfect being« turned out to be very influential for the philosophical discussions of the 18th century,

[3] Cf. GP 4, p. 424; translation by Bennett [7]. Cf. also GP 4, p. 359.
[4] Cf., e.g., GP 4, p. 427: "le nombre le plus grand de tous [...] aussi bien que la plus grande de toutes les figures, impliquent contradiction".
[5] Cf. GP 4, 359, fn. **. The corresponding argument concerning the existence of the »necessary man« ("homo necessarius") is to be found in A II, 1, p. 587.
[6] Cf. A VI, 3, pp. 578–579. An English translation may be found in LOE, pp. 167–168. Cf. also chapter 4 of Adams [6] which is entirely devoted to "The Ens Perfectissimum".

and, indeed, even for the revival of the ontological proof in the 20th century, notably by Kurt Gödel[7].

The present paper, however, concentrates on the second version of the proof which relies of the conception of a »necessary being«. Leibniz dealt with this topic mainly in the paper "Probatio existentiae DEI ex eius essentia" where he provided not only a short argument in favour of

(1_B) The necessary being is possible,

but also an interesting, detailed proof of the conditional proposition

(2_B) If the necessary being is possible, then it exists.

This proposition, which was praised by Leibniz as "one of the best fruits of the entire logic"[8], will be examined in the subsequent Sect. 2.

2. "Probatio Existentiae DEI ex Eius Essentia"

During his correspondence with Henning Huthmann, probably in January 1678, Leibniz devised three different versions of a "*Derivation of the Existence of God from his Essence*". One version was published already in 1926, while the other two variants appeared only in 2006.[9] The most interesting variant runs as follows:

> Si Ens necessarium est possibile, actu existet.
> Nam ponamus non existere, inde ratiocinabor hoc modo:
> (i) [10] Ens Necessarium non existit, ex hypothesi.
> (ii) Quicquid non existit, illud possibile est non existere
> (iii) Quicquid possibile est non-existere illud falso dicitur non posse non-existere
> (iv) Quicquid falso dicitur non posse non existere, illud falso dicitur esse necessarium. Nam necessarium est quod non potest non existere.
> (v) Ergo Ens necessarium falso dicitur esse necessarium.
> (vi) Quae conclusio est vel vera vel falsa.
> (vii) Si est vera, sequitur quod Ens necessarium implicet contradictionem, seu sit impossibile, quia de eo demonstrantur contradictoria, scilicet quod non sit necessarium. Conclusio enim contradictoria non nisi de re contradictionem implicante ostendi potest.

[7] Cf., e.g., Benzmüller [8] and the critical analysis of Gödel's proof in Kutschera [9].
[8] Cf. GP 4, p. 406 where Leibniz speaks of "un des meilleurs fruits de toute la Logique". Similarly the same proposition is characterized in GP 4, p. 402 as "la proposition la plus belle [...] et la plus importante de la doctrine des modales" and in A II, 1, p. 587 as "fastigium doctrinae Modalium", i.e. the pediment of the doctrine of the modal notions.
[9] See the first edition of Volume 1 of series II of the Academy-edition, pp. 390–393, and the revised 2nd edition as published in 2006 by the Leibniz Forschungsstelle Münster, pp. 585–591. The latter edition is available for download at https://www.uni-muenster.de-/Leibniz-/db_new_032009.html.
[10] The numbers are not in the manuscript but have been inserted for ease of discussion.

(viii) Si est falsa, necesse est aliquam ex praemissis esse falsam. Sola autem ex praemissis falsa esse potest hypothesis, quod scilicet Ens necessarium non existat.
(ix) Ergo concludimus Ens necessarium vel esse impossibile, vel existere.
(x) Si ergo DEUM definiamus Ens a se, seu Ens ex cuius essentia sequitur existentia, seu Ens necessarium, sequitur DEUM si possibilis sit actu esse.

I translate this as follows:

If the necessary being is possible, then it actually exists.
For if we assume that it doesn‘t exist, one may reason as follows:
(i) By hypothesis, the necessary being doesn‘t exist.
(ii) Whenever something doesn't exist, it possibly doesn't exist.
(iii) Whenever something possibly doesn't exist, it is falsely maintained to be impossible not to exist.
(iv) Whenever something is falsely maintained to be impossible not to exist, then it is falsely maintained to be necessary. (For necessary is what is impossible not to exist.)
(v) Therefore the necessary being is falsely maintained to be necessary.
(vi) This conclusion is either true or false.
(vii) If it is true, it follows that the necessary being contains a contradiction, or is impossible, because contradictory assertions have been proved about it, namely that it is not necessary. For a contradictory conclusion can only be shown about a thing which implies a contradiction.
(viii) If it is false, necessarily one of the premises must be false. But the only premise that might be false is the hypothesis saying that the necessary being doesn't exist.
(ix) Hence we conclude that the necessary being either is impossible, or it exists.
(x) So if we define GOD as an "Ens a se", i.e. a being from whose essence its existence follows, i.e. a necessary being, it follows that GOD, if he is possible, actually exists.

3. Formalization and Logical Analysis of the "Probatio"

In order to analyze the validity of Leibniz's proof, it must first be clarified how the idea of the *possibility* of God and the idea of a *necessary being* may be represented within the framework of modern logic. The modal operators of necessity, $\Box$, and possibility, $\Diamond$, are usually applicable only to *propositions*, but not to *objects*. Now, as Leibniz explained at the beginning of the *Probatio*, the possibility of an *object*, x, shall be understood as the possibility of x's *existence* which in turn may also be equated with x's *essence*.[11] Since, moreover, Leibniz considers 'existence' as a normal property, it may be represented by a

[11] Cf. A II, 1, p. 588: "Existentia possibilis seu Possibilitas rei alicujus, et ejusdem rei essentia sunt inseparabiles."

distinguished predicate letter, say 'E'.[12] If the name 'God' is abbreviated by an individual constant, say 'g', then the proposition 'God exists' takes the form '$E(g)$', while the proposition 'God is possible', i.e. 'God possibly exists', can be formalized by '$\Diamond E(g)$'. In sum, then, Leibniz's "remarkable proposition" 'If God is possible, then he exists' may be transformed into formula

(2) $\Diamond E(g) \;\rightarrow\; E(g)$.

Now just like a possible object is interpreted as an object which *possibly* exists, the traditional idea of God as a »necessary being«has to be interpreted as a being which *necessarily exists.* Thus in a marginal note to the *Probatio*, Leibniz paraphrased the "Ens necessarium" as an "Ens necessario existens". Therefore one may define

(3) $g =_{\mathrm{df}} \iota x \Box E(x)$.

The crucial variant of (2), which Leibniz considered as the "best fruit of the entire logic", can hence be formalized as follows:

(4) $\Diamond E(\iota x \Box E(x)) \;\rightarrow\; E(\iota x \Box E(x))$.

This formula is in full accordance with Leibniz's paraphrase ,,*Si Ens necessario existens est possibile, utique actu existet*" (A II, 1, 588, fn. 1). Anyway, we are now in a position to formalize Leibniz's proof as follows:

(i) $\neg E(\iota x \Box E(x))$
"By hypothesis, the necessary being doesn't exist."

(ii) $\Lambda x(\neg E(x) \;\rightarrow\; \Diamond\neg E(x))$
"Whenever something doesn't exist, it possibly doesn't exist."

(iii) $\Lambda x(\Diamond\neg E(x) \;\rightarrow\; \neg(\neg\Diamond\neg E(x)))$
"Whenever something possibly doesn't exist, it is falsely maintained to be impossible not to exist."

(iv) $\Lambda x(\neg(\neg\Diamond\neg E(x)) \;\rightarrow\; \neg\Box E(x))$
"Whenever something is falsely maintained to be impossible not to exist, then it is falsely maintained to be necessary."

(v) $\neg\Box E(\iota x \Box E(x))$
"Therefore the necessary being is falsely maintained to be necessary."

This first part of the *Probatio* is logically impeccable. It starts with the assumption (i), from which (ii) may be derived according to the well-known principle that, what is a fact, or is *true*, also must be *possible* ("ab esse ad posse valet consequentia"). (iii) contains an application of the principle of double negation ("duplex negatio affirmat"), while (iv) is based on the equivalence between 'necessarily p' and 'not possibly not-p'. Hence

[12] This crucial feature of Leibniz's logic requires that the universe of discourse, U, is conceived of as the set of all *possible* objects while the extension of the predicate E (the set of all actually existing objects) is a subset of U. See Sect. 6.1 below.

(α) (i) logically entails (v).

However, (α) doesn't yet represent the desired proof of (4), for (α) only says (by logical contraposition) that the negation of (v), i.e. $\Box E(\iota x \Box E(x))$, entails the negation of (i), i.e. $E(\iota x \Box E(x))$. This inference constitutes an instance of the well-known schema $\Box\, p \;\rightarrow\; p$, i.e. the traditional principle "ab necesse ad esse valet consequentia". What has, however, to be shown is that the same conclusion, $E(\iota x \Box E(x))$, already follows from the much weaker premise $\Diamond E(\iota x \Box E(x))$, or (again by logical contraposition):

(β) (i) logically entails $\neg\Diamond E(\iota x \Box E(x))$.

Leibniz attempted to justify the stronger inference (β) as follows. According to the well-known principle "tertium non datur", one has:

(vi) $\neg\Box E(\iota x \Box E(x)) \;\vee\; \neg\neg\Box E(\iota x \Box E(x))$
"This conclusion [(v)] is either true or false."

Next it is argued:

(vii) $\neg\Box E(\iota x \Box E(x)) \;\rightarrow\;$ ($\iota x \Box E(x)$ is impossible)[13]
"If it [(v)] is true, it follows that the necessary being contains a contradiction, or is impossible".

(viii) $\neg\neg\Box E(\iota x \Box E(x)) \;\rightarrow\; E(\iota x \Box E(x))$
"If it [(v)] is false, necessarily one of the premises must be false. But the only premise that might be false is hypothesis [i] saying that the necessary being doesn't exist."

In view of (vi), the two results (vii) and (viii) taken together yield:

(ix) ($\iota x \Box E(x)$ is impossible) $\vee$ $E(\iota x \Box E(x))$
"Hence we conclude that the necessary being either is impossible, or it exists."

Finally, Leibniz rounds off his proof by paraphrasing (ix) as follows:

(x) ($\iota x \Box E(x)$ is possible) $\;\rightarrow\; E(\iota x \Box E(x))$
"So if we define GOD as [...] a necessary being, it follows that GOD, if he is possible, actually exists."

At first sight, also the second part of the *Probatio* appears to be logically correct, but upon closer inspection two problems become visible. First, step (vii) of the proof has not yet been sufficiently justified. It remains to be shown in which sense the assumption that the necessary being doesn't necessarily exist, $\neg\Box E(\iota x \Box E(x))$, entails a *contradiction*. This point will be scrutinized in Sect. 5.

Second, within the *Probatio* Leibniz uses the notion of *possibility* in an ambiguous way. According to the explanation given at the beginning of this section, the possibility of an entity x has to be understood in the sense of something which *possibly exists*, $\Diamond E(x)$. However, in connection with steps

[13] We here use the informal expression 'impossible' instead of the formal operator $\Diamond$ because the latter may only be applied to propositions, but not to singular terms!

(vii)–(x), Leibniz interprets the assumption that the necessary being is *impossible* in the diverging sense of an entity which *involves a contradiction*; and it is far from evident whether these two notions coincide with one another. The following section is devoted to the question whether the latter notion of an »impossible object« does make sense at all.

4. The Problem of »Impossible Objects«

In step (vii) of the above proof, Leibniz maintained that "a contradictory conclusion can only be shown about a thing which implies a contradiction". Towards the end of the *Probatio*, he explains more exactly that an *entity*, y, is impossible if and only if "contradictory propositions" are true about y, and he argues that some such »impossible objects« do really exist:

> It is worthwhile noting here that a conclusion which entails a contradiction can nevertheless be true, namely if it is about an impossible thing. E.g., 'A square circle is not a circle'. This proposition, although contradictory, is true, for it can be correctly derived from true premises as follows:
>
> A square is not a circle
>
> A square circle is a square
>
> Hence
>
> A square circle is not a circle.[14]

Let's take a closer look at this inference which admits of at least two different interpretations! It appears quite natural to understand both the premises and the conclusion as *universal* propositions. In this case the syllogism (or better: *quasi*-syllogism)[15] amounts to the following inference:

P_1 Whatever is a square is not a circle
P_2 Whatever is a square circle is a square
K Whatever is a square circle is not a circle

Letting 'S' and 'C' abbreviate the predicates 'is a square' and 'is a circle', respectively, the inference can be formalized as follows:

$$\begin{array}{ll} P_1 & \forall x(S(x) \rightarrow \neg C(x)) \\ \underline{P_2} & \underline{\forall x(S(x) \wedge C(x) \rightarrow S(x))} \\ K_1 & \forall x(S(x) \wedge C(x) \rightarrow \neg C(x)). \end{array}$$

This inference is logically valid and all its premises are true, for P_1 is an *analytic* truth while P_2 is a *tautology*. Furthermore, the conclusion K_1 is somehow contradictory where its contradictoriness becomes more explicit if

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[14] Cf. A II, 1, 586: "Notandum hic est, quod conclusio implicans contradictionem potest esse vera, si scilicet sit de re impossibili. V.g. circulus quadratus non est circulus. Quae propositio vera est, etsi contradictoria sit, nam ex veris legitime probatur hoc modo. Quadratus non est circulus, Circulus quadratus est quadratus, Ergo circulus quadratus non est circulus."

[15] The traditional theory of the syllogism usually takes only single concepts like 'square' and 'circle' into account but not a conjunctive concept like 'square circle'; otherwise the above inference has the logical structure of type "Celarent" where the universal negative premise P_1 'No square is a circle' is transformed into a universal affirmative proposition with a negated copula.

one considers the tautology $\forall x(S(x) \wedge C(x) \rightarrow C(x))$ which, in addition to K_1, yields the strengthened formula:

$$K_2 = \forall x(S(x) \wedge C(x) \rightarrow C(x) \wedge \neg C(x)).$$

So Leibniz's claim, that one may derive a contradictory conclusion from true premises, turns out to be, at least somehow, correct. However, proposition K_2 is not *strictly* contradictory, because it only maintains that *if there were* a square circle, y, then y would possess the contradictory properties of both being a circle and not being a circle. But since K_2 is only a *conditional*, it doesn't entail the existence an »impossible *object*«y such that $C(y) \wedge \neg C(y)$.

In order to obtain a genuine inconsistency, one might resort to an alternative interpretation of the syllogism where the second premise "circulus quadratus est quadratus" is now taken in the sense of the *particular* proposition 'Some square circles are squares'. In terms of first order logic, this inference (of type "Ferio") would have to be formalized as follows:

$$\begin{array}{ll} P_1 & \forall x(S(x) \rightarrow \neg C(x)) \\ \underline{P_3} & \underline{\exists x(S(x) \wedge C(x) \wedge S(x))} \\ K_3 & \exists x(S(x) \wedge C(x) \wedge \neg C(x)). \end{array}$$

In this case, K_3 represents an outright inconsistency, but this conclusion has no longer been inferred, as Leibniz maintained, entirely from "true propositions". Unlike P_2, premise P_3 is no longer a tautology. As a matter of fact, it is not true at all, for it entails the existence of an object, x, which would be both a square and a circle: $\exists x(S(x) \wedge C(x))$. But this existential proposition directly *negates* the content of the other premise P_1, namely, that whatever is a square can't (also) be a circle! Hence the above syllogism turns out to contain two logically *incompatible* premises, and it is small wonder that from this inconsistent pair of propositions another inconsistency, namely K_3, can be logically derived. So Leibniz' argument fails to show that there exists an »impossible object«y such that contradictory propositions would be true about y.

Let it be mentioned in passing that the assumption of »impossible objects« would also be in conflict with the basic principles of classical, two-valued logic which Leibniz adheres to in all his later writings. Thus, e.g., in a paper of around 1686, he defended the principle of non-contradiction as follows:

> Above all I assume all propositions (both the affirmative and the negative ones) to be *either* true *or* false. If an affirmation is true, then the negation is false, and if the negation is true, then the affirmation is false [...] All this is usually understood by the *Principle of Contradiction* (Cf. GP 7, 299).

Hence the assumption of a proposition which would be *both* true ('Square y is round') *and* false ('Square y is not round') contradicts this highest principle of reason. Furthermore, for Leibniz the principle of non-contradiction is an indispensable basis for his construction of logical proofs. Thus in the fragment "De Principiis" he concisely emphasized:

Identica sunt vera, et contradictionem implicantia sunt falsa, i.e. identical propositions are logically true, while propositions which entail a contradiction are logically false (Cf. C, 183).

5. Reconsideration of the "Probatio" (without Recourse to »Impossible Objects«)

As has already been stressed in Sect. 3, assumption (i), i.e. $\neg E(\iota x \Box E(x))$, logically entails (v), i.e. $\neg \Box E(\iota x \Box E(x))$, which Leibniz considers as contradictory. The (alleged) inconsistency of this proposition is then used by Leibniz to conclude that its *subject*, i.e. the necessary being, must be an »impossible object«. But in view of the discussion of the foregoing section, such a detour (from an impossible *proposition* to an »impossible *object*«) is extremely problematic and should better be avoided. Why shouldn't we just stop at the inconsistency of proposition (v) and infer—by "reductio ad absurdum"—that the hypothesis (i), from which (v) was derived, therefore must be false? Briefly speaking, one might be tempted to simplify the *Probatio* as follows:

(α) (i) logically entails (v)
(γ) (v) is contradictory because it attributes to the necessary being the property of not being necessary.
Hence (δ) Assumption (i), saying that the necessary being does *not* exist, must be false, i.e. the necessary being exists: $E(\iota x \Box E(x)$.

Alas, this argument—if valid—would not only constitute a proof of Leibniz's *conditional* thesis

(2_B) If the necessary being is possible, then it exists;

but even a proof of the *categorical* proposition that the necessary being exists. So what about Leibniz's notorious claim that the usual versions of the ontological proof are incomplete and have to be supplemented by a proof of premise (1_B) 'The necessary being is possible'?

Well, the point is that our shortened »proof« contains the same grave mistake that is also hidden in step (vii) of the *Probatio*, namely the claim that (v) is a *contradictory* proposition. As a matter of fact, (v) »only« maintains that *the necessarily existing being does not necessarily exist*, formally $\neg \Box E(\iota x \Box E(x))$. Although this proposition is rather *strange*, it is not strictly *self-contradictory*. In order to obtain a genuine contradiction, one would need an additional premise which Leibniz evidently took for granted, namely the assumption that *the necessarily existing being* (so-to-speak »by definition«) *exists necessarily*:

(xi) $\Box E(\iota x \Box E(x))$.

Now there are basically two strategies to cope with this situation. First, one might try to find an additional *proof* for this formula; or, second, one might treat (xi) as an additional *premise*. In the latter case, however, Leibniz's entire proof would become severely *circular*. After all, the aim of the *Probatio* was to show that if the necessary being is possible, then it exists. If one now introduces (xi) as an additional premise, this would mean that one *presupposes* that the necessary being *exists necessarily*. From this assumption, of course, one may trivially infer[16] that the necessary being *exists simpliciter*, $E(\iota x \Box E(x))$; and a fortiori one may derive Leibniz's conditional thesis (2_{B}), $\Diamond E(\iota x \Box E(x)) \rightarrow E(\iota x \Box E(x))$! Such an argument really doesn't deserve the name of a proof.

So let us see whether one may perhaps *prove* formula (xi)! After all, in modern calculi of first order logic with identity and definite descriptions, one usually has an axiom like

(DD) $\Phi(\iota x \Phi(x))$,

which guarantees that (under a certain presupposition not spelled out in DD) *the* Φ, i.e. the one and only thing with property Φ, trivially *does have* property Φ. So if we substitute for Φ the condition of necessary existence, $\Box E(x)$, we obtain the desired result (xi), $\Box E(\iota x \Box E(x))$. However, the crucial presupposition for the validity of DD says that the respective definite description term $\iota x \Phi(x)$ satisfies the *normal condition* which means that *there exists exactly one object* x with property Φ! In the case of $\Box E(x)$, this normal condition amounts to the existence and uniqueness of an x such that x necessarily exists. Notwithstanding the question how the *uniqueness* of a necessary being, i.e. $\forall \mathrm{x} \forall \mathrm{y}(\Box E(x) \wedge \Box E(y) \rightarrow x = y))$, might ever be proved, it seems clear that the requirement of the *existence* of a necessary being,

(xii) $\exists x(\Box E(x))$,

again renders Leibniz's proof *circular*.

However, *this* charge of circularity may perhaps turn out to be premature because there is a subtle *ambiguity* between the notion of *existence* as used in the consequent of (x), on the one hand, and in the additional premise (xii), needed to validate the application of DD, on the other hand. In the former case, God's existence, i.e. the existence of the "Ens necessarium", is formalized by means of the predicate of existence as $E((\iota x \Box E(x))$, while in the latter case an existential quantifier is used to express the existence of (at least one) entity x which falls under the predicate of necessary existence, $\exists x(\Box E(x))$. In order to investigate this point a bit further, we now introduce an entirely different interpretation of the *Probatio* as based on Leibniz's own logic.

[16] This inference follows the traditional principle "ab necesse ad esse valet consequentia": $\Box \alpha \rightarrow \alpha$.

6. Interpretation of the "Probatio" within the Framework of Leibniz's Logic

The text of the *Probatio* dates from 1676, while Leibniz's ripe logic of concepts was developed only between 1679 and 1690.[17] Let us briefly sketch the essentials of the algebra of concepts ($L1$) plus its quantifier extension ($L2$) before reconsidering Leibniz's main ideas about the ontological proof.

6.1. The Algebra of Concepts L1

The algebra of concepts grows out of the framework of 17th century syllogistic by three achievements. First, Leibniz drops the quantifier expression 'every' and formulates the universal affirmative proposition 'Every A is B' simply as 'A is B' or, equivalently, as '*A contains B'*. This fundamental proposition shall here be symbolized as $\mathrm{A} \in \mathrm{B}$ while its negation will be abbreviated as $\mathrm{A} \notin \mathrm{B}$. Second, Leibniz introduces an operator of conceptual *conjunction* which combines two concepts A and B into AB (sometimes also written as "A+B"). Third, Leibniz allows the unrestricted use of conceptual *negation* ("Not-A") which shall here be symbolized as $\sim \mathrm{A}$. Hence, in particular, one can form the inconsistent concept $\mathrm{A} \sim \mathrm{A}$ and its tautological counterpart $\sim (\mathrm{A} \sim \mathrm{A})$.

Identity or coincidence of concepts may be defined as mutual containment:

Def 1 $(\mathrm{A} = \mathrm{B}) =_{\mathrm{df}} (\mathrm{A} \in \mathrm{B}) \wedge (\mathrm{B} \in \mathrm{A})$.

Alternatively, the algebra of concepts might be built up with '=' as a primitive operator while '∈' is defined by

Def 2 $(\mathrm{A} \in \mathrm{B}) =_{\mathrm{df}} (\mathrm{A} = \mathrm{AB})$.

Another important operator may be introduced by definition. Concept B is *possible* if B does not contain a contradiction like $\mathrm{A} \sim \mathrm{A}$:

Def 3 $\mathrm{P(B)} =_{\mathrm{df}} (\mathrm{B} \notin \mathrm{A} \sim \mathrm{A})$.

Leibniz uses many different locutions to express the self-consistency of a concept. Instead of 'A is possible' ("A est possibile") he often says 'A is a thing' ("A est res"), or 'A is a being' ("A est ens"). In the opposite case of an *impossible* concept he sometimes calls A a »false term« ("terminus falsus").

Identity can be axiomatized by the law of reflexivity in conjunction with the rule of substitutivity:

Iden 1 $\mathrm{A} = \mathrm{A}$

Iden 2 If $\mathrm{A} = \mathrm{B}$, then $\alpha[\mathrm{A}] \leftrightarrow \alpha[\mathrm{B}]$.

The containment relation is characterized by the following laws of reflexivity and transitivity:

Cont 1 $\mathrm{A} \in \mathrm{A}$

Cont 2 $\mathrm{A} \in \mathrm{B} \wedge \mathrm{B} \in \mathrm{C} \rightarrow \mathrm{A} \in \mathrm{C}$.

[17] One of the first drafts of a »Universal Calculus« is the "Specimen Calculi universalis" of 1679; cf. GP 7, 218–227. The biggest advance towards a full algebra of concepts was achieved in the "General Inquiries" of 1686; cf. A VI, 4, 739–787. Some important subtleties of the theory of "indefinite concepts" have been developed in the fragments of August 1690; cf. C. 232–237 + 421–423.

The most fundamental principle for the operator of conceptual conjunction says: "That A contains B and A contains C is the same as that A contains BC" (cf. LLP, 58, fn. 4), i.e.:

Conj 1 $\mathrm{A} \in \mathrm{BC} \leftrightarrow \mathrm{A} \in \mathrm{B} \wedge \mathrm{A} \in \mathrm{C}$.

Conjunction then satisfies the following laws:

Conj 2 $\mathrm{AA} = \mathrm{A}$
Conj 3 $\mathrm{AB} = \mathrm{BA}$
Conj 4 $\mathrm{AB} \in \mathrm{A}$
Conj 5 $\mathrm{AB} \in \mathrm{B}$.

The next operator is conceptual negation. Leibniz had serious problems with finding the proper laws governing this operator. From the tradition, he knew little more than the "law of double negation":

Neg 1 $\sim\sim \mathrm{A} = \mathrm{A}$.

One important step towards a complete theory of conceptual negation was to transform the informal principle of contraposition, 'Every A is B, therefore Every Not-B is Not-A' into the following axiom of *L*1:

Neg 2 $\mathrm{A} \in \mathrm{B} \leftrightarrow \sim \mathrm{B} \in \sim \mathrm{A}$.

Furthermore Leibniz discovered various variants of a "law of consistency":

Neg 3 $\mathrm{A} \neq \sim \mathrm{A}$
Neg 4 $\mathrm{A} = \mathrm{B} \rightarrow \mathrm{A} \neq \sim \mathrm{B}$.
Neg 5* $\mathrm{A} \notin \sim \mathrm{A}$
Neg 6* $\mathrm{A} \in \mathrm{B} \rightarrow \mathrm{A} \notin \sim \mathrm{B}$.[18]

Principles Neg 5* and Neg 6* have been marked with a '*' in order to indicate that the laws as stated by Leibniz are not absolutely valid but have to be restricted to self-consistent terms:

Neg 5 $\mathrm{P(A)} \rightarrow \mathrm{A} \notin \sim \mathrm{A}$
Neg 6 $\mathrm{P(A)} \rightarrow (\mathrm{A} \in \mathrm{B} \rightarrow \mathrm{A} \notin \sim \mathrm{B})$.

The following two laws describe some characteristic relations between the possibility-operator P and the other operators of *L*1:

Poss 1 $\mathrm{A} \in \mathrm{B} \wedge \mathrm{P(A)} \rightarrow \mathrm{P(B)}$
Poss 2 $\mathrm{A} \in \mathrm{B} \leftrightarrow \neg \mathrm{P}(A \sim \mathrm{B})$.

All these principles have been discovered by Leibniz himself who thus provided an almost complete axiomatization of *L*1. As a matter of fact, the »intensional«algebra of concept can be proven to be equivalent to Boole's extensional algebra of sets provided that one adds the following counterpart of the "ex contradictorio quodlibet":

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[18] In the "General Inquiries" these principles are formulated as follows: "A proposition false in itself is 'A coincides with Not-A"' (§ 11); "If A = B, then A ≠ Not-B" (§ 171); "It is false that B contains Not-B, i.e. B doesn't contain Not-B" (§ 43); and "A is B, therefore A isn't Not-B" (§ 91).

NEG 7 $(A \sim A) \in B$.

As regards the fundamental relation $A \in B$, it is important to observe that Leibniz's standard formulation 'A contains B' expresses the so-called »*intensional*« view, while we here want to develop an *extensional* interpretation in terms of the sets of individuals that fall under the concepts. Leibniz explained the mutual relationship between these two points of view in the following passage from the "New Essays on Human Understanding"[19]:

> The common manner of statement concerns individuals, whereas Aristotle's refers rather to ideas or universals. For when I say 'Every man is an animal' I mean that all the men are included among all the animals; but at the same time I mean that the idea of animal is included in the idea of man. 'Animal' comprises more individuals than 'man' does, but 'man' comprises more ideas or more attributes: one has more instances, the other more degrees of reality; one has the greater extension, the other the greater intension.

If ,Int(A)' and ,Ext(A)' abbreviate the »intension« and the extension of a concept A, respectively, then the so-called law of reciprocity can be formalized as follows:

RECI $\text{Int}(A) \subseteq \text{Int}(B) \leftrightarrow \text{Ext}(A) \supseteq \text{Ext}(B)$.

From this it immediately follows that two concepts A, B have the same »intension« iff they have the same extension. This somewhat surprising result might seem to unveil an inadequacy of Leibniz's conception. However, » intensionality« in the sense of traditional logic must not be mixed up with intensionality in the modern sense. Furthermore, in Leibniz's view, the extension of a concept A is not just the set of *actually existing* individuals, but rather the set of all *possible* individuals that fall under concept A. Therefore one may define the concept of an extensional interpretation of $L1$ as follows:

DEF 4
Let U be a non-empty set (the domain of all possible individuals); let ϕ be a function such that $\phi(A) \subseteq U$ for each concept-letter A; and let V be a valuation function which assigns to each proposition α of $L1$ a truth-value $V(\alpha)$. Then $< U, \phi, V >$ is an *extensional interpretation* of $L1$ if and only if:

(1) $\phi(AB) = \phi(A) \cap \phi(B)$;
(2) $\phi(\sim A) = \overline{\phi(A)}$;
(3) $V(A \in B) =$ true iff $\phi(A) \subseteq \phi(B)$;
(4) $V(P(A)) =$ true iff $\phi(A) \neq \emptyset$.

According to (1), an individual x belongs to the extension of the conjunctive concept AB just in case x belongs to the extension of both concepts (and hence to their intersection). According to (2), the extension of the negative concept $\sim A$ is the set of all individuals which do not fall under concept A. Condition

[19] Cf. Book IV, ch. XVII, § 8 or the original version in GP 5, 469.

(3) is a straightforward formalization of the law of reciprocity, while (4) says that a concept A is possible if and only if it has a non-empty extension.

At first sight, the latter requirement might appear to be somewhat inadequate, since there are certain concepts—such as that of a unicorn—which happen to be empty but which may nevertheless be regarded as possible, i.e. not involving a contradiction. However, the universe of discourse underlying the extensional interpretation of $L1$ does not consist of *actually existing* objects only, but instead comprises all *possible individuals*. Therefore the non-emptiness of the extension of A is both necessary and sufficient for the self-consistency of A. Clearly, if A is possible, then there must be at least one possible individual x that falls under concept A.

It has often been noted that Leibniz's logic of concepts lacks the operator of *disjunction*. Although this is by and large correct, it doesn't imply any defect of the system $L1$ because the operator $\mathrm{A} \vee \mathrm{B}$ may be introduced by definition:

Def 5 $\mathrm{A} \vee \mathrm{B} =_{\mathrm{df}} \sim (\sim \mathrm{A} \sim \mathrm{B})$.

On the background of the above axioms of negation and conjunction, the following standard laws become provable:

Disj 1 $\mathrm{A} \in (\mathrm{A} \vee \mathrm{B})$
Disj 2 $\mathrm{B} \in (\mathrm{A} \vee \mathrm{B})$
Disj 3 $\mathrm{A} \in \mathrm{C} \wedge \mathrm{B} \in \mathrm{C} \rightarrow (\mathrm{A} \vee \mathrm{B}) \in \mathrm{C}$.

6.2. The Quantificational System L2

The quantifier logic $L2$ emerges from $L1$ by the introduction of so-called »indefinite concepts«. These concepts are symbolized by letters from the end of the alphabet X, Y, Z ..., and they function as *quantifiers* ranging over *concepts*. Thus in § 16 of the "General Inquiries" Leibniz explained:

> An affirmative proposition is 'A is B' or 'A contains B' [...]. That is, if we substitute the value for A, one obtains 'A coincides with BY'. For example, 'Man is an animal', i.e. 'Man' is the same as 'a ... animal' (namely, 'Man' is 'rational animal'). For by the sign 'Y' I mean something undetermined, so that 'BY' is the same as 'Some B', or 'A ... animal' [...], or 'A certain animal'. So 'A is B' is the same as 'A coincides with some B', i.e. 'A = BY'.

With the help of the modern symbol for the existential quantifier, the latter law can be expressed more precisely as follows:

Cont 3 $\mathrm{A} \in \mathrm{B} \leftrightarrow \exists \mathrm{Y}(\mathrm{A} = \mathrm{BY})$.

As Leibniz himself noted, the formalization of the UA according to Cont 3 is provably equivalent to the simpler representation according to Def 2.[20] On the one hand, according to the rule of existential generalization,

Exist 1 If $\alpha[\mathrm{A}]$, then $\exists \mathrm{Y} \alpha[\mathrm{Y}]$,

[20] Cf. C, 366, or LLP, 56, fn. 1: "It is noteworthy that for 'A = BY' one can also say 'A = AB' so that there is no need to introduce a new letter".

A = AB immediately entails $\exists$Y(A = YB). On the other hand, if there exists some Y such that A = YB, then trivially also AB = YBB, i.e. AB = YB and hence (because of the premise A = YB) AB = A.[21]

Next observe that Leibniz often used to formalize the PA 'Some A is B' by means of the indefinite concept Y as 'YA $\in$ B'. In view of CONT 3, this representation may be transformed into the (elliptic) equation YA = ZB. However, both formalizations are somewhat inadequate because they are easily seen to be *theorems* of *L*2! According to CONJ 4, BA $\in$ B, hence by EXIST 1:

CONJ 6 $\exists Y(YA \in B)$.

Similarly, since, according to CONJ 3, AB = BA, a twofold application of EXIST 1 yields:

CONJ 7 $\exists Y \exists Z(YA = BZ)$.

These *tautologies*, of course, cannot adequately represent the PA which for an appropriate choice of concepts A and B may become *false*! In order to resolve these difficulties, consider a draft of a calculus probably written between 1686 and 1690, where Leibniz proved principle:

NEG 8* $A \notin B \leftrightarrow \exists Y(YA \in \sim B)$.

On the one hand, it is interesting to see that after first formulating the right hand side of the equivalence, »as usual«, in the *elliptic* way 'YA is Not-B', Leibniz later paraphrased it by means of the *explicit* quantifier expression "there exists a Y such that YA is Not-B"[22]. On the other hand, Leibniz discovered that NEG 8* has to be improved by requiring more exactly that *YA is possible*, i.e. Y must be *compatible* with A:

NEG 8 $A \notin B \leftrightarrow \exists Y(P(YA) \wedge YA \in \sim B)$.[23]

In Leibniz's logical fragments there are only a few passages where indefinite concepts function as *universal* quantifiers. E.g., in C, 260 Leibniz put forward principle "(15) 'A is B' is the same as 'If Y is A, it follows that Y is B"' which clearly has to be understood as:

CONT 4 $A \in B \leftrightarrow \forall Y(Y \in A \rightarrow Y \in B)$.

Furthermore, in § 32 GI, Leibniz at least vaguely recognized that just as A $\in$ B (according to CONJ 6) is equivalent to $\exists$Y(A = YB), so the negation A $\notin$ B means that, for *any* indefinite concept Y, A $\neq$ BY:

CONT 5 $A \notin B \leftrightarrow \forall Y(A \neq YB)$.[24]

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[21] This proof was given by Leibniz in the important paper "Primaria Calculi Logic Fundamenta" of August 1690; cf. C, 235.

[22] Cf. C, 259-261, or the text-critical edition in A VI, 4, 807-813.

[23] Leibniz's proof of this important law (cf. C, 261) is quite remarkable: "(18) [...] to say 'A isn't B' is the same as to say 'there exists a Y such that YA is Not-B'. If 'A is B' is false, then 'A Not-B' is possible by [POSS 2]. 'Not-B' shall be called 'Y'. Hence YA is possible. Hence YA is Not-B. Therefore we have shown that, if it is false that A is B, then QA is Not-B. Conversely, let us show that if QA is Not-B, 'A is B' is false. For if 'A is B' would be true, 'B' could be substituted for 'A' and we would obtain 'QB is Not-B' which is absurd."

[24] Cf. A VI, 4, 753: "(32) *Propositio Negativa*. A non continet B, seu A esse (continere) B falsum est, seu A non coincidit BY". Unfortunately, the last passage 'seu A non coincidit BY' had been overlooked by Couturat and it is therefore also missing in LLP!

Anyway, with the help of the universal quantifier '∀' one can formalize Leibniz's conception of *individual concepts* as *maximally-consistent* concepts in the following way:

DEF 6 $\mathrm{Ind(A)} \leftrightarrow_{\mathrm{df}} \mathrm{P(A)} \wedge \forall \mathrm{Y(P(AY)} \ \rightarrow \ \mathrm{A} \in \mathrm{Y)}$.

Hence A is an individual concept iff A is self-consistent and A contains every concept Y which is compatible with A. The underlying idea of the *completeness* of individual concepts had been formulated in § 72 GI as follows:

> So if BY is [»being«], and the indefinite term Y is superfluous, i.e., in the way that 'a certain Alexander the Great' and 'Alexander the Great' are the same, then B is an *individual*. If the term BA is [»being«] and if B is an individual, then A will be superfluous; or if BA=C, then B=C.[25]

It should be noted that DEF 6 can be simplified by requiring that, for each concept Y, A either contains Y or contains ∼ Y:

IND 1 $\mathrm{Ind(A)} \leftrightarrow \forall \mathrm{Y(A} \in\sim \mathrm{Y} \leftrightarrow \ \mathrm{A} \notin \mathrm{Y)}$.

As a corollary it follows that the *invalid* principle

NEG 9* $\mathrm{A} \notin \mathrm{B} \rightarrow \mathrm{A} \in\sim \mathrm{B}$,

which Leibniz again and again had considered as valid, in fact holds only for individual concepts:

NEG 9 $\mathrm{Ind(A)} \rightarrow (\mathrm{A} \notin \mathrm{B} \rightarrow \mathrm{A} \in\sim \mathrm{B})$.[26]

The definition of individual concepts, DEF 6, is semantically correct. If the idea of an extensional interpretation of $L1$ according to DEF 4 is duly extended to the quantifier logic $L2$, then the following condition becomes provable:

$$\mathrm{V(Ind(A))} = \text{true iff there exists a } \mathrm{x}\varepsilon\mathrm{U} \text{ such that } \Phi(\mathrm{A}) = \{\mathrm{x}\}.$$

Hence the extension of an *individual-concept* A is just a unit-set containing the corresponding (possible) *individual* aεU.

With the help of the operator 'Ind', a *second sort of quantifiers* ranging over »*individuals*« (i.e., more exactly, over individual-concepts) may be introduced as follows:

DEF 7 $\mathrm{VX}\alpha =_{\mathrm{df}} \exists \mathrm{X(Ind(X)} \wedge \alpha)$
$\Lambda \mathrm{X}\alpha =_{\mathrm{df}} \forall \mathrm{X(Ind(X)} \rightarrow \alpha)$.

E.g., the proposition $\mathrm{VX(X} \in \mathrm{B)}$ now says that there is at least one individual concept X such that X contains B. This condition holds whenever concept B is self-consistent:

POSS 3 $\mathrm{P(B)} \leftrightarrow \mathrm{VX(X} \in \mathrm{B)}$.

[25] Cf. LLP 65, Sect. 72 + fn. 1; for a closer interpretation of this idea see LENZEN [12].
[26] The long story of Leibniz's cardinal mistake of mixing up 'A isn't B' and 'A is not-B' is analyzed in detail in LENZEN [10].

Principle Poss 3 syntactically mirrors the semantic postulate (4) of Def 4 according to which concept B is possible if and only if there is at least one *possible* individual x which belongs to the extension of B.

Note that whenever A is an *individual concept*, then formula $A \in C$ is so-to-speak the »intensional counterpart« of the »extensional« formula of first order logic, C(a), which attributes *property* C to the *individual* a. Furthermore, the universal affirmative proposition $B \in C$ may not only be paraphrased by $\forall X(X \in B \rightarrow X \in C)$ (see principle Cont 4 above), but also by means of the new »object« quantifier 'Λ' as follows:

Cont 5 $B \in C \leftrightarrow \Lambda X(X \in B \rightarrow X \in C)$.

Again, the formula on the right hand side, $\Lambda X(X \in B \rightarrow X \in C)$, represents the »intensional counterpart« of the corresponding »extensional« formula of first order logic, $\Lambda x(B(x) \rightarrow C(x))$.

6.3. Reconstruction of Leibniz's Ontological Proof within L2

In Sects. 1–5 of this paper, the expression 'God' has been interpreted as a *singular term* which therefore had to be represented, in the framework of first order logic, by an individual constant like '*g*'. Within the framework of Leibniz's concept logic, however, 'God' rather has to be interpreted as a *general term* and thus be represented by a concept letter, say *G*. Although, intuitively, 'God' will be considered as an *individual* concept like 'Adam', 'Aristotle', etc., one may not simply *postulate* that concept G is an individual-concept, because in view of Def 6 the assumption Ind(G) entails in particular that concept G is free from contradiction, P(G). In the context of the ontological proof, however, such an assumption may not simply be taken for granted! As was stressed at the beginning of this paper, Leibniz had repeatedly pointed out that a *complete* proof of the existence of God requires a *demonstration* of the premise that God is possible:

God 1 P(G).

Since, according to Poss 3, this premise is equivalent to the formula $\mathrm{V}X(X \in G)$, it might seem that a proof of God 1 is already *sufficient* for the demonstration of the existence of God. After all, this formula is normally understood as saying that *there* »*exists*« an individual-concept X such that X is God. However, the range of the quantifier '$\mathrm{V}X$' is the universe of all *possible* individuals and not just the domain of all *existing* objects! Therefore '$\mathrm{V}X(X \in G)$' only means that there is some possible object X which has the property of a God while the issue of the ontological proof is whether such a *possible* God *actually exists*!

Within the framework of *L*2, the *real existence* of God will rather have to be expressed by means of the »predicate« (or concept) of existence, E, in the form '$G \in E$'. The second part of the ontological proof can then be formalized as follows:

God 2 $P(G) \rightarrow G \in E$.

It turned out, somewhat surprisingly, that GOD 2 is indeed a *theorem of L2*, provided that the concept of God is more specifically interpreted as an "Ens necessarium", i.e. as a being which necessarily exists. This traditional conception of God can be captured within $L2$ by the following definition:

GOD 3 $\Lambda X(X \in G \leftrightarrow_{df} \Box(X \in E))$.

We must abstain from reproducing the proof of GOD 2 here because it rests on some further logical constructions (such as the idea of possible worlds as maximal collections of compossible individuals) which clearly exceed the scope of this paper.[27] Suffice it to say that the other part of the proof, GOD 1, turns out *not* to be a theorem of $L2$! So in the end, Leibniz's sophisticated ontological proof suffers the same fate as its various predecessors (and followers): There is no *logical* guarantee that God—whether conceived of as a "Ens necessarium" or as a "Ens perfectissimum—*actually exists*!

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