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Meir–Keeler type contractions on JS-metric spaces and related fixed point theorems

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Abstract. We introduce two classes of Meir–Keeler type contractions in the framework of JS-metric spaces introduced by Jleli and Samet (2015). For each class, a fixed point result is derived. Some interesting consequences which follow from our obtained results are discussed.

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1. Introduction

The Banach contraction principle is one of the most famous results on metric fixed point theory. There have been many generalizations and extensions of this principle in the literature. One of the important and remarkable generalizations is due to Meir and Keeler [11]. Their result can be stated as follows: let (X, d) be a complete metric space and let $T: X \to X$ be a given mapping. Suppose that for every $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$(x, y) \in X \times X,\ \varepsilon \le d(x, y) < \varepsilon + \delta(\varepsilon) \implies d(Tx, Ty) < \varepsilon. \qquad 2.1$$

Then, T has a unique fixed point $x^* \in X$. Moreover, for any $x \in X$, the Picard sequence $\{T^n x\}$ converges to x^*.

The class of Meir–Keeler contractions includes the class of Banach contractions and many other classes of nonlinear contractions (see, for example [4,10,16]). Meir and Keeler's theorem was source of further investigations in metric fixed point theory. For more details, we refer the reader to [1,3,7,10,14,18,22,23], and the references therein.

Recently, Jleli and Samet [8] introduced the notion of JS-metric spaces, which extends a number of abstract metric spaces: b-metric spaces [5], dislocated metric spaces [6], modular spaces with the Fatou property [12,13], etc. They also established some fixed point theorems in such spaces including the Banach contraction principle. Since then, the study of fixed points in

JS-metric spaces attracted the attention of some researchers. In [9], Karapinar et al. established some fixed point results under more general contractive conditions using a reflexive and transitive binary relation. In [20], Senapati et al. generalized the notion of F-contraction introduced by Wardowski [24] to JS-metric spaces. For other related results, see, for example [2,17,21], and the references therein.

In this paper, our aim is to obtain some extensions of the Meir–Keeler fixed point theorem to JS-metric spaces. We introduce two classes of Meir–Keeler type contractions and, for each class, we provide sufficient conditions for the existence of fixed points. Next, some interesting consequences are derived from our main results.

The paper is organized as follows. In Sect. 2, we recall the notion of JS-metric spaces and introduce two classes of Meir–Keeler type contractions in the framework of such spaces. Some examples of mappings that belong to the suggested classes are presented. Section 3 is devoted to state and prove the main results of this paper. In Sect. 4, some particular cases are discussed.

2. Preliminaries

Through this paper, we denote by $\mathbb{N}$ the set of natural numbers, that is, $\mathbb{N} = \{0, 1, 2, \ldots\}$. We denote by $\mathbb{N}^*$ the set $\mathbb{N}\backslash\{0\}$. We denote by $\mathbb{Z}$ the set of integers, that is, $\mathbb{Z} = \mathbb{N} \cup (-\mathbb{N})$.

We start this section with recapitulating some essential points of the concept of JS-metric spaces introduced in [8].

Let X be a nonempty set and let $D\colon X \times X \to [0, +\infty]$ be a given mapping. For every $x \in X$, we define the set

$$\mathcal{C}(D, X, x) = \left\{\{x_n\} \subset X\colon \lim_{n\to\infty} D(x_n, x) = 0\right\}.$$

Definition 2.1. We say that D is a JS-metric on X if the following conditions are satisfied:

(D1) $(x, y) \in X \times X,\ D(x, y) = 0 \implies x = y$.
(D2) $D(x, y) = D(y, x)$, for all $(x, y) \in X \times X$.
(D3) There exists $C > 0$ such that

$$(x, y) \in X \times X,\ \{x_n\} \in \mathcal{C}(D, X, x) \implies D(x, y) \leq C \limsup_{n\to+\infty} D(x_n, y).$$

In this case, the pair (X, D) is said to be a JS-metric space.

Definition 2.2. Let (X, D) be a JS-metric space.

(i) A sequence $\{x_n\} \subset X$ is said to be D-convergent to $x \in X$ if $\{x_n\} \in \mathcal{C}(D, X, x)$.
(ii) A sequence $\{x_n\} \subset X$ is said to be D-Cauchy if

$$\lim_{n,m\to+\infty} D(x_n, x_m) = 0.$$

(iii) (X, D) is D-complete if every D-Cauchy sequence in X is D-convergent to some element in X.

It was proved in [8] that the limit of a D-convergent sequence is unique, that is, for all $(x,y)\in X\times X$, we have

$$\mathcal{C}(D,X,x)\cap\mathcal{C}(D,X,y)\neq\emptyset \implies x=y.$$

Let (X,D) be a JS-metric space, and let Y be a nonempty subset of X. We denote by $\overline{Y}$ the closure of Y, that is,

$$y\in\overline{Y}\Leftrightarrow\exists\,\{y_n\}\subset Y\colon\ \lim_{n\to+\infty}D(y_n,y)=0.$$

Let $T\colon X\to X$ be a given mapping. We say that T is continuous on Y if

$$\{y_n\}\subset Y,\ \lim_{n\to+\infty}D(y_n,y)=0,\ y\in Y \implies \lim_{n\to+\infty}D(Ty_n,Ty)=0.$$

A large list of abstract metric spaces that can be seen as particular cases of JS-metric spaces can be found in [8]. For other examples, we refer to [9]. Now, we add another example of JS-metric spaces that will be used later.

Example 2.1. Let $X=\mathbb{N}^*$, and let $D\colon X\times X\to[0,+\infty]$ be defined as follows:

$$i\le j \implies D(i,j)=\begin{cases}\frac{1}{1+2i-j} & \text{if } j\le 2i,\\ 2-\frac{1}{1+j-2i} & \text{if } j>2i;\end{cases}$$

and

$$D(i,j)=D(j,i),\quad (i,j)\in X\times X.$$

We claim that

$$D(i,j)\ge\frac{1}{1+\min\{i,j\}},\quad (i,j)\in X\times X. \tag{2.1}$$

First, suppose that $i\le j$. If $j\le 2i$, then

$$D(i,j)=\frac{1}{1+2i-j}=\frac{1}{1+i+(i-j)}\ge\frac{1}{1+i}=\frac{1}{1+\min\{i,j\}}.$$

If $j>2i$, then

$$D(i,j)=2-\frac{1}{1+j-2i}\ge 1\ge\frac{1}{1+i}=\frac{1}{1+\min\{i,j\}}.$$

Therefore, (2.1) holds for every $(i,j)\in X\times X$ with $i\le j$. Now, if $i>j$, by symmetry, we have

$$D(i,j)=D(j,i)\ge\frac{1}{1+\min\{i,j\}}.$$

Hence, (2.1) holds for every pair $(i,j)\in X\times X$.

Next, we shall prove that (X,D) is a JS-metric space. It is clear that $D(i,j)>0$, for all $(i,j)\in X\times X$. Therefore, the condition (D1) is satisfied. The condition (D2) is satisfied by the definition of the mapping D. Further, let $i\in X$ be fixed, and suppose that $\{i_n\}\in\mathcal{C}(D,X,i)$, that is, $\{i_n\}$ is a sequence in X such that

$$\lim_{n\to+\infty}D(i_n,i)=0.$$

Then, for n large enough, we have

$$D(i_n, i) < \frac{1}{1+i}.$$

On the other hand, from (2.1), we have

$$D(i_n, i) \geq \frac{1}{1+\min\{i_n, i\}} \geq \frac{1}{1+i},$$

for all n, which is a contradiction. Hence, we deduce that $\mathcal{C}(D, X, i) = \emptyset$, for every $i \in X$. Then, the condition (D3) is also satisfied. Thus, we proved that (X, D) is a JS-metric space.

Next, we shall prove that X has no D-Cauchy sequences. We argue by contradiction by supposing that there exists a certain D-Cauchy sequence $\{i_n\}$ in X. We divide the proof into two cases.

Case 1: There exists k such that

$$i_n \leq 2i_k, \quad n > k.$$

In this case, the sequence $\{i_n\}$ has only finite different terms. Without loss of generality, we may assume that the finite pairwise distinct terms are $\{r_1, r_2, \ldots, r_p\}$. So, we obtain

$$D(i_n, i_m) \geq \min_{i,j=1,2,\ldots,p} D(r_i, r_j) > 0, \quad (n, m) \in \mathbb{N} \times \mathbb{N},$$

which leads to a contradiction.

Case 2: For any k, there exists $n_k > k$ such that

$$i_{n_k} > 2i_k.$$

By the definition of D, for all k, we have

$$\begin{aligned} D(i_k, i_{n_k}) &= 2 - \frac{1}{1+i_{n_k} - 2i_k} \\ &\geq 1, \end{aligned}$$

which leads to a contradiction.

Consequently, we conclude that X has no D-Cauchy sequences.

From the above study, we deduce that (X, D) is a D-complete JS-metric space.

Further, we shall prove additional properties of the JS-metric D that will be used later. First, we shall prove that

$$\{D(i, j)\colon\ i, j \in X\} = \{a_k\}_{k \in \mathbb{Z}}, \tag{2.2}$$

where

$$a_k = \begin{cases} \frac{1}{k+1} & \text{if } \ k \geq 0, \\ 1 + \frac{k}{k-1} & \text{if } \ k < 0. \end{cases}$$

From the definition of D, it can be easily seen that

$$\{D(i, j)\colon\ i, j \in X\} \subset \{a_k\}_{k \in \mathbb{Z}}.$$

Next, let $k \in \mathbb{Z}$ be fixed. If $k = 0$, then

$$a_0 = 1 = D(1, 2) \in \{D(i, j)\colon\ i, j \in X\}.$$

If $k > 0$, then

$$a_k = \frac{1}{k+1} = D(k,k) \in \{D(i,j)\colon\ i,j \in X\}.$$

If $k < 0$, then

$$a_k = 1 + \frac{k}{k-1} = D(1,-k+2) \in \{D(i,j)\colon\ i,j \in X\}.$$

Hence,

$$\{a_k\}_{k\in\mathbb{Z}} \subset \{D(i,j)\colon\ i,j \in X\}.$$

Therefore, (2.2) holds.

Next, we shall establish that

$$(i,j) \in X \times X,\ k \in \mathbb{Z},\ D(i,j) = a_k \implies D(i+1,j+1) = a_{k+1}. \tag{2.3}$$

Let $(i,j) \in X \times X$ be such that $D(i,j) = a_k$, $k \in \mathbb{Z}$. Without restriction of the generality, we may suppose that $i \le j$. We divide the proof into three cases.

Case 1: $k \ge 0$. If $j > 2i$, then

$$D(i,j) = 2 - \frac{1}{1+j-2i} = \frac{1}{k+1},$$

which yields

$$k = \frac{2i-j}{1+2j-4i} < 0,$$

which is a contradiction. Therefore, we have $j \le 2i$ and

$$D(i,j) = \frac{1}{1+2i-j} = a_k = \frac{1}{k+1}.$$

So, $2i - j = k$ and $0 \le k \le i$. Note that $i+1 \le j+1 \le 2(i+1)$ and $2(i+1)-(j+1) = k+1$. Thus, we have

$$D(i+1,j+1) = \frac{1}{1+2(i+1)-(j+1)} = \frac{1}{(k+1)+1} = a_{k+1}.$$

Case 2: $k \le -2$. In this case, $j > 2i$ and

$$2 - \frac{1}{1+j-2i} = D(i,j) = a_k = 1 + \frac{k}{k-1} = 2 + \frac{1}{k-1}.$$

So, $j - 2i = -k \ge 2$. Note that $j+1 > 2(i+1)$ and $2(i+1)-(j+1) = k+1$. Then

$$\begin{aligned} D(i+1,j+1) &= 2 - \frac{1}{1+(j+1)-2(i+1)} \\ &= 2 - \frac{1}{1-(k+1)} = 2 + \frac{1}{k} = a_{k+1}. \end{aligned}$$

Case 3: $k = -1$. In this case, $j > 2i$ and

$$2-\frac{1}{1+j-2i}=D(i,j)=a_{-1}=1+\frac{-1}{-1-1}=\frac{3}{2}.$$

So, $j-2i=1$. Note that $j+1=2(i+1)$. Therefore,

$$D(i+1,j+1)=2-\frac{1}{1+(j+1)-2(i+1)}=1=a_0.$$

Therefore, (2.3) holds.

Let (X,D) be a JS-metric space. Let $T\colon X\to X$ be a certain self-mapping on X. For $n\in\mathbb{N}$, we denote by T^n the nth iterates of T (it is supposed that T^0 is the identity mapping on X).

We introduce the following concepts that will be used later.

Definition 2.3. We say that $T\colon X\to X$ is a strong Meir–Keeler contraction on a nonempty subset Y of X, if there exists a mapping $\delta\colon(0,+\infty)\to(0,+\infty)$ such that for every $\varepsilon>0$, we have

$$(x,y)\in Y\times Y,\ \varepsilon\le D(x,y)<\varepsilon+\delta(\varepsilon)\implies D(Tx,Ty)<\varepsilon \tag{2.4}$$

and

$$R>0\implies 0<\liminf_{r\uparrow R}\delta(r)<+\infty. \tag{2.5}$$

We have the following property concerning the class of strong Meir–Keeler contraction mappings.

Lemma 2.1. *Let $T\colon X\to X$ be a strong Meir–Keeler contraction on a nonepmty subset Y of X. Then*

$$(x,y)\in Y\times Y,\, 0<D(x,y)\le+\infty\implies D(Tx,Ty)\le D(x,y).$$

Proof. Let $(x,y)\in Y\times Y$. If $D(x,y)=+\infty$, then the desired inequality is trivial. If $0<D(x,y)<+\infty$, taking $\varepsilon=D(x,y)$, and since

$$\varepsilon\le D(x,y)<\varepsilon+\delta(\varepsilon),$$

we obtain from (2.4) that

$$D(Tx,Ty)<\varepsilon=D(x,y).$$

□

Further, we present some examples of strong Meir–Keeler contractions.

Example 2.2. Let $T\colon X\to X$ be a k-contraction on a certain nonempty subset Y of X, that is,

$$D(Tx,Ty)\le kD(x,y),\quad (x,y)\in Y\times Y,$$

where $0<k<1$. Then, T is a strong Meir–Keeler contraction on Y. Indeed, for any $\varepsilon>0$, we have

$$(x,y)\in Y\times Y,\ \varepsilon\le D(x,y)<\varepsilon+\left(\frac{1}{k}-1\right)\varepsilon\implies D(Tx,Ty)<\varepsilon.$$

Therefore, (2.4) is satisfied with

$$\delta(\varepsilon)=\left(\frac{1}{k}-1\right)\varepsilon,\quad \varepsilon>0.$$

Moreover, for any $R > 0$, we have

$$\liminf_{r \uparrow R} \delta(r) = \left(\frac{1}{k} - 1\right) R > 0.$$

Therefore, (2.5) is satisfied.

Example 2.3. Let Φ be the set of functions $\varphi\colon [0, +\infty) \to [0, +\infty)$ satisfying the following conditions:

(Φ_1) $\varphi(0) = 0$.
(Φ_2) $\varphi(t) > 0$, for all $t > 0$.
(Φ_3) There exists $\delta\colon (0, +\infty) \to (0, +\infty)$ such that

$$R > 0 \implies 0 < \liminf_{r \uparrow R} \delta(r) < +\infty,$$

and for every $s > 0$,

$$s \leq t \leq s + \delta(s) \implies \varphi(t) \leq s.$$

Note that Φ is a subset of the class of L-functions introduced by Lim [10]. Next, let $T\colon X \to X$ be a mapping satisfying

$$(x, y) \in Y \times Y,\ 0 < D(x, y) < +\infty \implies D(Tx, Ty) < \varphi(D(x, y)),$$

where Y is a nonempty subset of X and $\varphi \in \Phi$. We claim that T is a strong Meir–Keeler contraction on Y. In order to prove this claim, let us fix a certain $\varepsilon > 0$. Let $(x, y) \in Y \times Y$ be such that

$$\varepsilon \leq D(x, y) < \varepsilon + \delta(\varepsilon).$$

Then

$$\varphi(D(x, y)) \leq \varepsilon,$$

which yields

$$D(Tx, Ty) < \varphi(D(x, y)) \leq \varepsilon.$$

Therefore, T is a strong Meir–Keeler contraction on Y.

Remark 2.1. As an example of functions that belong to the set Φ, we can take

$$\varphi(t) = kt, \quad t \geq 0,$$

where $0 < k < 1$ is a constant. It can be seen that the conditions (Φ_1), (Φ_2) and (Φ_3) are satisfied with

$$\delta(s) = \left(\frac{1}{k} - 1\right) s, \quad s > 0.$$

Given $p \in \mathbb{N}^*$, let

$$\begin{aligned} &M_{T,p}(x, y) \\ &= \max\{D(T^i x, T^j y), D(T^i x, T^j x), D(T^i y, T^j y)\colon\ 0 \leq i, j \leq p\}, \end{aligned}$$

for all $(x, y) \in X \times X$.

Definition 2.4. We say that $T\colon X \to X$ is a strong generalized Meir–Keeler contraction on a nonempty subset Y of X, if there exists a mapping $\delta\colon (0,+\infty) \to (0,+\infty)$ such that for every $\varepsilon > 0$, we have

$$(x,y) \in Y \times Y,\ \varepsilon \le M_{T,p}(x,y) < \varepsilon + \delta(\varepsilon) \implies D(T^p x, T^p y) < \varepsilon \tag{2.6}$$

where $p = 1, 2, \ldots,$ and

$$R > 0 \implies 0 < \liminf_{r \uparrow R} \delta(r) < +\infty.$$

Lemma 2.2. *Let $T\colon X \to X$ be a strong generalized Meir–Keeler contraction on a nonepmty subset Y of X. Then*

$$D(T^p x, T^p y) \le M_{T,p}(x,y), \quad (x,y) \in Y \times Y.$$

Proof. Let $(x,y) \in Y \times Y$. We discuss three possible cases.

Case 1: $M_{T,p}(x,y) = +\infty$. In this case, obviously, we have

$$D(T^p x, T^p y) \le +\infty = M_{T,p}(x,y).$$

Case 2: $M_{T,p}(x,y) = 0$. In this case, we have $D(x,y) = 0$, which implies from the property (D1) that $x = y$. Therefore, we have

$$D(T^p x, T^p y) = D(T^p x, T^p x) \le M_{T,p}(x,y).$$

Case 3: $0 < M_{T,p}(x,y) < +\infty$. In this case, taking $\varepsilon = M_{T,p}(x,y)$, and since

$$\varepsilon \le M_{T,p}(x,y) < \varepsilon + \delta(\varepsilon),$$

we obtain

$$D(T^p x, T^p y) < \varepsilon = M_{T,p}(x,y).$$

□

Example 2.4. Let $T\colon X \to X$ be a generalized k-contraction on a certain nonempty subset Y of X, that is,

$$D(T^p x, T^p y) \le k M_{T,p}(x,y), \quad (x,y) \in Y \times Y,$$

where $0 < k < 1$. Then, T is a strong generalized Meir–Keeler contraction on Y.

Example 2.5. Let $T\colon X \to X$ be a mapping satisfying

$$(x,y) \in Y \times Y,\, 0 < M_{T,p}(x,y) < +\infty \implies D(T^p x, T^p y) < \varphi(M_{T,p}(x,y)),$$

where Y is a nonempty subset of X and $\varphi \in \Phi$. Then, T is a strong generalized Meir–Keeler contraction on Y. The proof is similar to that given in Example 2.3.

3. Main results

In this section, we state and prove our main results. First, let us fix some notations that will be used through this section.

Let $T\colon X \to X$ be a given mapping and let $x_0 \in X$. We denote by $O_T(x_0)$ the subset of X defined by

$$O_T(x_0) = \{T^n x_0 \colon\ n \in \mathbb{N}\}.$$

Let

$$\delta(D, T, x_0) = \sup\{D(T^i x_0, T^j x_0) \colon\ i, j \in \mathbb{N}\}.$$

For $n \in \mathbb{N}$, let

$$\delta_n = \sup\{D(T^i x_0, T^j x_0) \colon\ i, j \geq n\}.$$

3.1. Existence of fixed points for the class of strong Meir–Keeler contractions

The following lemma will be useful later.

Lemma 3.1. *Let (X, D) be a JS-metric space and let $T\colon X \longrightarrow X$ be a strong Meir–Keeler contraction on $O_T(x_0)$, where $x_0 \in X$. Suppose that*

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ D(x, y) = 0 \implies D(Tx, Ty) = 0. \tag{3.1}$$

If $\delta_n \not\to 0$ as $n \to +\infty$, then there exists $N \in \mathbb{N}^*$ and $\Delta \in (0, +\infty]$ such that

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq N.$$

Proof. We divide the proof into several cases.

Case 1: For any $n \in \mathbb{N}$, $\delta_n = +\infty$. In this case, we only need to take $N = 1$ and $\Delta = +\infty$.

Case 2: There exists $n_0 \in \mathbb{N}$ such that $\delta_{n_0} < +\infty$. In this case, we have

$$\delta_{n+1} \leq \delta_n \leq \delta_{n_0} < +\infty, \quad n \geq n_0.$$

Then, there exists some $\Delta > 0$ such that

$$\delta_n \downarrow \Delta \text{ as } n \to +\infty. \tag{3.2}$$

Since T is a strong Meir–Keeler contraction on $O_T(x_0)$, there exists $\delta := \delta_\Delta > 0$ such that

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ \Delta \leq D(x, y) < \Delta + \delta \implies D(Tx, Ty) < \Delta,$$

that is,

$$\Delta \leq D(T^k x_0, T^l x_0) < \Delta + \delta \implies D(T^{k+1} x_0, T^{l+1} x_0) < \Delta, \quad (k, l) \in \mathbb{N} \times \mathbb{N}. \tag{3.3}$$

On the other hand, from (3.2), there exists $n_1 > n_0$ such that

$$\Delta \leq \delta_n < \Delta + \delta, \quad n \geq n_1.$$

Case 2.1: $\delta_{n_1} = \Delta$. In this case, we have

$$\Delta \leq \delta_n \leq \delta_{n_1} = \Delta, \quad n \geq n_1.$$

Therefore,

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq n_1.$$

Case 2.2: $\Delta < \delta_{n_1} < \Delta + \delta$. Let

$$A_{n_1} = \{(k,l)\colon\ \Delta < D(T^k x_0, T^l x_0) < \Delta + \delta,\ k \geq n_1,\ l \geq n_1\},$$
$$B_{n_1} = \{(k,l)\colon\ 0 < D(T^k x_0, T^l x_0) \leq \Delta,\ k \geq n_1,\ l \geq n_1\},$$

and

$$C_{n_1} = \{(k,l)\colon\ D(T^k x_0, T^l x_0) = 0,\ k \geq n_1,\ l \geq n_1\},$$

Let $(k,l) \in \mathbb{N} \times \mathbb{N}$ be such that $k \geq n_1$ and $l \geq n_1$. If $(k,l) \in A_{n_1}$, then from (3.3) we have

$$D(T^{k+1} x_0, T^{l+1} x_0) < \Delta.$$

If $(k,l) \in B_{n_1}$, by Lemma 2.1, we have

$$D(T^{k+1} x_0, T^{l+1} x_0) \leq D(T^k x_0, T^l x_0) \leq \Delta.$$

If $(k,l) \in C_{n_1}$, then by (3.1), we have

$$D(T^{k+1} x_0, T^{l+1} x_0) = 0 < \Delta.$$

Therefore,

$$D(T^i x_0, T^j x_0) \leq \Delta, \quad i,j \geq n_1 + 1,$$

which yields

$$\delta_{n_1+1} = \Delta,$$

and thus by the same reason stated in Case 2.1, we have

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq n_1 + 1.$$

The lemma is proved. □

We have the following fixed point result concerning the class of strong Meir–Keeler contractions.

Theorem 3.1. *Let (X,D) be a complete JS-metric space and let $T\colon X \to X$ be a strong Meir–Keeler contraction on $\overline{O_T(x_0)}$, for some $x_0 \in X$, satisfying*

$$(x,y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)},\ D(x,y) = 0 \implies D(Tx,Ty) = 0. \tag{3.4}$$

Suppose that $\delta(D,T,x_0) < +\infty$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T. Moreover, if $\omega' \in \overline{O_T(x_0)}$ is a fixed point of T such that $D(\omega,\omega') < +\infty$, then $\omega = \omega'$.

Proof. We claim that

$$\lim_{n\to+\infty} \delta_n = 0. \tag{3.5}$$

Suppose the contrary, then by Lemma 3.1, there exists $N \in \mathbb{N}^*$ and $\varepsilon \in (0,+\infty]$ such that

$$\delta_n = \delta_{n+1} = \cdots = \varepsilon, \quad n \geq N. \tag{3.6}$$

Since $\delta(D,T,x_0) < +\infty$, then $0 < \varepsilon < +\infty$. Let

$$c = \liminf_{r\uparrow\varepsilon} \delta(r).$$

From (2.5), we know that $0 < c < +\infty$. Then, there exists some $\mu > 0$ such that

$$\delta(r) > \frac{c}{2}, \quad r \in (\varepsilon - \mu, \varepsilon) \cap (0, +\infty).$$

Let $\delta' = \min\left\{\mu, \frac{c}{2}\right\}$. Then, for any $r \in (\varepsilon - \delta', \varepsilon) \cap (0, +\infty)$, we have

$$\delta' < \delta(r),$$

and thus

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ r \le D(x, y) < r + \delta' \implies D(Tx, Ty) < r. \quad (3.7)$$

Now, let $n \ge N$ be fixed, and let $r \in (\varepsilon - \delta', \varepsilon) \cap (0, +\infty)$ be fixed. Let $(k, l) \in \mathbb{N}^* \times \mathbb{N}^*$ be such that $k \ge n$ and $l \ge n$. From (3.6), we have

$$D(T^k x_0, T^l x_0) \le \varepsilon.$$

We distinguish two cases.

Case 1: $r < D(T^k x_0, T^l x_0) \le \varepsilon$. In this case, from (3.7), we obtain

$$D(T^{k+1} x_0, T^{l+1} x_0) < r.$$

Case 2: $D(T^k x_0, T^l x_0) \le r$. In this case, using Lemmas 2.1 and (3.4), we obtain

$$D(T^{k+1} x_0, T^{l+1} x_0) \le D(T^k x_0, T^l x_0) \le r.$$

In consequence, we deduce that

$$\delta_{n+1} \le r < \varepsilon, \quad n \ge N,$$

which contradicts (3.6). Hence, (3.5) holds.

Next, let $\alpha > 0$ be fixed. From (3.5), there exists some $q \in \mathbb{N}$ such that

$$n \ge q \implies \delta_n < \alpha.$$

Therefore, we have

$$i, j \ge q \implies D(T^i x_0, T^j x_0) \le \delta_q < \alpha.$$

Then

$$\lim_{i,j \to +\infty} D(T^i x_0, T^j x_0) = 0,$$

which proves that $\{T^n x_0\}$ is a D-Cauchy sequence. Since (X, D) is D-complete, there exists some $\omega \in \overline{O_T(x_0)}$ such that

$$\lim_{n \to +\infty} D(T^n x_0, \omega) = 0. \quad (3.8)$$

On the other hand, by Lemmas 2.1 and (3.4), we have

$$0 \le D(T^{n+1} x_0, T\omega) \le D(T^n x_0, \omega), \quad n \in \mathbb{N}.$$

Passing to the limit as $n \to +\infty$ and using (3.8), we get

$$\lim_{n \to +\infty} D(T^{n+1} x_0, T\omega) = 0.$$

By uniqueness of the limit, we obtain $\omega = T\omega$, i.e., ω is a fixed point of T.

Next, suppose that $\omega' \in \overline{O_T(x_0)}$ is a fixed point of T such that $D(\omega, \omega') < +\infty$. If $D(\omega, \omega') > 0$, then by Lemma 2.1 (see the proof of Lemma 2.1 in the case $0 < D(x, y) < +\infty$) we have

$$D(\omega, \omega') = D(T\omega, T\omega') < D(\omega, \omega'),$$

which is a contradiction. Therefore, $D(\omega, \omega') = 0$, which implies from the property (D1) that $\omega = \omega'$. □

Note that in the absence of the condition (2.5), the result given by Theorem 3.1 is not valid. The following example shows this fact.

Example 3.1. Let (X, D) be the JS-metric space defined in Example 2.1. We proved previously that (X, D) is a D-complete JS-metric space. Define the mapping $T: X \to X$ by

$$Ti = i + 1, \quad i \in X.$$

Note that for any $(i, j) \in X \times X$, we have $D(i, j) > 0$. Therefore, (3.4) is satisfied. Moreover, for any $(i, j) \in X \times X$, we have $D(i, j) < 2$. Therefore,

$$\delta(D, T, i) < 2 < +\infty, \quad i \in X.$$

Next, let $\varepsilon > 0$ be fixed. We consider two possible cases.

Case 1: $\varepsilon \geq 2$. In this case, there are no $(i, j) \in X \times X$ satisfying $D(i, j) \geq \varepsilon$.

Case 2: $0 < \varepsilon < 2$. In this case, it can be easily seen that

$$a_{k(\varepsilon)+1} < \varepsilon \leq a_{k(\varepsilon)},$$

where $k(\varepsilon) \in \mathbb{Z}$ is given by

$$k(\varepsilon) = \begin{cases} \left[\frac{1}{\varepsilon}\right] - 1 & \text{if } 0 < \varepsilon \leq 1, \\ \left[1 - \frac{1}{2-\varepsilon}\right] & \text{if } 1 < \varepsilon < 2. \end{cases}$$

Let $\delta: (0, +\infty) \to (0, +\infty)$ be the function defined by

$$\delta(r) = \begin{cases} a_{k(r)-1} - r & \text{if } 0 < r < 2, \\ 3 - r & \text{if } 2 \leq r < 3, \\ c & \text{if } r > 3, \end{cases}$$

where $c > 0$ is a fixed real number. Next, if

$$\varepsilon \leq D(i, j) < \varepsilon + \delta(\varepsilon) = a_{k(\varepsilon)-1},$$

then $D(i, j) = a_{k(\varepsilon)}$, which implies from (2.3) that

$$D(Ti, Tj) = D(i + 1, j + 1) = a_{k(\varepsilon)+1} < \varepsilon.$$

In consequence, we deduce that for any $\varepsilon > 0$, we have

$$(i, j) \in X \times X, \, \varepsilon \leq D(i, j) < \varepsilon + \delta(\varepsilon) \implies D(Ti, Tj) < \varepsilon.$$

Therefore, (2.4) is satisfied with $Y = \overline{O_T(i)}$, for any $i \in X$. On the other hand, we have

$$\liminf_{r \uparrow 3} \delta(r) = 0.$$

Thus, the condition (2.5) is not satisfied. Note that the set of fixed points of T is empty.

3.2. Existence of fixed points for the class of strong generalized Meir–Keeler contractions

Lemma 3.2. *Let (X, D) be a JS-metric space and let $T: X \longrightarrow X$ be a strong generalized Meir–Keeler contraction on $O_T(x_0)$, where $x_0 \in X$. If $\delta_n \not\to 0$ as $n \to +\infty$, then there exists $N \in \mathbb{N}^*$ and $\Delta \in (0, +\infty]$ such that*

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq N.$$

Proof. As in the proof of Lemma 3.1, without restriction of the generality, we may suppose that there exists $n_0 \in \mathbb{N}$ such that $\delta_{n_0} < +\infty$. In this case, we have

$$\delta_{n+1} \leq \delta_n \leq \delta_{n_0} < +\infty, \quad n \geq n_0.$$

Then, there exists some $\Delta > 0$ such that

$$\delta_n \downarrow \Delta \text{ as } n \to +\infty.$$

Since T is a strong generalized Meir–Keeler contraction on $O_T(x_0)$, there exists $\delta := \delta_\Delta > 0$ such that

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ \Delta \leq M_{T,p}(x, y) < \Delta + \delta \implies D(T^p x, T^p y) < \Delta,$$

that is, for all $(k, l) \in \mathbb{N} \times \mathbb{N}$,

$$\begin{aligned} &\Delta \leq \max\{D(T^i x_0, T^j x_0);\ (i, j) \in [k, k+p] \cup [l, l+p]\} \qquad < \Delta + \delta \\ &\implies D(T^{k+p} x_0, T^{l+p} x_0) < \Delta. \end{aligned} \tag{3.9}$$

On the other hand, there exists $n_1 > n_0$ such that

$$\Delta \leq \delta_n < \Delta + \delta, \quad n \geq n_1.$$

We distinguish two possible cases.

Case 1: $\delta_{n_1} = \Delta$. In this case, we have

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq n_1.$$

Case 2: $\Delta < \delta_{n_1} < \Delta + \delta$. Let

$$\begin{aligned} A_{n_1} &= \{(k, l):\ \Delta < M_{T,p}(T^k x_0, T^l x_0) < \Delta + \delta,\ k \geq n_1,\ l \geq n_1\}, \\ B_{n_1} &= \{(k, l):\ 0 \leq M_{T,p}(T^k x_0, T^l x_0) \leq \Delta,\ k \geq n_1,\ l \geq n_1\}. \end{aligned}$$

Let $(k, l) \in \mathbb{N} \times \mathbb{N}$ be such that $k \geq n_1$ and $l \geq n_1$. If $(k, l) \in A_{n_1}$, then from (3.9) we have

$$D(T^{k+p} x_0, T^{l+p} x_0) < \Delta.$$

If $(k, l) \in B_{n_1}$, by Lemma 2.2, we have

$$D(T^{k+p} x_0, T^{l+p} x_0) \leq M_{T,p}(T^k x_0, T^l x_0) \leq \Delta.$$

Therefore,

$$D(T^i x_0, T^j x_0) \leq \Delta, \quad i, j \geq n_1 + p,$$

which yields

$$\delta_{n_1+p} = \Delta,$$

and thus by the same reason stated in Case 1, we have

$$\delta_n = \delta_{n+1} = \cdots = \Delta, \quad n \geq n_1 + p.$$

The lemma is proved. □

We have the following fixed point result for the class of strong generalized Meir–Keeler contractions.

Theorem 3.2. *Let (X, D) be a complete JS-metric space, and let $T\colon X \to X$ be a strong generalized Meir–Keeler contraction on $O_T(x_0)$, for some $x_0 \in X$. Suppose that $\delta(D, T, x_0) < +\infty$ and T is continuous on $\overline{O_T(x_0)}$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T.*

Proof. Suppose that $\delta_n \not\to 0$ as $n \to +\infty$. Then by Lemma 3.2, there exists $N \in \mathbb{N}^*$ and $\varepsilon \in (0, +\infty]$ such that

$$\delta_n = \delta_{n+1} = \cdots = \varepsilon, \quad n \geq N.$$

Since $\delta(D, T, x_0) < +\infty$, then $0 < \varepsilon < +\infty$. Let

$$c = \liminf_{r \uparrow \varepsilon} \delta(r).$$

From (2.5), we know that $0 < c < +\infty$. Then, there exists some $\mu > 0$ such that

$$\delta(r) > \frac{c}{2}, \quad r \in (\varepsilon - \mu, \varepsilon) \cap (0, +\infty).$$

Let $\delta' = \min\left\{\mu, \frac{c}{2}\right\}$. Then, for any $r \in (\varepsilon - \delta', \varepsilon) \cap (0, +\infty)$, we have

$$\delta' < \delta(r),$$

and thus

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ r \leq M_{T,p}(x, y) < r + \delta' \implies D(T^p x, T^p y) < r.$$

Now, let $n \geq N$ be fixed, and let $r \in (\varepsilon - \delta', \varepsilon) \cap (0, +\infty)$ be fixed. Let $(k, l) \in \mathbb{N}^* \times \mathbb{N}^*$ be such that $k \geq n$ and $l \geq n$. Then, we have

$$M_{T,p}(T^k x_0, T^l x_0) \leq \delta_n = \varepsilon.$$

We distinguish two cases.

Case 1: $r < M_{T,p}(T^k x_0, T^l x_0) \leq \varepsilon$. In this case, we obtain

$$D(T^{k+p} x_0, T^{l+p} x_0) < r.$$

Case 2: $M_{T,p}(T^k x_0, T^l x_0) \leq r$. In this case, using Lemma 2.2, we obtain

$$D(T^{k+p} x_0, T^{l+p} x_0) \leq M_{T,p}(T^k x_0, T^l x_0) \leq r.$$

In consequence, we deduce that

$$\varepsilon = \delta_{n+p} \leq r < \varepsilon, \quad n \geq N,$$

which is a contradiction. Therefore, we proved that

$$\lim_{n \to +\infty} \delta_n = 0,$$

which implies that $\{T^n x_0\}$ is a D-Cauchy sequence. Since (X, D) is D-complete, there exists some $\omega \in \overline{O_T(x_0)}$ such that

$$\lim_{n\to+\infty} D(T^n x_0, \omega) = 0.$$

Since T is continuous on $\overline{O_T(x_0)}$, we have

$$\lim_{n\to+\infty} D(T^{n+1} x_0, T\omega) = 0,$$

ss which implies by the uniqueness of the limit that $\omega = T\omega$, i.e., ω is a fixed point of T. □

4. Some consequences

In this section, some fixed point results are deduced from the obtained results in Sect. 3.

The following results are consequences of Theorem 3.1.

Corollary 4.1. *Let (X, D) be a complete JS-metric space, and let $T: X \to X$ be a k-contraction on $\overline{O_T(x_0)}$, for some $x_0 \in X$, that is,*

$$D(Tx, Ty) \le kD(x, y), \quad (x, y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)},$$

where $k \in (0, 1)$ is a constant. Suppose that $\delta(D, T, x_0) < +\infty$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T. Moreover, if $\omega' \in \overline{O_T(x_0)}$ is a fixed point of T such that $D(\omega, \omega') < +\infty$, then $\omega = \omega'$.

Proof. First, it can be easily seen that the condition (3.4) of Theorem 3.1 is satisfied. On the other hand, from Example 2.2, we know that T is a strong Meir–Keeler contraction on $\overline{O_T(x_0)}$. Therefore, the desired results follow from Theorem 3.1. □

Remark 4.1. Corollary 4.1 is a generalization of [8, Theorem 3.3], where the contraction was supposed to be satisfied for every pair of points $(x, y) \in X \times X$.

Corollary 4.2. *Let (X, D) be a complete JS-metric space and let $T: X \to X$ be a mapping satisfying*

$$(x, y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)},\ D(x, y) = 0 \implies D(Tx, Ty) = 0$$

and

$$(x, y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)},\ 0 < D(x, y) < +\infty \implies D(Tx, Ty) < \varphi(D(x, y)),$$

where $x_0 \in X$ and $\varphi \in \Phi$ (the set of functions defined in Example 2.3*). Suppose that $\delta(D, T, x_0) < +\infty$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T. Moreover, if $\omega' \in \overline{O_T(x_0)}$ is a fixed point of T such that $D(\omega, \omega') < +\infty$, then $\omega = \omega'$.*

Proof. From Example 2.3, we know that T is a strong Meir–Keeler contraction on $\overline{O_T(x_0)}$. Therefore, applying Theorem 3.1, we obtain the desired results.

The next fixed point results follow from Theorem 3.2.

Corollary 4.3. *Let (X, D) be a complete JS-metric space and let $T: X \to X$ be a generalized k-contraction on $O_T(x_0)$, for some $x_0 \in X$, that is,*

$$D(T^p x, T^p y) \leq k M_{T,p}(x, y), \quad (x, y) \in O_T(x_0) \times O_T(x_0),$$

where $k \in (0, 1)$ is a constant. Suppose that $\delta(D, T, x_0) < +\infty$ and T is continuous on $\overline{O_T(x_0)}$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T.

Proof. From Example 2.4, we know that T is a strong generalized Meir–Keeler contraction on $O_T(x_0)$. Therefore, the result follows from Theorem 3.2. □

Corollary 4.4. *Let (X, D) be a complete JS-metric space and let $T: X \to X$ be a mapping satisfying*

$$(x, y) \in O_T(x_0) \times O_T(x_0),\ 0 < M_{T,p}(x, y) < +\infty \implies D(T^p x, T^p y) < \varphi(M_{T,p}(x, y)),$$

where $x_0 \in X$ and $\varphi \in \Phi$. Suppose that $\delta(D, T, x_0) < +\infty$ and T is continuous on $\overline{O_T(x_0)}$. Then, $\{T^n x_0\}$ is D-convergent to some $\omega \in \overline{O_T(x_0)}$, where ω is a fixed point of T.

Proof. From Example 2.5, we know that T is a strong generalized Meir–Keeler contraction on $O_T(x_0)$. Therefore, applying Theorem 3.2, we get the desired result. □

Next, using an argument of Samet [19], we will show that it is possible to deduce easily an extension of Ran-Reurings fixed point theorem [15] to a dislocated metric space, which is a particular JS-metric space.

First, recall that a mapping $d: X \times X \to [0, +\infty)$ is said to be a dislocated metric on X (see [6]) if the following conditions are satisfied:

(d1) $(x, y) \in X \times X,\ d(x, y) = 0 \implies x = y$.
(d2) $d(x, y) = d(y, x)$, for all $(x, y) \in X \times X$.
(d3) $d(x, y) \leq d(x, z) + d(z, y)$, for all $(x, y, z) \in X \times X \times X$.

Clearly, any dislocated metric on X is a JS-metric on X with $C = 1$ (see [8]). Moreover, we have

$$\lim_{n\to+\infty} d(x_n, x) = \lim_{n\to+\infty} d(y_n, y) = 0 \implies \lim_{n\to+\infty} d(x_n, y_n) = d(x, y).$$

Let (X, d) be a dislocated metric space and let $\preceq$ be a certain partial order on X.

We say that $T: X \to X$ is non-decreasing with respect to $\preceq$ if

$$(x, y) \in X \times X,\ x \preceq y \implies Tx \preceq Ty.$$

We have the following result, which can be deduced from Corollary 4.1.

Corollary 4.5. *Let (X, d) be a complete dislocated metric space. Suppose that X is partially ordered by a certain binary relation $\preceq$. Let $T: X \to X$ be a continuous mapping and non-decreasing with respect to $\preceq$. Suppose that there exists $0 < k < 1$ such that*

$$(x, y) \in X \times X,\ x \preceq y \implies d(Tx, Ty) \leq kd(x, y).$$

Suppose also that there exists a certain $x_0 \in X$ such that $x_0 \preceq Tx_0$. Then, $\{T^n x_0\}$ is d-convergent to a fixed point of T.

Proof. Let $(x, y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)}$. There exists a sequence $\{T^{n_k} x_0\} \subset O_T(x_0)$ such that

$$\lim_{k \to +\infty} d(T^{n_k} x_0, x) = 0.$$

Similarly, there exists a sequence $\{T^{n_l} x_0\} \subset O_T(x_0)$ such that

$$\lim_{l \to +\infty} d(T^{n_l} x_0, y) = 0.$$

fyin, since T is non-decreasing with respect to $\preceq$, and $x_0 \preceq Tx_0$, we have

$$x_0 \preceq Tx_0 \preceq T^2 x_0 \preceq \cdots \preceq T^n x_0 \preceq T^{n+1} x_0 \preceq \cdots$$

Therefore, for every $(k, l) \in \mathbb{N} \times \mathbb{N}$, we have

$$T^{n_k} x_0 \preceq T^{n_l} x_0 \quad \text{or} \quad T^{n_l} x_0 \preceq T^{n_k} x_0.$$

Hence, by symmetry of d (see the condition (d2)), we have

$$d(T^{n_k+1} x_0, T^{n_l+1} x_0) \leq k d(T^{n_k} x_0, T^{n_l} x_0), \quad (k, l) \in \mathbb{N} \times \mathbb{N}.$$

Fixing $l \in \mathbb{N}$, and passing to the limit as $k \to +\infty$, by the continuity of T, we obtain

$$d(Tx, T^{n_l+1} x_0) \leq k d(x, T^{n_l} x_0), \quad l \in \mathbb{N}.$$

Next, passing to the limit as $k \to +\infty$, and using again the continuity of T, we obtain

$$d(Tx, Ty) \leq k d(x, y).$$

Therefore, we have

$$d(Tx, Ty) \leq k d(x, y), \quad (x, y) \in \overline{O_T(x_0)} \times \overline{O_T(x_0)}.$$

Moreover, using the condition (d3), it can be easily seen that

$$\delta(d, T, x_0) < +\infty.$$

Finally, applying Corollary 4.1, we obtain the desired result. □

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