Log. Univers. 9 (2015), 523–544

1661-8297/15/040523-22, *published online* November 2, 2015
DOI 10.1007/s11787-015-0131-y

Logica Universalis

Liberating Paraconsistency from Contradiction

Jonas R. Becker Arenhart

Abstract. In this paper we propose to take seriously the claim that at least some kinds of paraconsistent negations are subcontrariety forming operators. We shall argue that from an intuitive point of view, by considering paraconsistent negations as formalizing that particular kind of opposition, one needs not worry with issues about the meaning of true contradictions and the like, given that "true contradictions" are not involved in these paraconsistent logics. Our strategy will consist in showing that, on the one hand, the natural translation for subcontrariety in formal languages is not a contradiction in natural language, and on the other, translating alleged cases of contradiction in natural language to paraconsistent formal systems works only provided we transform them into a subcontrariety. Transforming contradictions into subcontrariety shall provide for an intuitive interpretation for paraconsistent negation, which we also discuss here. By putting all those pieces together, we hope a clearer sense of paraconsistency can be made, one which may liberate us from the need to tame contradictions.

Mathematics Subject Classification. Primary 03B53; Secondary 03A05.

Keywords. Paraconsistency, negation, contradiction, subcontrariety.

1. Introduction

Possibly everyone has already heard the rough characterization of paraconsistent logics as those systems in which a contradiction does not necessarily entail every formula of the language, or some characterization very close to that. So, the general idea seems to be that paraconsistent logics allow us to deal with contradictions,—tame them, as it were—without leading us to triviality. Now, this idea has got its own difficulties and critics. A recurring theme

I would like to thank two anonymous referees of the journal for their comments which helped improve a previous version of the paper. The remaining mistakes and infelicities are my sole responsibility, of course.

concerning systems of paraconsistent logic is whether a paraconsistent negation really is a negation. Ever since Slater [20] raised doubts about the existence of paraconsistent logics, friends of paraconsistency have hurried to argue that a paraconsistent negation is a real negation (see Béziau [4]). The general line of response concerns arguing that even though a paraconsistent negation does not generate contradictions, but rather at most subcontraries, it is still to be taken as a kind of negation, because a negation is not to be defined exclusively as an operation that generates contradictions.

However, even if one agrees with the fact that there are negations for everyone, so that a paraconsistent negation is a real negation, one point that stems from this controversy is still not very clear: assuming that $\sim$ is a paraconsistent negation, what does a pair of expressions of the kind P and $\sim P$ stand for intuitively? Well, ins't it a contradiction? But recall that in order to rebut Slater's argument, it is agreed that a paraconsistent negation does not generate contradictions, but it is rather to be understood as being, at best, a subcontrariety forming operator. So, can we still hold that P and $\sim P$ is a contradiction? If we could, there would be no dispute with Slater to begin with! The issue seems to be rarely addressed, given that it is still very common to find characterizations of paraconsistent systems as those systems that allow one to reason in the presence of contradictions (the most recent of such claims is perhaps to be found in Šešelja and Straßer [19]). More generally, there are at least three features that paraconsistent systems of logic are said to accomplish:

- Paraconsistent logics may be characterized as the underlying logics of inconsistent but non-trivial theories (see, for instance, [14, p. 791]);
- Applications of paraconsistent logics in philosophy may help us understand contradictory objects, such as the famous round–square (see [11, p. 13]);
- Paraconsistent logics help us violate the Principle of Non-Contradiction while keeping rationality in the presence of inconsistencies.

Of course, there is still much more to the characterization and application of such systems, but these are the main claims about what paraconsistent logics do. Given those claims, it is not surprising that most of the disputes concerning paraconsistent logics have focused on showing that a paraconsistent negation is a negation, as we have mentioned, and on how we are to understand properly a "true contradiction" (see for instance Carnielli and Rodrigues [9] and Carnielli and Coniglio [10]). However, something is missing in this picture: as we mentioned, in the defense of a paraconsistent negation as a real negation, we were led to acknowledge that a paraconsistent negation is at best a subcontrariety forming operator. On the other hand, paradoxically, some logicians and philosophers are still defending the plausibility of paraconsistent logics against charges that true contradictions do not exist or that contradictions are unintelligible. Now, given that expressions such as P and $\sim P$ are not contradictories, but at best subcontraries, that is a strange kind of worry (see also Béziau [3,5] and [7] for enlightening remarks about the relation of distinct kinds of negation to traditional oppositions).

Our aim in this paper is to try to address precisely this issue. We shall take seriously the claim that at least some paraconsistent logics deal with subcontrariety forming operators, and not with contradictory forming operators, restricting most of the discussion to those systems (although, as we shall argue later, this seems to be no harm to the generality of the argument). We shall illustrate how this fact is to be understood intuitively by employing intuitive translations from formal language to informal language, and vice versa. Our aim is to show that the friends of (at least) some kinds of paraconsistent logics need not worry with the status of contradictions, given that those systems are not really directly concerned with contradictions, after all. The effect of this move is at the same time disturbing and liberating. It is disturbing because we were always told that paraconsistent logics deal with contradictions, and now it is strange to be told that they do not directly deal with contradictions. It is liberating, because once we realize that they do not deal with contradictions, we don't have to feel the need to address issues such as how to make sense of (true?) contradictions, and how to reason when 'The Principle' of non-contradiction is not generally valid.

The outcome of these discussions will be also twofold. On the one hand, we may conjecture that we were induced to believe that expressions like P and $\sim P$ are contradictory due to their syntactical form: it resembles perfectly a classical contradiction. Perhaps this is a case of a form that does not fit the content; it looks like a contradiction, but it is not. Also, in the second place, as we shall see, the fact that contradictions are not directly involved in paraconsistent logics allows us to provide for intuitive interpretations of the paraconsistent negation that fit quite well Carnielli and Rodrigues' [9] *epistemic interpretation of contradictions*. The only fact to be taken into account is that there are no contradictions anymore (at least not directly, as we shall argue).

This paper is divided into two main parts. In Sect. 2 we discuss the meaning of "contradiction" and "subcontrariety" that shall be employed throughout the paper. We employ the terminology of Béziau ([3] and [5]), having in mind the distinctions captured by the traditional square of opposition. Our aim is to isolate as clearly as possible the specific kind of paraconsistent negation we shall be concerned with in this paper; in particular, we hope to distinguish it sharply from classical negation. This will be crucial for our analysis of the intended aims of paraconsistent logics. In Sect. 3 we examine how typical paraconsistent logic applications fare in the face of such an analysis and how discussions about the meaning of paraconsistent logics may be dealt with under those circumstances. We argue that for most of the applications of paraconsistent logics, no essential use of the notion of contradiction is required. That is a fact to be welcomed by paraconsistentists, because as they have already acknowledged, paraconsistent logic really deals at most with subcontraries when 'true contradictions' come in. We conclude in Sect. 4.

2. Oppositions and Contradictions

There seems to be an obvious and straightforward way to characterize a contradiction. In general, in a language in which there is a negation sign $\neg$, sentences P and $\neg P$ are said to be *contradictory*. A set Γ deriving a pair of contradictory sentences is said to be *inconsistent*. However, this is not enough to characterize contradiction and inconsistency, given that inconsistent sentences, as we have just defined, may belong to different systems of logic, having thus distinct meanings. Or, at least we shall argue so.

To begin our attempt on a clarification of what a contradiction is, we could follow da Costa [11] (and also the approach in da Costa, Krause and Bueno [14]), who has already advanced the thesis that a symbol such as a negation sign gains its meaning by its overall role in the deductive system to which it belongs (a Hilbert-style axiomatic system, natural deduction system, and so on). This is a syntactical approach to the meaning of a logical constant, and we shall call it, only for ease in reference, *inferential semantics*. So, according to inferential semantics, a negation sign of a paraconsistent system of logic has surely a distinct meaning than a negation sign of a classical or intuitionist system of logic. This, of course, is no novelty, but should already warn us that in inferential semantics "contradiction" and "inconsistency" are contextually dependent on which particular system we are considering.

For the sake of generality, we may classify negations in three broad categories (following, for instance, Béziau [3]):

- Classical negation ($\neg_c$), appearing in classical systems of logic.
- Paracomplete negation ($\neg_i$), appearing in paracomplete logics.
- Paraconsistent negation ($\neg_p$), appearing in paraconsistent logics.

This classification is very rough, of course, but preserves some features of those negations that are most relevant for our discussion. Classical systems of logic have as theses the corresponding law of non-contradiction ($\neg_c(P \wedge \neg_c P)$) and of the excluded middle ($P \vee \neg_c P$), among many others (classical *reductio ad absurdum*, double negation elimination, and so on). Paracomplete logics obey a version of the law of non-contradiction (that is, ($\neg_i(P \wedge \neg_i P)$ holds), but do not obey a version of the excluded middle ($P \vee \neg_i P$ does not hold). As a dual to paracomplete logics, one characterization of paraconsistent logics involves characterizing paraconsistent negations as an operator that violates a version of the law of the non-contradiction ($\neg_p(P \wedge \neg_p P)$ does not hold), while a version of the excluded middle holds ($P \vee \neg_p P$). Of course, by accepting inferential semantics the result is that conjunction and disjunction also have distinct meanings in these distinct systems. As a consequence, the definitions of contradiction and inconsistency in each of these kinds of systems will have distinct meanings (thence the condition 'a version of' in front of each formulation of the principles). A paraconsistent contradiction, for instance, a pair P and $\neg_p P$, means something different from a classical contradiction P and $\neg_c P$.

Now, while this characterization of paraconsistent logics is interesting, it is not to be taken as involving a strict definition of paraconsistent negation. On the one hand, there are also the so-called "paraconsistent classical logics" in which the law of non-contradiction holds, even though they are not trivialized by a pair of expressions P and $\neg P$ (see for instance Sylvan and Urbas [21]). An example of such systems is LP, Priest's *Logic of Paradox*. On the other hand, there are also "paranormal logics" in which negation does not obey neither the law of non-contradiction nor the law of excluded middle, still being paraconsistent (see Béziau [7]). For the arguments on the sections to follow, we shall confine our discussions to paraconsistent systems accepting the law of excluded middle and accepting that expressions like P and $\neg P$ may have a model; that is all we require. This is compatible with both the acceptance and rejection the law of non-contradiction.

As it may be clear from what has just been said, it seems to follow that the meanings of paraconsistent and classical negations, for instance, are distinct. One then has no common grounds on which to discuss the issue of a system violating the laws of the other, or being a restriction of the other. They are simply not comparable as to their meanings. This indicates that an inferential semantics is not the best guide for the kind of task we are attempting at here. Furthermore, an inferential semantics is still not enough for us to understand the proper meaning of an expression such as $(P \wedge \neg_p P)$ and its relation to what we take as a contradiction in ordinary discourse (which is what we set ourselves to investigate in the first place).

Here, to set the stage for our arguments in the next section, we must also have the resources of a traditional truth-value attribution semantics. In this demand, we follow da Costa [12], who makes it clear that a system of logic demands not only a formal apparatus but also an interpretation that suits it in its purposes (a remark to be found also in [14, p. 791]). There are good philosophical reasons for that requirement of "purpose adequacy", even though we shall not discuss them in detail here. The main idea is that only when this kind of semantics is taken into account the main differences between distinct systems of logic are made clear. For instance, from a purely formal point of view intuitionistic propositional logic may be seen as a subsystem of classical propositional logic; from the point of view of their intuitive semantics and their purposes, they cannot be viewed like that.[1] From an intuitionist point of view, classical logic gets things completely wrong! Something similar holds for the relation of classical logic with paraconsistent logic, as we shall discuss later. So, we shall give precedence to the truth-value attribution semantics in this paper and try to relate it to an intuitive interpretation. It shall provide for a common ground from which to discuss our points.

Let us now consider a semantically minimal definition of the three distinct kinds of negations already presented. The idea is that, semantically, these three negations have properties that are characteristic of three kinds of opposition on the traditional square of opposition (once again, we refer the reader to Béziau [3,7]). That is:

[1] For discussions on the intuitive semantics for intuitionistic logic, see da Costa [12, pp. 161–166].

Classical negation: α and $\neg_c\alpha$ can be neither both true, nor both false.
Paracomplete negation: α and $\neg_i\alpha$ can be both false, but not both true.
Paraconsistent negation: α and $\neg_p\alpha$ can be both true, but not both false.

So, in terms of the traditional square of opposition, P and $\neg_c P$ are called *contradictory*, P and $\neg_i P$ are called *contraries* and P and $\neg_p P$ are called *subcontraries.* Of course, this terminology applies when we consider that the relations of opposition encapsulated by these negations mimic the relations of opposition described in terms of the traditional square. In this sense, there is only one pair of propositions that deserve to be called "contradictories", *viz.* P and $\neg_c P$, for any proposition P. In fact, according to tradition, the semantic relations obeyed by both paraconsistent and paracomplete negations do not generate a contradiction, but rather the relations of subcontrariety and contrariety, respectively. However, this does not mean that these are not also to be understood as negations, even though we are not going to enter the merit of such a dispute by now (see again Béziau [4,7] and on the plurality of negations, in formal systems and even in natural language, see Horn and Wansing [17]). It only shows that these distinct negations do not always work to generate contradictions.

Anyway, aside from such semantic concerns, the usual procedure in logic texts is to keep calling an expression of the form $(P \wedge \neg_p P)$ a contradiction (it could be called a "paraconsistent contradiction"), and the same is usually done for paracomplete negation. What is relevant is that we do not forget that these "contradictions" have distinct meanings: $(P \wedge \neg_p P)$ may be true, but $(P \wedge \neg_c P)$ is never true. The same holds for the definition of an inconsistent set of formulas: it depends on what kind of contradiction is being taken into account. In short:

- Classical contradiction: P and $\neg_c P$.
- Paracomplete contradiction: P and $\neg_i P$.
- Paraconsistent contradiction: P and $\neg_p P$.

These will be the crucial distinctions for our work. In strict terms, a contradiction only arises when we use classical negation.

We are aware, of course, that particular systems of logic, for instance, particular systems of paraconsistent logics, may vary in the meaning attributed to negation (for instance, the distinct C_n systems of da Costa differ from Priest's *Logic of Paradox*), but in overall, we shall deal with paraconsistent logics in which the general classification presented here is respected, with variations appearing in finer details attributed to the specific system under study. As it is clear, we shall be concerned here with paraconsistent negations characterized semantically so that they may allow for what may be called true "paraconsistent contradictions", or, for paraconsistent negation behaving as a subcontrariety-forming operator. In terms of valuations, such negations allow for valuations obeying the condition that, for some valuation v, we may have $v(\neg_p P) = 1 = v(P)$ (where 1 represents truth). Furthermore, we shall be mainly concerned with cases in which we do indeed have $v(\neg_p P) = 1 = v(P)$, even though subcontrariety allows for $v(\neg_p P) \neq v(P)$.

Of course, as we have already mentioned, this is not the only possible characterization of a paraconsistent system. There are systems of paraconsistent logics preserving classical tautologies, while other systems are not even two valued (so it would be more appropriate to speak of propositions receiving the *designated value* or the *undesignated value* instead of truth or falsehood; for discussion on three-valued paraconsistent systems violating the law of non-contradiction and the principle of Explosion, see Béziau and Franceschetto [8]). Paranormal systems are paraconsistent while having a negation that is not even a subcontrariety forming operator, given its rejection of the law of excluded middle. On what follows our arguments are restricted to systems characterized as encompassing negation as a subcontrariety forming operator, but the conclusions could be framed so as to encompass such paranormal systems also: given that one accepts that negation in such systems is not even subcontrariety, then one should not have trouble accepting that it does not represent a contradiction! On the other hand, the case of some paraconsistent classical logics such as Priest's LP involves other kinds of considerations. Some such systems seem to involve ambiguities in the treatment of their truth-values, as Béziau [4] and [7] has already pointed out. There is a kind of switch between 'Truth' and 'Designated value' which allows Priest in LP, for instance, to preserve the law of non-contradiction while rejecting explosion. As Slater [20] pointed, if we take 'Designated value' as meaning 'Truth', then LP's negation is also a subcontrariety forming operator, because expressions such as P and $\neg_p P$ may both have a model (and so the discussion of our paper applies). As Béziau [4] pointed out, however, if we confine ourselves and take as designated only what Priest calls 'Truth', then LP is not paraconsistent (because then P and $\neg_p P$ cannot have a model). So, by taking the designated values as encapsulating truth somehow, the discussion to follow applies to LP as well.

Bearing all these characterizations in mind, we now go on to discuss the peculiar applications of paraconsistent logics and check how well these logics deal with alleged cases of contradiction in natural languages. It is our goal to argue that even though one may properly call a pair of propositions P and $\neg_p P$ a contradiction from a syntactical point of view, a "paraconsistent contradiction" should be better understood as a *façon de parler* when it comes to deal with the extreme case of a "true contradiction", given that it does really capture a weaker kind of opposition: a subcontrariety. In fact, perhaps the syntactical form of a contradiction has been confused with another kind of opposition; in much less clear words, but with more impact: the form does not correspond to the content in this case.

3. Paraconsistency and Contradictions

3.1. Requirements for Successful Application

The general prototypical situation when a paraconsistent logic is thought to be required concerns cases where, in natural language, we are facing a contradiction, or something looking very much like a contradiction. Given that classical negation is explosive in the presence of contradictions, and, apparently, natural language is not trivial, a paraconsistent logic is called to deal more appropriately with the situation. That is, in order to keep rationality and to keep closer to the real world practices of speakers, we keep contradictions and avoid triviality by regimenting the natural language in a paraconsistent logic.

Notice that so far nothing has been said of a contradictory reality. In fact, if there are real contradictions then a paraconsistent logic also seems to be required, at least if we take into account the usual characterization of a paraconsistent logic mentioned before. But it seems that less is already enough. For instance, when we have a scientific theory with contradictory propositions, then, even if that theory is known to be false, or at least not to be the whole truth, we may need a paraconsistent logic to employ it without trivialization. Or so the argument goes.

To put the issue in a more precise way: consider that there is a natural language situation requiring logical regimentation (for the purpose of serious metaphysical work, for instance, or for other philosophical reasons). The situation in case comprises in general an argumentative or scientific context where a contradiction appears or seems to appear in natural or scientific discourse. Obviously, natural language may be regimented in formal languages in a plurality of ways. However, when contradictions appear, it seems, the required regimentation seems to demand that a paraconsistent negation be employed in the formalization of the contradiction. This would be very close to the real facts, because it accomplishes to encompass in the formal system two features of natural language in these cases:

Requirement 1: the regimentation keeps the contradiction and
Requirement 2: the regimentation keeps non-triviality.

That is, there is a natural flow of the informal information from natural language to the paraconsistent logic, and also from the paraconsistent logic to the natural language, without loss of this feature on either side. In particular, requirement 1 states that an expression that is deemed a contradiction in informal language is properly translated as a "paraconsistent contradiction" in an appropriate language for a paraconsistent system. In the same vein, the translations preserve information when going from formal language to natural languages. That is, the contradictory character of an expression or statement is preserved when we employ a paraconsistent negation.

But is that condition satisfied?

Our arguments in this section will be advanced to the purpose of showing that, even if at the surface of the grammar of formal languages expressions such as P and $\neg_p P$ seem to be a contradiction, they are not the best way to represent contradictions in natural languages and scientific contexts. We shall provide for examples of going in and out of formal languages preserving the properties of classical contradiction (that is, a pair P and $\neg_c P$) and paraconsistent "contradiction" (that is, expressions like P and $\neg_p P$). Our claim will be that at the formal level, the syntactical expression of paraconsistent contradictions looks very much like a contradiction, but when we analyze what was being required, they do not seem to capture the intuitive informal contradiction. In the end, as we have already mentioned, this will work for the benefit of paraconsistent logics, because an informal interpretation of paraconsistent negation will emerge, one which does not demand that we make sense of "true contradictions".

3.2. Becoming Paraconsistent

To begin with our analysis, we discuss the *first requirement* stated a few paragraphs ago, according to which paraconsistent logics really represent contradictions and preserve their meaning when we go from natural language to formal languages. We shall employ some of the definitions from Sect. 2. Recall that a contradiction involves two propositions, and in its semantic definition it was required that the propositions must have opposite truth values. Recall also that a paraconsistent negation does not generate a contradiction in the traditional sense, but rather a subcontrariety (which some may call a "paraconsistent contradiction", as we mentioned). In this sense, one could claim that a paraconsistent expression that resembles a contradiction, that is, an expression like P and $\neg_p P$, is in fact an extrapolation of the classical case by attempting only to syntactical similarities. That is, it could be argued that we are mistaken in taking such a pair of propositions as representing a contradiction, so that this representation is not close enough to the facts of natural language. By following this line of argument we agree with Béziau [6,7], who proposes that a paraconsistent logic is not a formal tool for reasoning in the presence of contradictions, but rather for systems in which explosion does not hold in general. That is, it does not deal primarily with contradictions, but with a restriction of the classical law that from a contradiction everything follows:

$$P, \neg_p P \nvdash Q.$$

Here, Q is any proposition whatever.

Let us examine now with more detail the suggestion encapsulated in the second requirement, that a paraconsistent negation does encompass a contradiction and restrict triviality. Obviously, the suggestion of the previous paragraph does have a broader impact on the role of paraconsistency and the associated demands of reason for non-triviality (as suggested by many authors, more recently by Šešelja and Straßer [19]). In fact, if the previous paragraph is correct, then paraconsistent logics do not really allow us to reason without triviality in the presence of contradictions. They do not really involve contradictions with expressions such as P and $\neg_p P$ (see, furthermore, Novaes [18]). There is no contradiction to begin with. What is really there is a restriction on explosion: when we have expressions such as P and $\neg_p P$, we do not always have that everything follows. But the meaning of the opposition holding between P and $\neg_p P$ does not behave as the required meaning for a contradiction. So, there is not a restriction of explosion *under contradictions* because we are not under a contradiction in this case.

In fact, recall that in some paraconsistent logics explosion is restored when we have a well-behaved proposition, or a consistent one. That is, when a paraconsistent negation is working as a classical negation explosion obtains. That works because in this case we do have a contradiction. Of course, the explanation is that this happens when the valuation under consideration is such that it is never the case that $v(P) = v(\neg_p P) = 1$. Recall that a paraconsistent valuation allows for such a situation, but does not require it. In cases when $v(P) = v(\neg_p P) = 1$ does not obtain for no valuation, the valuations for P and $\neg_p P$ are just like a classical valuation, and we do have a contradiction between P and $\neg_p P$. In fact, the classical negation is introduced in these paraconsistent systems by inserting a paraconsistent negation in front of a proposition and by claiming that the proposition is well-behaved (or consistent). So, in a sense we could say that when explosion obtains we do have a contradiction forming operator (from the semantical point of view), and when it does not, we do not have a contradiction, but a subcontrary (and explosion does not hold). Anyway, we do not reason under contradictions without triviality. Contradictions always imply triviality because when a contradiction obtains we are back to the classical case.

As we mentioned, those are pretty odd things to say about paraconsistency, given that everyone is already used to think about paraconsistent logics associated with taming contradictions. However, it is precisely this odd fact which will allow us later to make precise sense of the "epistemic interpretation" of contradictions by Carnielli and Rodrigues [9] and extend it in a plausible way. The effect, as we mentioned, is a liberation from worries about real or true contradictions.

Now, to make clearer the point that a paraconsistent negation does not really deal with true contradictions, we shall consider some cases of translating from natural language to formal language and back, to illustrate what the difference is between obtaining a contradiction and a subcontrariety. We hope this kind of move shall ground our previous claims and illustrate intuitively the difference between a paraconsistent negation and a contradiction in informal terms.

Suppose we are formalizing an argument that employs only the classical Aristotelian categorical propositions A, E, I and O. Furthermore, suppose someone says that in the argument we are supposed to be dealing with, two of the premises are contradictory, for instance, the following could be the premises:

P1: "Some man is sick" and

P2: "No man is sick".

If we were to use a propositional language (even acknowledging the limitations of such languages to deal with quantifiers), then, were someone to translate $P1$ by P, we could reasonably translate $P2$ by $\neg_c P$, with classical negation. That would settle the matter and provide for the correct formalization. On the other hand, had we formalized that argument with $P1$ as P and $P2$ as $\neg_p P$, with a paraconsistent negation, then, knowing that P stands for "some man is sick", we would have no choice but to read back $\neg_p P$ from the formal language to the natural language as standing for "some man is not sick", because *that reading* would preserve the correct semantical properties of the paraconsistent negation. However, as it is clear, that is no longer a contradiction, but rather subcontrariety. So, the contradiction is formalized when we use classical negation; when a paraconsistent negation is employed, another kind of opposition obtains, or, in other words, it works for the purpose of formalizing other kind of opposition.

That is, in going from a claim in natural language that would be properly called a contradiction, we do not seem advised to employ a paraconsistent negation to do the correct job. So, in going from natural language to a formal language the contradiction—in case there was one in natural language—is lost. On the other hand, by sticking to the case in hand, when we go from a "paraconsistent contradiction" in the formal language back to the natural language, the most natural translation (i.e. the one that would preserve the properties of paraconsistent negation), we do not return to a contradiction in natural language. So, in going from formal language to natural language we do not obtain a contradiction. Again, the formal and the informal are not in tune if we are to stick with contradictions everywhere.

But perhaps it is unfair to use propositional variables to translate such categorical propositions, and the above result could be thought as an artifact of this limitation. However, that is not the case. If we translate $P1$ by $\exists x(Mx \wedge Sx)$, how should we translate the contradictory of this proposition? Certainly by $\neg_c \exists x(Mx \wedge Sx)$, or, what is logically equivalent in classical logic, $\forall x(Mx \rightarrow \neg_c Sx)$. This could be justified by the fact that we want the properties characterizing contradiction to hold. How would we obtain the intuitive meaning of the paraconsistent contradiction of $P1$, that is, $\neg_p \exists x(Mx \wedge Sx)$? Well, to keep subcontrariety working, it would have to be understood as the corresponding O proposition, formalized traditionally as $\exists x(Mx \wedge \neg_c Sx)$. Of course, when we check the intuitive meanings of these propositions, we immediately see that there is no contradiction in the paraconsistent case. This reading is justified by the fact that in some quantificational paraconsistent logics, such as $C_1^\star$, the equivalence $\neg_p \exists x \alpha(x) \leftrightarrow \forall x \neg_p \alpha(x)$ does not hold, and the same happens for the other classical equivalences between existencial and universal quantifiers (see [14, p. 814]). So, the paraconsistent negation of a quantified expression like a categorical proposition does not take it to its contradictory, but rather to its subcontrary (the equivalences hold only for well-behaved propositions, that is, when negation is classical negation). As we see, the use of such categorical proposition may help us discerning between the syntactical expression and the intuitive meaning of the expression. Syntactically we seem to have a contradiction, but informally we do not have it, and the formalism of systems such as $C_1^\star$ seems to underwrite our interpretation.

Continuing with the case in hand, let us suppose that we have a particular case of a proposition A, such as "Every man is mortal" and a particular case of a proposition O, such as "Some man is not mortal". This is a contradiction, according to the traditional terminology, and as we have argued in the previous paragraph, it involves classical negation. Does everything follow from it in the theory of syllogism? That is, does explosion hold? Not really, if those two specific propositions are used as premisses, then we commit a fallacy of not employing three terms, and nothing whatever follows. So, classical syllogism does not allow us to infer that everything follows from a contradiction. That is,

$$A, O \nvdash_{Syll} P, \quad \text{for any proposition} \quad P, \quad \text{whatever.}$$

Also,

$$E, I \nvdash_{Syll} P, \quad \text{for any proposition} \quad P, \quad \text{whatever.}$$

Consider the paraconsistent analogue. Suppose again that we have propositions I like "Some man is mortal" and O like "some man is not mortal". This is the proper paraconsistent opposition. Again, not everything follows from this "paraconsistent contradiction", because of the same reason already exposed, and also because in classical syllogisms nothing follows from two existential premisses.

$$I, O \nvdash_{Syll} P, \quad \text{for any proposition} \quad P, \quad \text{whatever.}$$

What is really relevant for us here is that to have a classical contradiction when dealing with categorical propositions we must have a pair such as A and O or E and I. To have a "paraconsistent contradiction", on the other hand, we must have I and O. These are clearly distinct cases, and corroborate intuitively our thesis that a "paraconsistent contradiction" does not capture the meaning of a contradiction in the intuitive sense, at least not the classical contradiction we wish to formalize in most cases. So, when facing a contradiction we are not well-advised to formalize the contradictory propositions by using a paraconsistent negation, at least not, so far, when the propositions involved are categorical propositions. That is, requirement 1, as stated before, is not satisfied in this case. We shall soon try to make this a more general case by removing the restriction to categorical propositions.

Before we move ahead, we should make it clear that we are aware that this example with categorical propositions follows from the definitions presented in Sect. 2, but it serves the explicit purpose of providing determinate intuitive content to the oppositions captured by each kind of negation. As we are claiming, when one is faced with a contradiction in such a simple vocabulary as the one comprised by the four Aristotelian categorical propositions, one is in general talking about the typical contradictory pairs, and not about the subcontraries. However, the paraconsistent opposition captures only the last case.

Consider now another example, which will also help us in making our point clearer. Suppose we have two quantified propositions in natural language (an informal scientific theory or an informal argument, perhaps), "every object is such that A" and "some objects are such that A", for a property A whatever. Now, taking into account the semantic relations of Sect. 2, what would be the contradictory of "every object is such that A"? Surely "Not every object is such that A", or, "some objects are not A". This is the classical negation in action. What is the paraconsistent negation of "some objects are such that A"? Well, to keep the semantical properties of paraconsistent negation (subcontrariety) it cannot be "no objects are A" again (this would be classical negation), but rather "some objects are not A". This preserves the subcontrariety property (it keeps the possibility of both being true); however, this is again not a contradiction. That is, paraconsistent negations do not revert contradictions in natural language into contradictions in a formal language when they act as a subcontrariety forming operator; they have other purposes. If one wants to formalize a contradiction in natural language one must use classical negation, as these intuitive examples show.

Let us put some formal language in this example, just to make sure it will work correctly. Let us translate "Some objects are A" by the first-order sentence $\exists xA$. Now, we claim that the contradictory to this proposition is translated naturally as $\neg_c\exists xA$, which would be translated back to natural language as "No objects are A". Given that, let us consider the formula $\neg_p\exists xA$. In order to keep the subcontrariety property of paraconsistent negations, this formula would have to be translated as "Some objects are not A", which would not be a contradiction with the original "Some objects are A". So, even though from a syntactical point of view both $\neg_c\exists xA$ and $\neg_p\exists xA$ seem to be contradictory to $\exists xA$, only the first one really keeps the intuitive property of being a contradiction. That is, to translate contradictions, again considering this particular example, we are advised to choose classical negation, and not paraconsistent negation. So, once again, the requirement 1, stated above, seems to be violated. That is, in going from a contradiction in natural language to a syntactical expression that looks like a contradiction in a paraconsistent logic, one does not preserve the required feature of taking an informal contradiction to the formal domain. In a schematic fashion:

Affirmative: Some objects are A.
Contradiction: No objects are A.
Subcontrariety: Some objects are not A.

Those propositions are translated as:

Affirmative: $\exists xA$.
Contradiction: $\neg_c\exists xA$.
Subcontrariety: $\neg_p\exists xA$.

As we have already suggested, perhaps it is this syntactic form in common between contradiction and subcontrariety which prompts many to suppose that a paraconsistent negation generates a contradiction too.

One could have some doubts now concerning this last example. That is: how can we make sure that the negations above are the correct ones in each translation? In other words: how can we be so sure that when a contradiction is found in natural language, the correct way to formalize it is by using classical negation, and not the paraconsistent one? Aren't we begging the question? It doesn't seem so. Besides the fact that in $C_1^\star$, the equivalence $\neg_p\exists x\alpha(x) \leftrightarrow \forall x\neg_p\alpha(x)$ does not hold, we have another related argument for the plausibility of that translation. The new clue to our answer is the following one: as is well-known, statements using quantifiers, such as the ones employed in the previous examples, are the natural first-order correspondents for simple modal claims, but now translated in terms of the semantics of possible worlds. Recall that the modal proposition $\Diamond P$ has its truth values stated in terms of possible worlds in a Kripke-style semantics, so that the claim that this proposition is true is naturally translated in a first-order language as "there exists a possible world w in which P is the case" (considering the accessibility relation of $S5$, for instance). Similar translations relate the universal quantifier with the necessity operator. Now, as we know, from the modal square of opposition, $\Diamond P$ and $\Diamond\neg_c P$ are the corresponding subcontraries, so that $\Diamond\neg_c P$ represents a paraconsistent negation of $\Diamond P$ (see once again Béziau [3]). In this sense, our claim that the translations above are correct comes from an analogy with the modal case. In order to represent a subcontrariety, we must attain ourselves to pairs such as $\Diamond P$ and $\Diamond\neg_c P$, so that the negation is now internal to the modal operator. This corresponds naturally to conditions stated in quantified terms, so that, just as in the case of modal operators, the quantifier comes *before* the negation (which is classical, but in the scope of an operator or quantifier, it behaves like an outer paraconsistent negation; more on this soon). So, in the end the claim that there are no paraconsistent contradictions is correct, we always shift back to subcontrariety by introducing an internal negation, not an external one. It is the syntactic form of the formal apparatus that makes it look like a contradiction.

Before moving ahead, we must complete the exposition of the previous example with a remark on the case for the universal quantifier (or, for modal operators, for the necessity operator). Why have we spoken just about existential case? Well, as is well known, the case for the universal quantifier deals with paracomplete negation. In terms of the modal square, propositions $\Box P$ and $\Box\neg_c P$ are contraries, not contradictories and not subcontraries. The translation into universal quantifiers is made just as in the case of the existential quantifier before.

These arguments work to the purpose of showing that even though at the syntactical level expressions such as P and $\neg_p P$ look like a contradiction, they are not really a contradiction, rather encapsulating other kinds of semantical properties. Of course, one may, for the sake of generality, call such expressions as a "paraconsistent contradiction", but one must also not forget that they do not work properly when it comes to formalize intuitive contradictions. In this sense, the requirement 1, according to which a paraconsistent logic may be used fruitfully to translate contradictory statements fails. But more than that, what those arguments show us is how to provide for an intuitive semantics for paraconsistent logics. The next subsection shall elaborate this suggestion, while still furnishing evidence that paraconsistent negations do not deal with contradictions.

In some intuitive sense, in order to preserve the subcontrariety property, a paraconsistent pair P and $\neg_p P$ must be understood as involving some kind of external sentential operator, so that the negation is internal when the sentence is understood intuitively. That is, in general, external negations, such as the negation of "John said Mary is coming" *viz.* "it is not the case that John said

Mary is coming", are thought of as providing for contradictory propositions, so that they are better represented by a classical negation, or as a pair P and $\neg_c P$. On the other hand, internal negations are better represented by paraconsistent negations. Consider the sentences "John said Mary is coming" and "John said Mary is not coming". Now, this last pair could be better represented by P and $\neg_p P$, instead of classical negation P and $\neg_c P$. Once again we see the same pattern as before, repeating itself: when the negation is inside the scope of some operator or quantifier, it behaves in such a way that it could be formally translated as an external negation, but now encompassing an opposition that is weaker than contradiction, that is, subcontrariety. In schematic form, using $\natural$ as an operator for "John said that" and P for "Mary is coming":

Natural language	Formal translation
John said Mary is coming	$\natural P$
It is not the case that John said Mary is coming	$\neg_c \natural P$
John said Mary is not coming	$\natural \neg_c P$, or $\neg_p \natural P$

3.3. Interpreting Paraconsistent Negation

So, as a general suggestion, what we would take that atomic sentences such as "it rains" or "Plato is a philosopher" do not have natural paraconsistent negations (they are just too well-behaved for that), or else their paraconsistent negations turn out to be classical negations in those cases, because the scope of the negations is the same. The point is that they just cannot be the sentences that will form a true pair of P and $\neg_p P$ for an atomic P. On the other hand, sentences that are formalized as atomic propositions along with their paraconsistent negations have more structure than their formal counterparts allow us to see; *they always involve some external sentential operator or else some kind of quantifier, so that the negation is internal to these operators or quantifiers* (in the sense of being in the scope of these operators or quantifiers). This move allows us to keep the subcontrariety quality of the propositions involved. In this case, the idealization of the formal representation "veils" some of the structure of the sentence in natural language. It is as if there were always an attribution of a specific context to a proposition that is ignored or veiled by the formalization process in paraconsistent language.

This suggestion is made clear again when we analyze cases said to encompass contradictions and to require a paraconsistent logic, such as the one presented by Béziau [2].[2] Suppose dr. Bouvard tells John he has got cancer. Now, if the diagnostic is correct, John is going to die in three months. However, before going completely desperate, John goes on to hear from another doctor a second opinion, and he asks dr. Pécuchet whether that is really the case. Now, to make matters more complicated, dr. Pécuchet tells John that he has got no cancer. Are we facing a contradiction? Not at all. We have once again the internal negation operating in these propositions. The proposition involved is

[2] This paper is in French; the ideas presented in it are outlined in English in da Costa, Béziau and Bueno [13].

not the atomic "John has cancer", so that the negation of this sentence in this case is not "it is not the case that John has cancer" (or "John doesn't have cancer"). Rather, the following more involved propositions are in play now, including a context of utterance:

"*dr. Bouvard said that* John has cancer",

and

"*dr. Pécuchet said that* John does not have cancer".

So, by using two kinds of sentential operators like "dr. Bouvard said that ..." and "dr. Pecuchet said that ..." we may deal with the issue without having to invoke contradictions in the context of paraconsistency. Indeed, it would be unfair to the facts to translate the situation as involving the atomic claims "John has cancer" and "John doesn't have cancer", and it seems no one would think that this could be the case (a curious medical condition, even for contradiction lovers). However, given that the negation is internal to the mentioned operators, another option is to shift to a paraconsistent negation and use some paraconsistent logic, as the authors of the mentioned paper have done. The whole point, however, is that there is no contradiction to be dealt with by a paraconsistent logic, but it becomes a case of subcontrariety when negation is internal to some operator or quantifier as the case suggests. In this case, the opposition becomes subcontrariety, and a paraconsistent negation is applicable. Again, the syntactical form of the statements in the formal language makes it look like there is a contradiction involved, but there isn't. Also, the informal case only superficially involves a contradiction: the contradictory diagnoses come from distinct sources (more on this soon). As a remark: the contradictory from "dr. Bouvard said that John has cancer" is "it is not the case that dr. Bouvard said that John has cancer", and not "dr. Pécuchet said that John does not have cancer".

For a similar case in fiction (the above story is also fiction, of course, but it resembles close enough real situations), consider the famous story of Pinocchio.[3] In chapter 16, Pinocchio lays in bed between life and dead, and the Fairy calls for three doctors to have an accurate diagnostic. Two of the doctors, the Crow and the Owl, come to what seems to be a contradiction: the Crow declared him dead, while the Owl declared him alive. Of course, the diagnostic of the Owl contradicts the diagnostic of Crow, but is this a case in which we have Pinocchio both dead and alive? Not really. The statements in case involve something like a more specific context of utterance, like "the Crow declared Pinocchio dead" and "The Owl declared Pinocchio alive". These statements are not contradictory; both are true (inside the fiction, anyway), so that they form a subcontrary pair, amenable to paraconsistent treatment.

[3] The text may be found in http://it.wikisource.org/wiki/Le_avventure_di_Pinocchio/Capitolo_16.

Also, as we discover when the Talking Cricket pronounces himself, Pinocchio is in fact alive.[4]

As we mentioned, what this interpretational *manoeuvre* attests to is the plausibility of the "epistemic interpretation" of contradictions, suggested by Carnielli and Rodrigues [9, sec. 3]. Their suggestion, in general lines, is that a contradiction reflects merely our epistemic limitations in a given time. So, if we have contradictory information coming from distinct sources, this is not a case of a real contradiction, but a problem prompting further investigation. For instance, when we have claims like "Our theory $T1$ says that P" and "Our theory $T2$ says that not-P", we just recognize our epistemic limitations and continue to investigate what the case is. More investigation is required in order to put the issue on clear grounds, but provisorily we may employ a paraconsistent logic to deal with the situation. Notice, the "contradiction" is not really a contradiction, again, but mere subcontrariety: it is this fact that allows us to use a paraconsistent negation to capture what is going on. For instance, their suggestion that we take a pair of propositions P and $\neg_p P$ as intuitively claiming that "there is evidence that P is the case" and "there is evidence that P is not the case" really transforms a contradiction into a subcontrariety, and it is *that transformation* which makes paraconsistency applicable. So, the paraconsistent logics we are considering here do not naturally deal with contradiction, but rather with subcontrariety (as we already knew from the controversy with Slater, anyway).[5] There seems to be always a context or more inside which the contradiction appears, transforming it into a subcontrariety.

This interpretation of a paraconsistent negation as an internal negation encompasses even the discussion by Carnielli and Rodrigues [9] of paraconsistent logic as a logic to deal with group discussion, or a logic for collective reasoning. In fact, in a group discussion distinct members may have contradictory opinions, but, when considered as a group, we must take into account that it is distinct members that are having the contradictory opinions (of course, this was the main idea presented by Jaśkowski's discussive logic, see [14]). Now, the fact that we take into account the specific member that is expressing a specific opinion allows us to transform what would look like a contradiction into a subcontrariety, and so paraconsistent logic is applicable. That is, we do not go from simple statements P and not-P, but rather "member n says P" and "member m says not-P" (with $n \neq m$). Of course, this is not a contradiction, both statements can be true. Notice, the possibility of application of paraconsistent negation requires that we shift to a case of subcontrariety. The example of a group of referees deciding on a selection of papers fits our discussion quite well (see [9, p. 16]). When one referee pronounces herself as

[4] The idea that something can be dead and alive at the same time appears frequently in discussions of quantum superposition, with the case of Schrödinger's cat. While sealed inside a box, the cat is pronounced by some to be both dead and alive; see [15] for a paraconsistent approach to superpositions, and [16] for the same interpretation involving contradiction. For a critical discussion, see [1].

[5] Recall our delimitation in page 7.

favorable to a paper, and another is not-favorable, then, do we have a contradiction? Again, there is a subcontrariety: one referee is favorable, the *other* is not; that fixes the contexts of utterances of the contradictions, which then become a subcontrariety. No one is both favorable and not-favorable (*that* would be a contradiction).

Of course, this suggestion allows us to deal with the associated claim that we must use paraconsistent logics to cope with inconsistent information about the world, even if the latter itself is not contradictory. That claim falls precisely in our above treatment of internal negation: the claims involved in "contradictory information" are "we have information that P" and "we have information that not-P". Again, this is not really contradictory, but only subcontrariety. Suppose, for instance, that we have two sources of information, S_1 and S_2. The fact that they provide for contradictory information does not mean that we are dealing with contradictions. In fact, the propositions "According to S_1, the plane is on fire", and "According to S_2, the plane is not on fire". This is not a contradiction, as we have already mentioned,[6] but it may look like one when we formalize such claims such that we ignore the use of operators encompassing the source of the information (they are "abstracted away") and use a paraconsistent negation to deal with the issue. And this, of course, somehow hides the fact that the propositions are coming from distinct sources, putting negation as an external operator. However, as we suggested, introducing negation as an internal operator may suggest in many cases an informal semantics for paraconsistent logics.

Now, these claims placing paraconsistent logics in another context (where subcontrariety obtains, instead of contradiction) could also be invoked to evaluate the claim that some paraconsistent logics allow us to revise or refute the Aristotelian principle of non-contradiction. Leaving aside the fact that there is much discussion on what that principle really means and to what exactly does it apply to (see the discussion in da Costa [12, pp. 113–133]), one thing is certain: it says that contradictions are not true. So, when we provide for an analysis in terms of the square of opposition by attempting to the semantic properties of a contradiction, as we have seen in Sect. 2, it is classical negation that captures contradictions, and, by definition, they are not true. Use of the resources of a paraconsistent logic does not address the real issue, given that it deals with subcontrary propositions. So, to say that a paraconsistent pair such as P and $\neg_p P$ are a contradiction is to fall prey to an equivocation (recall our previous discussion of the restriction of the law of explosion: it is not the case that in the presence of a contradiction not everything follows, because when a contradiction obtains we are back to the classical case). Notice that this alleviates the burden on the paraconsistentist: she does not have to provide for examples of true contradictions to convince the sceptic. This is a challenge that needs not to be met, because it is not what is really at stake.

---

[6] Again, the contradictory to "According to S_1, the plane is on fire" is "It is not the case that according to S_1, the plane is on fire".

According to our informal interpretation of paraconsistent negation, also, there is always the introduction of a context to shift from a contradiction to a subcontrariety. This is precisely what is taken care of by the usual formulations of the law of non-contradiction: it is not possible for an object o to have and not to have a property P at the same time and under the same circumstances (see again da Costa [12, pp. 113–133]). Time and circumstances play the role of the context which we have been stressing, and they work for the purposes of avoiding contradiction. So, there is no violation of this law.

Perhaps one could object that by adhering to the right metaphysical thesis, some atomic propositions may turn out to be contradictory and the principle of non-contradiction may be seen as false; that is, it is all a matter of finding true contradictions. Dialetheists, in particular, would like to hold that the world violates the law of non-contradiction. However, as we have already anticipated, once such things were to be found, we would then discover that both a proposition and its negation are true. However, if that were the case, then, immediately we could be sure that we would not be facing a contradiction, but a subcontrariety: the negation must be paraconsistent in order for that to make sense. Adhering to a strict terminology makes life hard to Dialetheists: any instance favoring their thesis would be a change in subject. An analogy may make it clearer. Consider first-order logic with identity. As it is well-known, some models for such logic exist in which the identity sign is not interpreted as the diagonal of the domain of interpretation. However, no one claims that with that fact identity is violated; it is simply no longer identity *from a semantical point of view*, even though the identity sign is there in the formal language. The same should hold for a paraconsistent pair P and $\neg_p P$: it is just not contradictory, so that the law of non-contradiction is not violated when an expression like $(P \wedge \neg_p P)$ appears.

To complete this discussion on the law of non-contradiction and the Dialetheist, there is a further curious point. Dialetheists are eager to prove that some contradictions may be true. Also, most of them accept a paraconsistent classical logic as their favorite logic: Priest's LP. Now, in LP we have $\neg(P \wedge \neg P)$ as valid. So, how can some contradictions be true? Aren't they ruled out by the law of non-contradiction? Again, we face the ambiguity between "True" and "Designated value" (see Béziau [4]). While $(P \wedge \neg P)$ is never True in LP, it may sometimes receive a designated value, not being an anti-tautology (and neither a tautology). So, strictly speaking, from a syntactical point of view there is no violation of the law of non-contradiction in this case, but in the semantic sense, when we allow such a switching between Truth and Designated value, there may be an attribution to $(P \wedge \neg P)$ in which it is Designated, but not in which it is True. The Dialetheist slips from "having a designated value" to "being true", and so, it seems that the law is violated. However, as we have already mentioned, when we restrict ourselves to the Designated values LP's negation is only a subcontrariety forming operator.

4. Conclusion

We have argued for the thesis that paraconsistent logics do not deal directly with contradictions, in an intuitive but precise sense. We have provided for an intuitive grounding for this thesis by considering what would be the most natural translation of some expressions in formal languages to natural languages, and from natural languages to formal languages. In particular, we have argued that, in order to preserve the subcontrariety character of an expression like P and $\neg_p P$, we should not translate it as a contradiction. On the other direction, to go from a natural language assertion that seems to comprise a contradiction to a paraconsistent language, one seems required to make clear some contexts of utterance that transform the alleged contradiction in a subcontrariety; that accounts for the usual claim of dealing with "true contradictions". Of course, here "context of utterance" had to be understood very broadly.

Perhaps the main reason for people to keep thinking that a paraconsistent logic deals with contradictions comes from the use of expressions like P and $\neg_p P$ in formal languages. As Béziau [6] and [7] has emphatically remarked, a paraconsistent negation does not generate a contradiction, and perhaps paraconsistent logics should not be defined as dealing with contradictions anymore. To avoid confusion between a formal expression and its intended meaning, perhaps it could be useful to proceed as Horn and Wansing [17], who introduce a new symbol ⓒ for a contrary forming operator, a symbol that does not resemble classical negation, so that we are not tempted to call P and ⓒP a contradiction, even though ⓒP is still to be considered as a kind of negation of P. The same could be useful for paraconsistent negation, even though this would be considered by some as being just a cosmetic change. The benefits of this kind of move would be that we would not be tempted to associate an improper intuitive interpretation to expressions that look like a classical contradiction, but are not such from a semantical point of view.

In fact, keeping the idea of a contradiction always clear is also important when we evaluate the need for the application of a paraconsistent logic. This may help in curing us from paraconsistencitis, a disease famous for making people see contradiction everywhere. As a matter of fact, as part of the therapy, Béziau [7] has argued that the paradigmatic contradictory object, the infamous round–square, is not even a contradictory object; in fact, the attributions "x is square" and "x is round" encompass rather the opposition of contrariety. In the same vein, some attempts at clarifying the meaning of contradictions by invoking Buddhism, with statements like "all sentient beings have a Buddha nature" and "all sentient beings lack a Buddha nature" (see [10, p. 39]) are also just a case of contrariety. The same could be said of a famous box that is said to be both empty and full. Keeping in mind the correct distinctions may help us avoiding the labor of trying to deal with apparent contradictions.

As we hope to have shown, given that paraconsistent logics deal at best with subcontrariety, the need to transform what looks as contradictory statements into subcontrary ones by bringing to light the contexts of utterance is of utmost importance. By proceeding that way we keep a good match between the informal discourse and its formal counterpart, and we don't put contradictions where there are none. So, if that is correct, maybe contradictions will lose their prominence in the foundations of at least some of the paraconsistent logics, and we don't have to worry neither with providing an interpretation for them, nor with a metaphysics of contradiction. In the urge to show that a paraconsistent negation is really a negation paraconsistent logicians have conceded that it is at most a subcontrariety forming operator, but have not stopped talking about contradictions. Perhaps it is time to take that step and move ahead.

References

[1] Arenhart, J.R.B., Krause, D.: Contradiction, quantum mechanics, and the square of opposition. Forthcoming in Logique et Analyse (2015)

[2] Béziau, J.-Y.: La logique paraconsistante. In: N.C.A. da Costa, Logiques Classiques et Non Classiques (pp. 237–255) (French translation of [12] by J.-Y.Béziau with a preface and two appendices by the translator). Masson, Paris (1997)

[3] Béziau, J.-Y.: New light on the square of oppositions and its nameless corners. Log. Investig. **10**:218–232 (2003)

[4] Béziau, J.-Y.: Paraconsistent Logics! A reply to Slater. Sorites **17**:17–25 (2006)

[5] Béziau, J.-Y.: The power of the hexagon. Logica Universalis **6**(1–2):1–43 (2012)

[6] Béziau, J.-Y.: Contradiction and Negation. Talk delivered at the XVII EBL, Petrópolis, Brazil (2014)

[7] Béziau, J.-Y.: Round squares are no contradictions (Tutorial on Negation, Contradiction, and Opposition). In: Béziau, J.-Y., Chakraborty, M., Dutta, S. (eds.) New Directions in Paraconsistent Logic. Springer, New Delhi (2015)

[8] Béziau, J.-Y., Franceschetto, A.: Strong three-valued paraconsistent logics. In: Béziau, J.-Y., Chakraborty, M., Dutta, S. (eds.) New Directions in Paraconsistent Logic. Springer, New Delhi (2015)

[9] Carnielli, W., Rodrigues, A.: What contradictions say (and what they say not). Cle e-prints **12**(2) (2012)

[10] Carnielli, W., Coniglio, M.E.: On discourses addressed by infidel logicians. In: Tanaka, K., Berto, F., Mares, E., Paoli, F. (eds.) Paraconsistency: Logic and Applications (pp. 27–42). Logic, epistemology, and the unity of science 26. Springer, Dordrecht (2013)

[11] da Costa, N.C.A., The philosophical import of paraconsistent logics. CLE Manuscr. (1989)

[12] da Costa, N.C.A.: Ensaio sobre os fundamentos da lógica. 3rd ed (first edition: 1980). Hucitec, São Paulo (2008)

[13] da Costa, N.C.A., Béziau, J.-Y., Bueno, O.: Aspects of paraconsistent logic. Bull. IGPL **3**(4):597–614 (1995)

[14] da Costa, N.C.A., Krause, D., Bueno, O.: Paraconcistent logic and paraconsistency. In: Handbook of the Philosophy of Science (pp. 791–911). Volume 5: Philosophy of Logic. Volume editor: Dale Jacquette. Handbook editors: Dov M. Gabbay, Paul Thagard and and John Woods. Elsevier, Amsterdam (2006)

[15] da Costa, N.C.A., de Ronde, C.: The paraconsistent logic of superpositions. Found. Phys. **43**:854–858 (2013)

[16] Horn, L.R.: Contradiction. The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/spr2014/entries/contradiction/ (2014)

[17] Horn, L.R., Wansing, H.: Negation. In: The Stanford Encyclopedia of Philosophy (Spring 2015 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/spr2015/entries/negation/ (2015)

[18] Novaes, C.D.: Contradiction: the real philosophical challenge for paraconsistent logic. In: Béziau, J.-Y., Carnielli, W., Gabbay, D. (eds.) Handbook of Paraconsistency (pp. 465–480). Elsevier, Amsterdam (2008)

[19] Šešelja, D., Straßer, C.: Concerning Peter Vicker's recent treatment of 'Paraconsistencitis'. Int. Stud. Philos. Sci. **28**(3):325–340 (2014)

[20] Slater, B.H.: Paraconsistent logics? J. Philos. Logic **24**:451–454 (1995)

[21] Sylvan, R., Urbas, I.: Paraconsistent classical logic. Logique Anal. **141**(142):3–24 (1993)

Jonas R. Becker Arenhart
Department of Philosophy
Federal University of Santa Catarina
Florianópolis, Santa Catarina
Brazil
e-mail: jonas.becker2@gmail.com

Revised: July 1, 2015.
Accepted: October 6, 2015.