Log. Univers. 10 (2016), 215–231

1661-8297/16/020215-17, *published online* March 24, 2016
DOI 10.1007/s11787-016-0145-0

Logica Universalis

CrossMark

Singular Propositions, Negation and the Square of Opposition

Lopamudra Choudhury and Mihir Kumar Chakraborty

Abstract. This paper contains two traditions of diagrammatic studies namely one, the Euler–Venn–Peirce diagram and the other, following tradition of Aristotle, the square of oppositions. We put together both the traditions to study representations of singular propositions (through a diagram system Venn-i, involving constants), their negations and the inter relationship between the two. Along with classical negation we have incorporated negation of another kind viz. absence (taking a cue from the notion of 'abhãva' existing in ancient Indian knowledge system). We have also considered the changes that take place in the context of open universe.

Mathematics Subject Classification. Primary 03B99; Secondary 00A66.

Keywords. Square of oppositions, singular propositions, open universe, Abhãva (absence).

1. Introduction

In this work two traditions of diagrammatic studies have been combined in a kind of symbiosis. One is the tradition of square of opposition to represent the relation between categorical propositions and the other, Euler–Venn–Peirce diagrams, to represent emptiness/non-emptiness of sets and their relations. The categorical propositions viz. **A** (All men are mortal), **E** (No man is mortal), **I** (Some men are mortal), **O** (Some men are not mortal), a major concern of Aristotelian logic, had been represented in a diagram by Boetheus in the middle ages based on the formulation of Apulieus (cf. [17]). Afterwards many logicians followed his work. Of these, Parson's diagram has been the most popular one and is widely used in logic texts (cf. [17]). However, further innovation or study of the square had not been the agendum of logicians for a long time. Similarly, the development in the studies of diagrams of the second kind that passed through outstanding mathematician-logicians like Euler [8] and Venn [24], even

Birkhäuser

after insightful contributions by the philosopher-logician Peirce [15], had ceased unfortunately. For an introduction to this history one may consult [18]. Both the forms of diagrams, the square of opposition for categorical propositions and Euler–Venn–Peirce diagrams for sets, are widely used in pedagogy. But diagrams are treated as an aid to the dissemination of knowledge not as an essentiality–not as a subject of research per se. Yet the fact is that sometimes diagrams express more [13].

Interestingly, both the traditions have stepped into a new life during the past three decades. Shin published her PhD dissertation, 'The Logical Status of Diagrams', in 1994 [18]. Almost simultaneously, contributions of Barwise (1990) and Hammer (1995) had been published [1,9]. Since then diagram studies within this tradition has gathered momentum ([6,7,19–22] and many others). The first conference on diagrams was organized in 2000 and subsequently, many other events took place. On the other hand, the first world congress on the square of oppositions was held in 2007 followed by several successive conferences. The list of publications in this field is growing fast, some references are included here that represent only a small fragment [2,3,17]. In this paper we put together both the traditions to study representation of singular propositions, their negations and the interrelationship between the two. Taking a cue from the notion of 'abhãva' [14] existing in ancient Indian knowledge system, we have incorporated a negation called 'absence' and have interpreted it in three ways of which one coincides with the classical negation. It has been observed that the notion of 'absence' as a negation becomes more relevant in the context of 'open universe', that is a universe without a boundary (to be explained in Sect. 4). The notion, in turn, gives rise to several philosophical, mathematical and computational issues. However, we have not delved into them; rather our concern is with representation through diagrams, diagrammatic language and computation with diagrams.

Since we shall use the language of diagram system Venn-i (Venn diagram with individuals [5]) it would be necessary for the readers to have some familiarity with the language of this system.

The diagrammatic language of Venn-i is as follows:

Primitive symbols :

Rectangle : the universe;
Closed curve : monadic predicate;
Shading : emptiness;
x : non emptiness;
$a_1, a_2, a_3, \ldots, a_n$: names of individuals;
$\bar{a}_i$: absence of individual named a_i;
P, Q, ... : names for closed curves representing monadic predicates;
—— : lines connecting crosses (lcc) representing inclusive disjunction of non-emptiness of regions; in the degenerate case an lcc reduces to x;
------- : broken lines connecting individuals a_i(lci) representing exclusive disjunction of existence of an individual in regions; in the degenerate case an lci reduces to a_i.

Diagrammatic objects are the following:
x-sequence,
a_i-sequence,
shading *and*
$\bar{a}_i$.

The x's (a_i's) in a sequence of x's (a_i's) in an lcc (lci) connected by – (-------) are called nodes.

Definition 1.1. Any closed curve without any diagrammatic object is called a blank closed curve.

Definition 1.2. Well formed diagram (wfd):

Type I: A single blank closed curve P within a rectangle is a wfd.

A single closed curve with a finite number of diagrammatic objects inscribed within it or outside it but inside the rectangle is a wfd.

Following (Fig. 1) are some examples:
The third picture says a is P, the fourth says something is P, fifth says absence of a is P i.e. a is not P and the sixth says a is P and P is empty which is a contradiction. Similarly other pictures have direct interpretations.

Type II: This type consists of a rectangle containing more than one closed curve such that each closed curve cuts each other closed curve in exactly two points. The minimal regions so formed may have or may not have entries of diagrammatic objects. The closed curve must not pass through the signs x, a or P. The diagram may also contain lcc or lci with the restriction that none of the nodes appear more than once in the same minimal region. If there is shading it has to cover an entire region. Some examples of type-II diagram are given in Fig. 2:

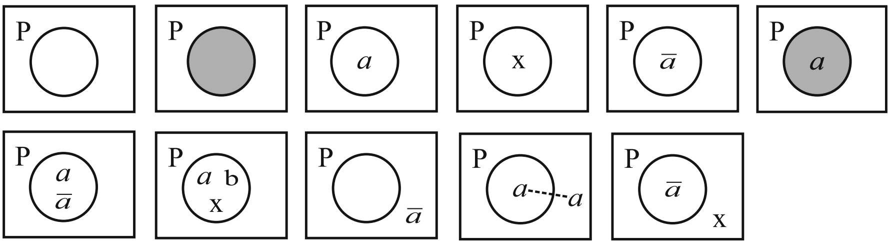

Figure 1.

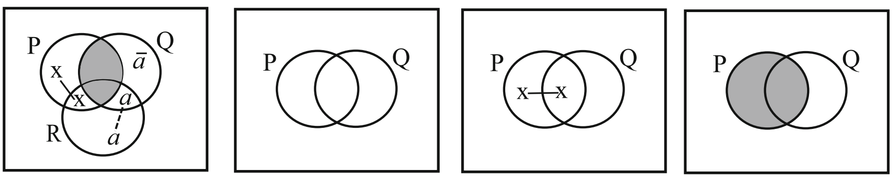

Figure 2.

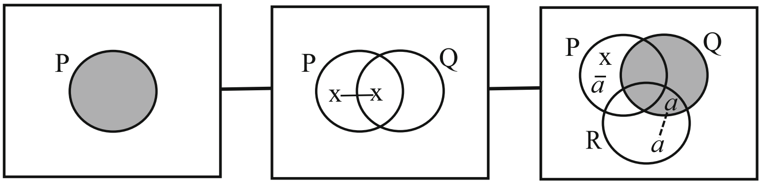

Figure 3.

Type II diagrams are either tautologies (second picture) or conjunctions of informations relative to more than one predication. For example diagram one of Fig. 2 says (P ∩ Q is empty) and (either P\(Q ∪ R) is non-empty or (P ∩ R)\Q is non-empty) and (either a is in (Q ∩ R)\P or R\(P ∪ Q)) and (a is absent in Q\(P ∪ R)).

Type III: If $D_1, D_2, \ldots, D_n$ are diagrams of type I or type II and D' results from joining $D_1, D_2, \ldots, D_n$ by straight lines (written as $D_1 - D_2 - \cdots - D_n$) then D' is a wfd. Each D_i of D' is called a component of the diagram. An example of type III diagram is given below (Fig. 3).

These diagrams are disjunctions of conjunctions.

For transformation rules, soundness and completeness results view [5]. Venn-i allows us to represent singular propositions, negations of singular propositions in addition to the representation of the monadic predicate as is done in Venn II of Shin (1994).

This paper is organized as follows. The Sect. 2 deals with the relationship between singular affirmative, universal affirmative and particular affirmative propositions. In the Sect. 3 we discuss about representation of singular propositions and their negations. Section 4 deals with open universe and its specific properties. The concluding section contains discussion of what has surfaced out of this study and points at future directions.

2. Singular Affirmative, Universal Affirmative and Particular Affirmative Propositions

In logical discourse the following three types of sentences are considered as basic:

(a) a is Q (Singular affirmative)
(b) All P is Q (Universal affirmative)
(c) Some P is Q (Particular affirmative)

Taking (b) and (c) as two corners **A** and **I** and their negations **O** and **E** respectively in Fig. 4, the so called 'square of opposition' is constituted. The relation between **A** and **E** is **contrary** and that of **I** and **O** is **subcontrary**. The relations between (**A, I**) and (**E, O**) are **subalternation**. The relations between (**A, O**), and (**E, I**) are **contradictory**.

The relationships between the pairs of corners viz. **contradictory, contrary, subcontrary** and **subalternation**, however, do not hold if the existential import

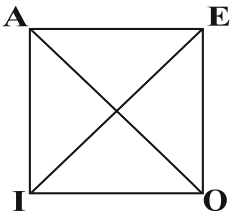

Figure 4.

for the subject and predicate terms P and Q are not presumed. For a discussion on this problem we refer to [17]. Propositions of type (a) are considered in traditional Aristotelian logic as subsumed under type (b) "since in every singular proposition the affirmation or denial is of the whole of the subject" [23] and from this respect cardinality of P does not matter. This assumption has faced serious criticisms dating back from the 13th century in favour of considering (a) and (b) as separate types (vide [12] for a discussion). Firstly as indicated in [23], negation of a universal proposition becomes particular whereas by negating a singular proposition we obtain another singular proposition. Secondly, two propositions are **contrary** if they cannot be true together but can be false together. Propositions of the forms all P is Q (**A**) and no P is Q (**E**) are **contrary**. Now, if P is a singleton a, one gets a proposition of the form (a) viz. a is Q. In this case the counterpart of 'no P is Q' turns out to be 'no a is Q' which is equivalent to 'a is not Q' and this is contradictory to 'a is Q'. Thus in case of singleton P, sentences 'all P is Q' and 'no P is Q' cannot be false together and hence the treatment of (a) and (b) alike misses this difference. This does not, of course, mean to say that singular affirmatives have no **contraries**. For a detail study on the peculiarities of singular propositions, we refer to [23]. In this paper Cześowski Tadeuz considered above three types of categorical propositions. He has used the phrase 'This P' instead of a proper name in order to enable the singular proposition to enter the opposition square. We quote from [23]; "a distinction ought to be made between singular and universal propositions and that trichotomy into universal (All P is Q), singular (This P is Q) and particular (Some P is Q) propositions should be introduced in place of the customary dichotomy according to quantity, into universal and particular propositions". Thus emerged a hexagon of opposition (Fig. 5) as an extension of the traditional square as given below.

The new relations involving **SA** (singular affirmation) and **SN** (singular negation) are the following:

SA − **SN** : contradictory	
A − **SA** : subaltern	**SN** − **A** : contrary
SA − **E** : contrary	**E** − **SN** : subaltern
SA − **I** : subaltern	**SN** − **I** : subcontrary
SA − **O** : subcontrary	**SN** − **O** : subaltern

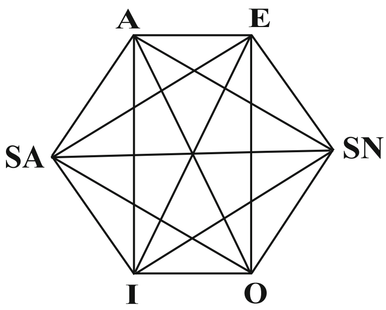

Figure 5.

In the present study, since diagrammatic representation would be the main focus, considering singular propositions as a separate type would be most natural, simple and convenient. 'a' within a circle 'P' (as in the third picture of Fig. 1) is automatically cognized as the fact that the object 'a' is one in the extension P of some property. One of our main concerns is, however, representation of the negation of singular propositions of type (a). Representing negation of a singular proposition of type (a) seems impossible in Venn-like diagrams for which we have to make use of the diagram language Venn-i. This is particularly so as we shall take up some non-classical interpretations of the negation of singular propositions (vide Sect. 3). Various types of diagrammatic representations at the four corners of the square have been studied in [2] of which we pick only the Venn representation. Enhancing that diagram by adding pictures for the corners **SA** and **SN** using methods adopted in Venn-i we get Fig. 6.
'a' may be considered as the proper name for 'This P' and $\bar{a}$ for 'absence of This P'. This is to be noted that as customary in mathematical practices (and formalized by Shin [18]), we have adopted a rectangle representing the universe. This helps us in representing the negation of a singular proposition by the Fig. 11(II) of Sect. 3. In Fig. 6 classical interpretation of $\bar{a}$ is to be taken (see Sect. 3). However, with respect to another interpretation (second one, sec 3) of $\bar{a}$ there are four possibilities (see Fig. 13). Bernhard in [2] rightly mentions that "The representation of the four categorical propositions by different diagram systems allows a deeper insight into this structure".

Since two more corners are now added in diagram 6 (viz. **SA** and **SN**), it would be imperative to see the links between the representation of new (singular) propositions and their relations with the existing square. We follow Bernhard's method in this regard with Venn diagram as base [2]. Following Czeżowski [23], depicting the singular name 'This P' (or this object which is P) by the letter 'a' within the circle (P) we have the following all possible pictures corresponding to Gergonne relations [2]:

While the sentence 'This P is Q' is true in Fig. 7(I) it is false in Fig. 7(II). Thus representations of **SA** and **SN** have no diagram common and together exhaust all possibilities, hence these nodes are contradictory. It is to be noted

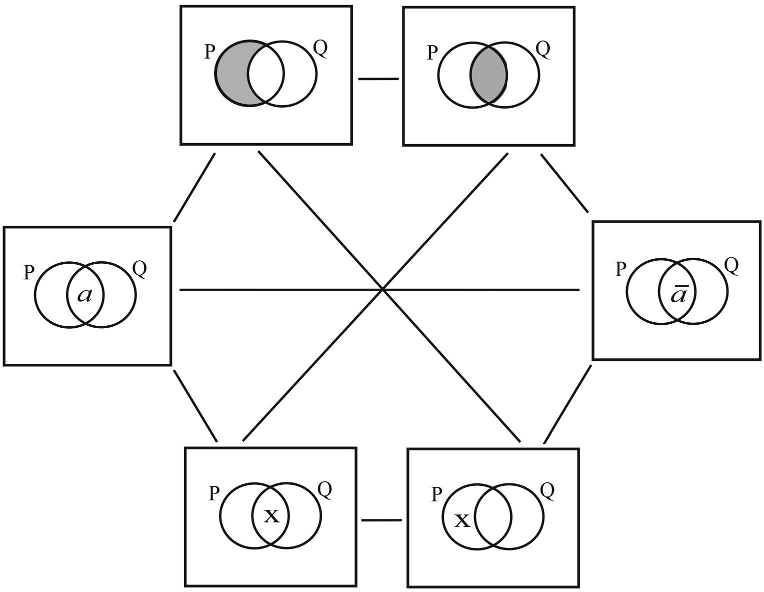

Figure 6.

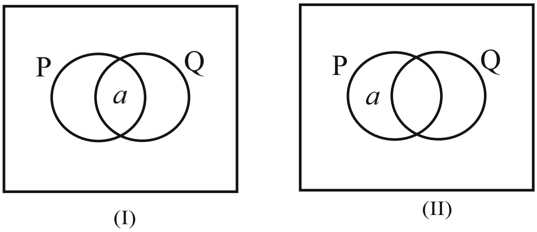

Figure 7.

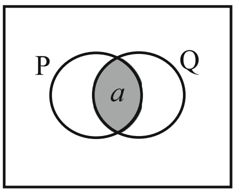

Figure 8.

that this second picture is equivalent to the picture at the corner **SN** of Fig. 6 with respect to the classical interpretation of $\bar{a}$. On the other hand the node **SA** and **E** cannot be true together since then we get Fig. 8.

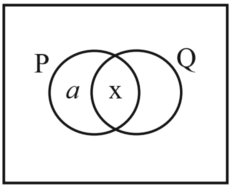

Figure 9.

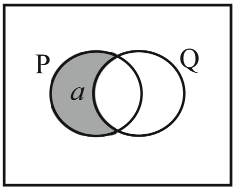

Figure 10.

which say that P∩Q is empty and contains 'a'. From our cognitive stand point this cannot be. But **SA** and **E** can be made false by the situation depicted in Fig. 9.

So, **SA** and **E** are **contrary**. Similarly, **SA** and **O** are **sub-contrary** since while they can be true together, they cannot be false for in that case, we have the diagram.

Figure 10 depicts a contradiction. We can similarly show pictorially the **subalternate** relationship between some other pairs.

Note: In all the above pictures a stands for a name for 'This P'. All the explanations are based on the assumptions that a is P and P is nonempty. We, however, are interested in more general cases when 'a' is an unqualified proper name which stands for an object without any qualifying property. This will be evident in subsequent pictures. In the case of open universe, of course, we shall assume that when an object is mentioned, it is mentioned with at least one qualifying property. Pictorially any letter 'a' is placed always within some circle(s) in the case of open universe.

Types of negation.

Let us now clarify the types of negation that have been dealt with in this paper.

First type (the classical): here we assume that the sentences are about the objects of a non-empty universe X. For any subset P of X and an element 'a' of X, negation of '$a \in$ P' is the same as $a \in \mathrm{P}^c$, P^c being the complement of P in X. Thus the sentence 'a is not P' is true if and only if 'a' is in P^c. Negation of quantified sentences are as in the classical logic.

Second type: here as before, there is a non-empty universe. But negation of '$a \in$ P' does not necessarily imply $a \in \mathrm{P}^c$. This negation has a constructive flavour and may be obtained when P is a recursively enumerable set. Existence of the absence of 'a' in P may not always imply that the object 'a' would be locatable in P^c by some search procedure. Other motivations for adoption of this position from real-life angles are discussed in Sect. 3.

Third type: here the universe is assumed to be of open boundary. This openness may arise from two kinds of situations viz. the boundary does not exist or it exists but is unknown. This point of view is not the God's eye-view which sees everything but of an explorer who does not know the limit of the field being explored or even whether the limit exists. In either case from the knowledge that 'a' is not in P and the information that the object 'a' exists the explorer cannot infer that 'a' is in the complement P^c since P^c does not exist *to the explorer* (in either of the above situations). Thus for the present discussion the difference between the two ontological situations for openness of the universe does not matter. Here too the negation of '$a \in P$' behaves non-classically as from its (the negation's) being true no inference about the location of 'a' may be ascertained.

In the following two sections we shall deal with singular affirmation and singular negation only. The distinguishing characteristics of these statements will be the only point of study. However, we shall also use Euler diagram in some cases towards the end.

3. Representation of Singular Propositions and Their Negations

As a first step let us mention that negation of the singular proposition 'a is P' (Fig. 11(I)) may be depicted in the system Venn-i by either of the figures Fig. 11(II) or Fig. 11(III).

Figure 11(III) depicts the 'absence of a' directly in the region P as Fig. 11(I) depicts the presence of a in the region. Figure 11(II) is a kind of derivative; if a is not in P, it must be present in the complement. Figure 11(II) and Fig 11(III) are considered to be equivalent from the classical point of view of first order logic and set theory; absence of a is in P implies that a is present in P^c and vice-versa. Under this interpretation of the sign $\bar{a}$, the only gain is in the simplicity of pictorial representation of the negation of the singular proposition 'a is P' (for details see [7]). It may also be noted that representing singular propositions as **A** propositions as depicted in the top left corner of Fig. 6 would be too cumbersome since that would require depiction of cardinality [19]. Let us show some pictures of more complex situations with two monadic predications to illustrate how the introduction of $\bar{a}$ helps in reduction of cluttering in the diagrammatic representation.

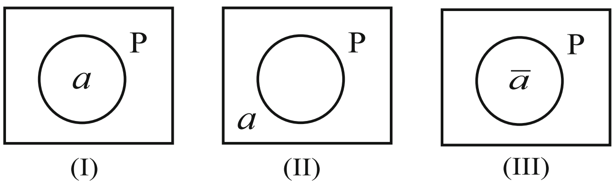

Figure 11.

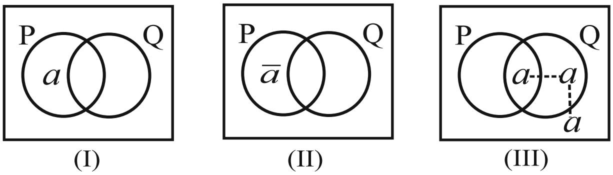

Figure 12.

Figure 12(I) says a is in P\Q. Its negation may be depicted equivalently by Figs. 12(II) and 12(III). The simplicity in representation of negation by Fig. 12(II) is quite visible.

Note: For the idea of depiction of absence of a in the picture we took cue from the notion of abhãva ([14], p.47) in the traditional Indian system. The Indian logicians (Nyãya Vaisesika thinkers) admit a distinct ontological category called abhãva (absence) with a view to accounting for negative statements. Absence is a relative concept – absence is always of something at some place (locus). Absence has to be admitted as the object of negative form of cognition. Accordingly, the symbol $\bar{a}$ is to be treated as a term or as an individual and read as 'absence of a'. Every cognition has an object which exists independently of the cognition and if the object was not there, the cognition could not occur. The object of perception acts as a cause of perception in case of veridical perception. Since we all have negative form of cognition and such forms of cognition have for their object some negative entity or absence, some philosophers admitted that there are negative entities in the world without which negative cognition could not be possible. Let us quote from ([14], p. 146) the central point of the concept: "Navya-Nyãya is not in favour of the affirmative-negative dichotomy of propositions. Instead , it speaks of contradictory pairs of qualifiers, viz., blue-colour and the absence of blue-colour, or pot (i.e., pot-presence) and pot-absence. Thus, the contradictory pairs of propositions (or qualificative cognitions, to use strictly the Nyãya terminology) are formulated with such contradictory pairs of qualifiers." These thinkers are in favour of accepting positive objects like a chair as well as negative objects like absence of a chair. They admit absence as a separate ontological category It should, however, be emphasized that we have only taken the cue not the idea in its totality. An excellent account on various aspects of negation and negative forms of cognition is available in [11]. Russell in his 'Philosophy of Logical Atomism' maintains similar view when he considers two kinds of atomic facts: positive atomic fact and negative atomic fact. To quote from Russell – "...I think you will find it better to take negative facts as ultimate. Otherwise you will find it so difficult to say what it is that corresponds to a proposition when, e.g., you have a false positive proposition, say 'Socrates is alive'. It is false because of the non-correspondence between Socrates being alive and the state of affair. A thing cannot be false except because of a fact, so that you find it extremely difficult to say what exactly happens when you make a positive assertion that it is false, unless you are going to admit negative facts." ([16], p.214).

The ideas of Naiyãyika (the Nyãya philosopher) and Russell almost synchronized on the acceptance of absence as an independent category and negative atomic fact.

In the next step the equivalence between Fig. 11(II) and (III) will be done away with.

The second interpretation of $\bar{a}$ has already been given in Sect. 2. Here it is presented more formally. All discourses take place, as before, within a fixed universe of objects denoted pictorially by the rectangle. Let the following conditions be assumed for the notion of 'absence of a', which will be denoted by $\bar{a}$.

(i) a is in P implies $\bar{a}$ is in P^c(the complement of P).
(ii) $\bar{a}$ is in P does not necessarily imply a is in P^c.
(iii) a is in P and $\bar{a}$ is in P are contradictory.
(iv) Whereas a cannot be present in more than one minimal regions, $\bar{a}$ can.
(v) Absence of absence of a is not a well formed expression of Venn-i. Putting two bars above a, $\bar{\bar{a}}$ is not a valid sign.

Note: If $\bar{a}$ in P is true or equivalently, a in P is false it becomes uncertain whether a is in the complement of P (relative to some universe) or not. Thus besides true (T) and false (F), a third category 'uncertain' (U) steps in. U can be interpreted as the set {T, F} meaning thereby that a statement may be true, may be false. The relation expressed through the traditional square of opposition is one of truth-falsity relation. The four assertions a is in P, $\bar{a}$ is in P, a is in P^c and $\bar{a}$ is in P^c now enter into a relationship as shown in the following diagram.

Some remarks on the condition (ii) may be helpful in understanding the idea. This is somewhat similar - but only similar - to the notion of recursively enumerable set [10] where the objects inside the set are locatable by a procedure but those outside the set are not. If it is supposed that actually a is not within P then there shall be no procedure to locate it within P^c. In our case of course, we assume that it may or may not be in P^c. In this sense ours is a more general notion.

Secondly, the following examples from real life situations may be helpful in clarifying the idea of $\bar{a}$ sought to be addressed here.

(1) The attendance register book of the students of a class is in one-to-one correspondence with the set of students. The representation of the absence of a student 'a' is marked by a symbol just as is done in case of the presence. We are not pointing at the aspect of administrative convenience, but at the cognitive impact of this practice. No mark corresponding to some student would bring to our mind the message 'no information'. Thus in the register absence of a student in class is shown by a mark (and not by his/her presence somewhere else).

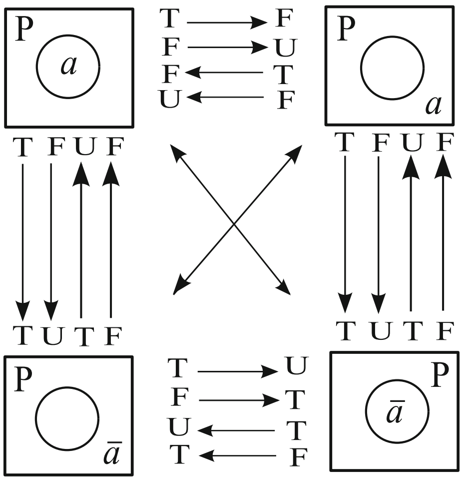

Figure 13.

(2) From the perspective of administration too it may not be enough to get the information that a is missing from a country, it may demand a police-record. This certificate is the sign of the absence of a. To the administration the only important point is the record that the person is missing from here, where she/he is at present or is alive at all does not matter.

In either of the cases (1) or (2), the marked absence of a in a location P (a class or a country) does not automatically imply a's presence outside the location.

Before proceeding further, let us consider the following case. If we fix P and Q to be disjoint and adopt the first classical interpretation of $\bar{a}$, an interesting situation (Fig. 14) arises which will help in future development. To depict P and Q to be disjoint we shift from Venn to Euler diagram. As the purpose here is to study various relationships among propositions depicted through diagrams, the problem of rigor in mixing the two modes of representation should not arise.
As before, U stands for uncertain, meaning thereby may be T, may be F.

The similarity in Figs. 13 and 14 (ignoring Q) is to be noticed. This similarity indicates that the second interpretation of $\bar{a}$ leads to considering P^c as just another Q disjoint from P. So, at this step, complement of P is not treated as the entirety outside P and which is known. In the next step we shall deal with the situation when the complement of P is unknown altogether or in other words the universe is open. Before that, let us include 'absence of a', that is $\bar{a}$, in the context of Fig. 14 i.e. two disjoint locations P and Q without stipulating the classical equivalence as shown in the lower row of Fig. 14. The following diagram shows all the possibilities (Fig. 15).

The diagram of opposition for all the nine cases mentioned above is now summed up in Fig. 16 (only the respective numbers are shown) with bold line representing **contradictory** relation, normal line representing **contrary** relation

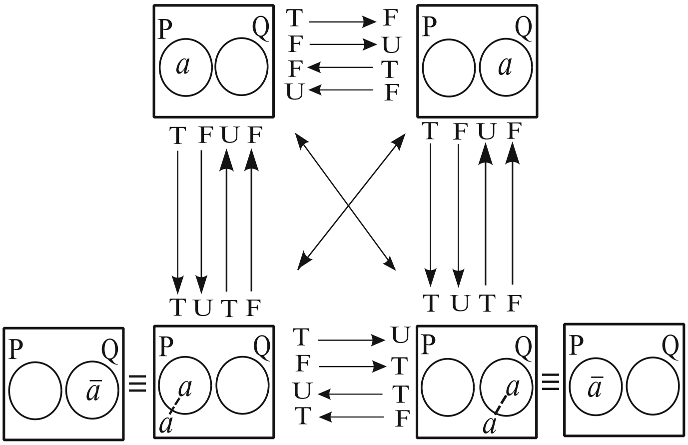

FIGURE 14.

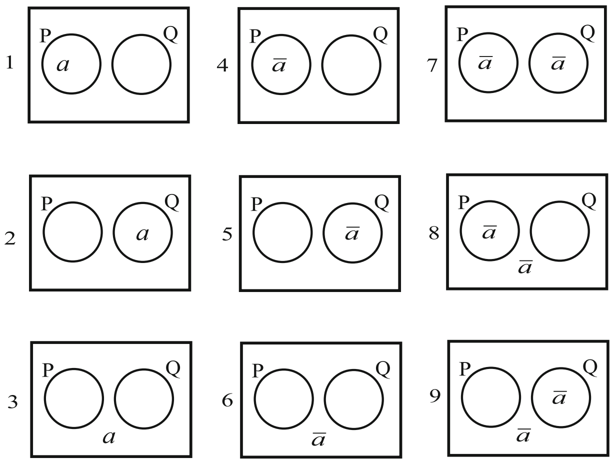

FIGURE 15.

and dotted line representing **sub-contrary** relation. However all the relations are not shown.

Next we take the final step into open universe.

4. Open Universe

In the context of open universe the notion of absence becomes more significant. The universe is not static. When an object appears, it appears with a property denoted by a predicate P and represented by a closed curve. Similarly when an object a disappears from the extension P of some property, $\bar{a}$ appears in P. However this dynamics is neither depicted in the diagram nor captured in the

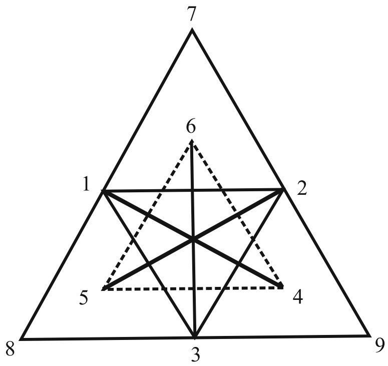

Figure 16.

formal theory that has been developed. We only consider a static slice of the changing universe in which absolute complement of a set is not meaningful. Pictorially this needs only to remove the boundary rectangle from the diagram and carry out necessary adjustments. It may be noticed that the categorical propositions **A, E, I, O** do not need the existence of a universal set for their cognitive import. Difficulty arises only in case of representing negations of singular propositions in the classical way that is by not using a sign for 'absence of a'. Since we have incorporated $\bar{a}$, for representing that a is not in P, existence of the boundary rectangle is not required either. In case of open universe absence of a in P does not imply that a is in the complement of P simply because the complement does not exist as the universe is open. But absence of a in P that is $\bar{a}$ in P implies two possibilities: a is in some region Q such that $\mathrm{P} \cap \mathrm{Q} = \emptyset$ or a is nowhere which in turn means that a is absent also in any other disjoint closed curve Q whenever Q is depicted. Thus, the assumptions here are:

(1) The boundary of the Universe is unknown.
(2) a and $\bar{a}$ (absence of a) are to be present always within some extensions (represented by closed curves).
(3) a cannot be present in two disjoint locations (represented by minimal regions).
(4) $\bar{a}$ can be present in two disjoint extensions and even in all known mutually disjoint locations (represented by mutually disjoint regions).
(5) $\bar{a}$ is in region P implies that a is in region Q and $\mathrm{P} \cap \mathrm{Q} = \emptyset$ or a is not depicted at all.

Figure 17 below shows the mutual relationship between various pairs with one individual 'a' and two disjoint predicates P and Q in open universe (following the same convention with lines and numbering as in Fig. 16).

The difference between Fig. 17 involving open universe and that of closed universe (Fig. 16) may be noted.

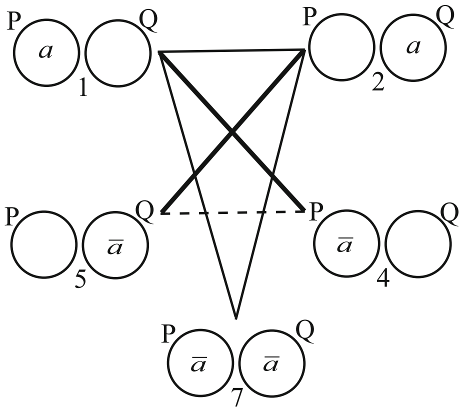

Figure 17.

5. Concluding Remarks

From the fact that $a \notin \mathrm{P}$ it is outright inferred in classical set theory and logic that $a \in \mathrm{P}^c$ irrespective of the fact whether a is locatable in P^c or not. However, the incorporation of open universe rules out the possibility that $a \in \mathrm{P}^c$ since the complement does not exist. Other than the universe of mathematical objects or closed world semantics it is cognitively almost impossible to inspect the complement of P. Thus we will have $a \in \mathrm{P}$, $\bar{a} \in \mathrm{P}$ as cases of direct cognition and we need to admit the third possibility, the 'know not' situation. Incorporation of open universe along with the absence of a particular will render the diagrammatic system more natural language friendly in the sense that we will be able to talk about fictitious objects, like ghosts or fairies, unidentified objects of science fiction like UFO or life in other planets etc.

A first attempt towards development of a formal diagrammatic language for open universe has been made in [4]. This is based on Venn-I system, no Euler type representation has been used. The rules have been sound with respect to the intended semantics. Method of derivation is presented.

Let us summarize the salient points of this work. These are

1. incorporation of absence of individual a in P that is $\bar{a} \in \mathrm{P}$ as a category separate from $a \notin \mathrm{P}$,
2. giving three interpretations of $\bar{a}$ viz. the classical one where $\bar{a} \in \mathrm{P}$ is equivalent to $a \notin \mathrm{P}$ assuming closed universe, secondly, $\bar{a} \in \mathrm{P}$ not being equal to $a \notin \mathrm{P}$, P being somewhat similar to recursively enumerable set, and thirdly, with respect to open universe assumption,
3. laying main focus on diagrammatic representations by squares and other rectilinear figures of opposition incorporating singular propositions.

Remarks 1 and 2 will be elaborately discussed in some of our future work. Completeness proofs with second interpretation of $\bar{a}$ (Sect. 3) and with respect to the open universe interpretation are ready for a submission.

Acknowledgements

This research is supported by DeitY funded NPPEII and UGC UPEII at the School of Cognitive Science Jadavpur University, Kolkata. We are thankful to Reetu Bhattacharya and Purbita Jana for preparing the diagrams for us.

References

[1] Barwise, J., Etchemendy, J.: Visual information and valid reasoning. In: Allwein, G., Barwise, J. (eds.) Logical Reasoning with Diagrams, pp. 3–25. Oxford University Press, Oxford (1990)

[2] Bernhard, P.: Visualization of the square of opposition. Logica Universalis **2**, 31–41 (2008)

[3] Béziau, J.Y., Payette, G. (eds.) The square of opposition. In: A General Framework for Cognition. Peter Lang, Bern (2012)

[4] Choudhury, L., Chakraborty, M.K.: On representing Open Universe. Stud. Logic **5**(1), 96–112 (2012)

[5] Choudhury, L., Chakraborty, M.K.: On extending Venn diagram by augmenting names of individuals. In: Blackwell, A., et al. (eds.) Diagrams 2004, LNAI 2980, pp. 142–146 (2004)

[6] Choudhury, L., Chakraborty, M.K.: Comparison between spider diagrams and venn diagrams with individuals. In: Proceedings of the Workshop Euler Diagrams 2005. INRIA, Paris (2005)

[7] Choudhury, L., Chakraborty, M.K.: Singular propositions and their negations in diagrams published in the proceedings of DLAC 2013. In: CEUR Workshop Proceedings, vol. 1132 (2013). http://ceur-ws.org/

[8] Euler, L.: Letters a une princesse d'allemagne, Sur divers sujets de physique et de philosophie. Letters No. 102–108, vol. 2. Basel Birkhauser (1761)

[9] Hammer, E.: Logic and Visual Information. CSLI Pubs., USA (1995)

[10] Hopcroft, J.E., Ullman, J.D.: Formal Languages and Their Relations to Automata. Addison-Wesley, London (1969)

[11] Horn, L.R.: A Natural History of Negation. CSLI Pub, USA (2001)

[12] Khomiskii, Y.: William of Sherwood, Singular Proposition and the Hexagon of Opposition. In: Béziau, J.Y., Payette, G. (eds.) The Square of Opposition. A general Framework for Cognition, pp. 43–57. Peter Lang, Bern (2012)

[13] Larkin, J.H., Simon, H.A.: Why a diagram is (Sometimes) Worth Ten Thousand Words. Cognit. Sci. **11**, 65–99 (1987)

[14] Krishna, M.B.: Logic, Language and Reality. Motilal Banarasidass. Pub, New Delhi (1990)

[15] Peirce, C.S.: Collected Papers of C.S.Peirce. **iv**, HUP (1933)

[16] Russell, B.: Philosophy of Logical Atomism: Logic and Knowledge. Unwin Hyman, London (1988)

[17] Sharma, S.S.: Interpreting square of oppositions with the help of diagrams. In: Béziau, J.Y., Payette, G. (eds.) The Square of Opposition. A general Framework for Cognition, pp. 174–192. Peter Lang, Bern (2012)

[18] Shin, S.J.: The Logical Status of Diagrams. CUP, Cambridge (1994)

[19] Stapleton, G., Taylor J., Thompson S., Howse, J.: The expressiveness of spider diagrams augmented with constants. J. Vis. Lang. Comput. **20**(1), 30–49 (2009)

[20] Stapleton, G., Howse, J., Taylor, J.: A decidable constraint diagram reasoning system. J. Logic Comput. **15**(6), 975–1008 (2005)

[21] Stapleton, G.: Incorporating negation into visual logics: a case study using Euler diagrams. Vis. Lang. Comput., 187–194 (2007)

[22] Swoboda, N., Allwein, G.: Heterogeneous reasoning with Euler/Venn diagrams containing named constants and FOL. Electron. Not. Theor. Comput. Sci. **134**, 153–187 (2005)

[23] Tadeusz, C.: On certain peculiarities of singular propositions. Mind **64**, 392–395 (1955)

[24] Venn, J.: On the diagrammatic and mechanical representation of propositions and reasoning. Philos. Mag. J. Sci. Ser. 5. **10**(59), 1–18 (1880)

Lopamudra Choudhury
Department of Philosophy, School of Cognitive Science
Jadavpur University
Jadavpur
Kolkata 700032
India
e-mail: choudhuryl@yahoo.com

Mihir Kumar Chakraborty
School of Cognitive Science
Jadavpur University
Jadavpur
Kolkata 700032
India
e-mail: mihirc4@gmail.com

Received: July 15, 2015.
Accepted: March 11, 2016.