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The Banach and Reich contractions in $b_v(s)$-metric spaces

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Abstract. In this paper, the concept of $b_v(s)$-metric space is introduced as a generalization of metric space, rectangular metric space, b-metric space, rectangular b-metric space and v-generalized metric space. We next give proofs of the Banach and Reich contraction principles in $b_v(s)$-metric spaces. Using a new result, we provide short proofs which are different from of the original ones in metric spaces. The results we obtain generalize many known results in fixed point theory. We also provide a solution to an open problem.

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Keywords. Fixed points, b-metric space, rectangular metric space, rectangular b-metric space.

1. Introduction and Preliminaries

Bakhtin [1] and Czerwik [3] introduced b-metric spaces (a generalization of metric spaces) and proved the contraction principle in this framework. In the last period many authors obtained fixed point results for single-valued or set-valued functions, in the setting of b-metric spaces.

Definition 1.1 (Bakhtin [1] and Czerwik [3]). Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is said to be a b-metric if and only if for all $x, y, z \in X$ the following conditions are satisfied:

(1) $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$;
(3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A triplet (X, d, s), is called a b-metric space with coefficient s.

In the sequel Branciari [2] introduced the concept of rectangular metric space (RMS) by replacing the sum on the right hand side of the triangular inequality in the definition of a metric space by a three-term expression and proved an analogue of the Banach contraction principle in such space.

Definition 1.2 [2]. Let X be a nonempty set and the mapping $d : X \times X \to [0, \infty)$ satisfying:

(RM1) $d(x, y) = 0$ if and only if $x = y$;
(RM2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(RM3) $d(x, y) \leq d(x, u)+d(u, v)+d(v, y)$ for all $x, y \in X$ and all distinct points $u, v \in X\backslash\{x, y\}$.

Then d is called a rectangular metric on X and (X, d) is called a rectangular metric space (in short RMS).

In the paper [6] George et al. introduce the concept of rectangular b-metric space, which is not necessarily Hausdorff and which generalizes the concept of metric space, rectangular metric space and b-metric space.

Definition 1.3 [6]. Let X be a nonempty set and the mapping $d : X \times X \to [0, \infty)$ satisfying:

(RbM1) $d(x, y) = 0$ if and only if $x = y$;
(RbM2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(RbM3) there exists a real number $s \geq 1$ such that $d(x, y) \leq s[d(x, u) + d(u, v) + d(v, y)]$ for all $x, y \in X$ and all distinct points $u, v \in X\backslash\{x, y\}$.

Then d is called a rectangular b-metric on X and (X, d) is called a rectangular b-metric space (in short RbMS) with coefficient s.

The main result in paper [6] is the following theorem (analogue of Banach contraction principle in rectangular b-metric space).

Theorem 1.4. *Let (X, d) be a complete rectangular b-metric space with coefficient $s > 1$ and $T : X \to X$ be a mapping satisfying:*

$$d(Tx, Ty) \leq \lambda d(x, y) \tag{1.1}$$

for all $x, y \in X$, where $\lambda \in [0, \frac{1}{s}]$. Then T has a unique fixed point.

In [6], the authors raised the following problem (Open Problem 1).

Problem. In Theorem 1.4, can we extent the range of λ to the case $\frac{1}{s} < \lambda < 1$?

In 2000, Branciari [2] introduced the following concept.

Definition 1.5 (Branciari [2]). Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$ and let $v \in \mathbb{N}$. Then (X, d) is said to be a v-generalized metric space if the following hold:

(N1) $d(x, y) = 0$ if and only if $x = y$;
(N2) $d(x, y) = d(y, x)$ for all $x, y \in X$,
(N3) $d(x, y) \leq d(x, u_1)+d(u_1, u_2)+\cdots+d(u_v, y)$ for all $x, u_1, u_2, \ldots, u_v$, $y \in X$ such that $x, u_1, u_2, \ldots, u_v, y$ are all different.

Suzuki et al. [11] give a proof of the following fixed point theorem which is a generalization of the Banach contraction principle in v-generalized metric spaces.

Theorem 1.6 (Suzuki et al. [11]). *Let (X, d) be a complete v-generalized metric space and let T be a contraction on X, that is, there exists $\lambda \in [0, 1)$ such that*

$$d(Tx, Ty) \leq \lambda d(x, y)$$

for any $x, y \in X$. Then T has a unique fixed point z. Moreover, for any $x \in X$, $\{T^n x\}$ converges to z.

In the paper Dominguez et al. [4] introduce a class N-polygonal K-metric space and proved fixed point result for Kannan type maps in the framework of a complete N-polygonal K-metric space.

Let $(V, ||\cdot||)$ be a Banach space. A set $K \subset V$ is called a cone if and only if:

(1) K is nonempty and $K \neq \{0_V\}$.
(2) If $\alpha, \beta \in K$ and $a, b \in [0, \infty)$, then $a\alpha + b\beta \in K$.
(3) $K \cap (-K) = \{0_V\}$.

For a given cone $K \subset V$, we can define a partial ordering $\preceq$ with respect to K by $\alpha \preceq \beta$ if and only if $\beta - \alpha \in K$. We shall write $\alpha \prec \beta$ to indicate that $\alpha \preceq \beta$ but $\alpha \neq \beta$. We will refer $(V, ||\cdot||, K)$ as an ordered Banach space. The cone K is called normal if there exists a number $\lambda \geq 1$ such that for all $\alpha, \beta \in V, 0_V \preceq \alpha \preceq \beta$ implies $||\alpha|| \leq \lambda ||\beta||$. The least positive number satisfying above is called the normal constant of K.

Definition 1.7 (Dominguez et al. [4]). Let X be a set and $d_K : X \times X \to K$ a mapping. We say that d_K is a N-polygonal K-metric, if for all $x, y \in X$ and for all distinct points $z_1, z_2, \ldots, z_N \in X$, each of them different from x and y, one has

(1) $d_K(x, y) = 0_V$ if and only if $x = y$;
(2) $d_K(x, y) = d_K(y, x)$;
(3) $d_K(x, y) \preceq d_K(x, z_1) + d_K(z_1, z_2) + \cdots + d_K(z_{N-1}, z_N) + d_K(z_N, y)$.

The pair (X, d_K) is said to be a N-polygonal K-metric space.

We introduce the concept of $b_v(s)$-metric space as follows.

Definition 1.8. Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$ and let $v \in \mathbb{N}$. Then (X, d) is said to be a $b_v(s)$-metric space if for all $x, y \in X$ and for all distinct points $u_1, u_2, \ldots, u_v \in X$, each of them different from x and y the following hold:

(B1) $d(x, y) = 0$ if and only if $x = y$;
(B2) $d(x, y) = d(y, x)$;
(B3) there exists a real number $s \geq 1$ such that

$$d(x, y) \leq s[d(x, u_1) + d(u_1, u_2) + \cdots + d(u_v, y)].$$

Note that:

- $b_1(1)$-metric space is usual metric space,
- $b_1(s)$-metric space is b-metric space with coefficient s of Bakhtin and Czerwik,
- $b_2(1)$-metric space is rectangular metric space,

- $b_2(s)$-metric space is rectangular b-metric space with coefficient s of George et al.,
- $b_v(1)$-metric space is v-generalized metric space of Branciari.
- Let (X, d_K) be a N-polygonal K-metric space over an ordered Banach space $(V, ||\cdot||, K)$ such that K is a closed normal cone with normal constant λ and the function $D : X \times X \to [0, \infty)$ defined by $D(x, y) = ||d_K(x, y)||$. Then (X, D) is $b_N(\lambda)$-metric space.

Definition 1.9. Let (X, d) be a $b_v(s)$-metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then

(a) The sequence $\{x_n\}$ is said to be convergent in (X, d) and converges to x, if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) < \varepsilon$ for all $n > n_0$ and this fact is represented by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.
(b) The sequence $\{x_n\}$ is said to be Cauchy sequence in (X, d) if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_{n+p}) < \varepsilon$ for all $n > n_0, p > 0$ or equivalently, if $\lim_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.
(c) (X, d) is said to be a complete $b_v(s)$-metric space if every Cauchy sequence in X converges to some $x \in X$.

The following three lemmas are new and useful in this framework.

Lemma 1.10. *If (X, d) is a $b_v(s)$-metric space, then (X, d) is a $b_{2v}(s^2)$-metric space.*

Proof. Let (X, d) be a $b_v(s)$-metric space. Let

$$d(x, y) \le s[d(x, u_1) + d(u_1, u_2) + \cdots + d(u_v, y)],$$

for all distinct points $x, u_1, u_2, \ldots, u_v, y$. Then for different $s_1, s_2, \ldots, s_v \in X\backslash\{x, u_1, \ldots, u_v, y\}$ we have

$$\begin{aligned} d(x, y) &\le s[d(x, u_1) + d(u_1, u_2) + \cdots + d(u_v, y)] \\ &\le s[d(x, u_1) + d(u_1, u_2) + \cdots + d(u_{v-1}, u_v) \\ &\quad + s[d(u_v, s_1) + d(s_1, s_2) + \cdots + d(s_{v-1}, s_v) + d(s_v, y)]] \\ &\le s^2[d(x, u_1) + d(u_1, u_2) + \cdots + d(u_{v-1}, u_v) \\ &\quad + d(u_v, s_1) + d(s_1, s_2) + \cdots + d(s_{v-1}, s_v) + d(s_v, y)]. \end{aligned}$$

So, (X, d) is a $b_{2v}(s^2)$-metric space. □

Lemma 1.11. *Let (X, d) be a $b_v(s)$-metric space $T : X \to X$ and let $\{x_n\}$ be a sequence in X defined by $x_0 \in X$ and $x_{n+1} = Tx_n$ such that $x_n \ne x_{n+1}$, $(n \ge 0)$. Suppose that $\lambda \in [0, 1)$ such that*

$$d(x_{n+1}, x_n) \le \lambda d(x_n, x_{n-1}) \quad \text{for all } n \in \mathbb{N}. \tag{1.2}$$

Then $x_n \ne x_m$ for all distinct $n, m \in \mathbb{N}$.

Proof. We will prove that $x_n \ne x_{n+k}$ for all $n \ge 0, k \ge 1$. Namely, if $x_n = x_{n+k}$ for some $n \ge 0$ and $k \ge 1$ we have that $Tx_n = Tx_{n+k}$ and $x_{n+1} = x_{n+k+1}$. Then (1.2) implies that

$$d(x_{n+1}, x_n) = d(x_{n+k+1}, x_{n+k}) \leq \lambda^k d(x_{n+1}, x_n) < d(x_{n+1}, x_n)$$

is a contradiction. Thus, we obtain that $x_n \neq x_m$ for all distinct $n, m \in \mathbb{N}$. □

The next lemma can be compared with recent interesting results from [8] (Lemma 2.2).

Lemma 1.12. *Let (X, d) be a $b_v(s)$-metric space and let $\{x_n\}$ be a sequence in X such that x_n $(n \geq 0)$ are all different. Suppose that $\lambda \in [0, 1)$ and c_1, c_2 are real nonnegative numbers such that*

$$d(x_m, x_n) \leq \lambda d(x_{m-1}, x_{n-1}) + c_1\lambda^m + c_2\lambda^n, \quad \text{for all } m, n \in \mathbb{N}. \tag{1.3}$$

Then $\{x_n\}$ is Cauchy.

Proof. If $\lambda = 0$ then the proof is trivial. Let $\lambda \in (0, 1)$. Since $\lim_{n\to\infty} \lambda^n = 0$, there exists a natural number n_0 such that

$$0 < \lambda^{n_0} \cdot s < 1, \tag{1.4}$$

is true. From condition (1.3) we obtain

$$d(x_{n+1}, x_n) \leq \lambda^n d(x_1, x_0) + n[c_1\lambda^{n+1} + c_2\lambda^n]. \tag{1.5}$$

So,

$$d(x_{n+1}, x_n) \leq \lambda^n d(x_1, x_0) + C_1 n\lambda^n, \tag{1.6}$$

where $C_1 = c_1\lambda + c_2$. Similarly, from (1.3) we have that

$$d(x_{m+k}, x_{n+k}) \leq \lambda^k d(x_m, x_n) + k\lambda^k[c_1\lambda^m + c_2\lambda^n] \text{ for all } k \geq 1. \tag{1.7}$$

We consider the following two cases:

- $v \geq 2$.
- $v = 1$.

Let $v \geq 2$. Since, (X, d) is $b_v(s)$-metric space, from condition (B3) we have

$$\begin{aligned} d(x_n, x_m) \leq s[&d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-3}, x_{n+v-2}) \\ &+ d(x_{n+v-2}, x_{n+n_0}) + d(x_{n+n_0}, x_{m+n_0}) + d(x_{m+n_0}, x_m)]. \end{aligned}$$

Then,

$$\begin{aligned} d(x_m, x_n) &\leq s[(\lambda^n + \lambda^{n+1} + \cdots + \lambda^{n+v-3})d(x_0, x_1) \\ &+ C_1(n\lambda^n + (n+1)\lambda^{n+1} + \cdots + (n+v-3)\lambda^{n+v-3}) \\ &+ \lambda^n d(x_{v-2}, x_{n_0}) + n\lambda^n(c_1\lambda^{v-2} + c_2\lambda^{n_0}) + \lambda^{n_0} d(x_n, x_m) \\ &+ n_0\lambda^{n_0}(c_1\lambda^m + c_2\lambda^n) + \lambda^m d(x_{n_0}, x_0) + m\lambda^m(c_1\lambda^{n_0} + c_2)]. \end{aligned}$$

and

$$\begin{aligned} d(x_m, x_n)(1 - \lambda^{n_0}s) &\leq s[(\lambda^n + \lambda^{n+1} + \cdots + \lambda^{n+v-3})d(x_0, x_1) \\ &+ C_1(n\lambda^n + (n+1)\lambda^{n+1} + \cdots + (n+v-3)\lambda^{n+v-3}) \\ &+ \lambda^n d(x_{v-2}, x_{n_0}) + n\lambda^n(c_1\lambda^{v-2} + c_2\lambda^{n_0}) + n_0\lambda^{n_0}(c_1\lambda^m + c_2\lambda^n) \\ &+ \lambda^m d(x_{n_0}, x_0) + m\lambda^m(c_1\lambda^{n_0} + c_2)]. \end{aligned}$$

From this, together with (1.4) as $m, n \to \infty$ we conclude that $d(x_m, x_n) \to 0$ and $\{x_n\}$ is a Cauchy sequence in X.

If $v = 1$ then proof follows from Lemma 1.10. □

In this paper, we give a proof of the Banach contraction principle in $b_v(s)$-metric spaces. The proof is short and different from the proof of the original Banach contraction principle in metric spaces.

2. Main Result

Theorem 2.1. *Let (X,d) be a complete $b_v(s)$-metric space and $T : X \to X$ be a mapping satisfying:*

$$d(Tx,Ty) \leq \lambda d(x,y) \tag{2.1}$$

for all $x, y \in X$, where $\lambda \in [0,1)$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$ be arbitrary. Define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_n = x_{n+1}$ then x_n is fixed point of T and the proof holds. So, suppose that $x_n \neq x_{n+1}$ for all $n \geq 0$. Then from Lemma 1.11 we obtain $x_n \neq x_m$ for all distinct $n, m \in \mathbb{N}$. From condition (2.1) we obtain

$$d(x_m, x_n) \leq \lambda d(x_{m-1}, x_{n-1}).$$

Now, from Lemma 1.12, (we can put $c_1 = 0, c_2 = 0$ for all $m, n \in \mathbb{N}$) we obtain that $\{x_n\}$ is a Cauchy sequence in X. By completeness of (X,d) there exists $x^* \in X$ such that

$$\lim_{n\to\infty} x_n = x^*. \tag{2.2}$$

Now we obtain that x^* is the unique fixed point of T. Namely, for any $n \in \mathbb{N}$ we have

$$\begin{aligned} d(x^*, Tx^*) &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_{n+3}) + \cdots \\ &\quad + d(x_{n+v-2}, x_{n+v-1}) + d(x_{n+v-1}, x_{n+v}) + d(x_{n+v}, Tx^*)] \\ &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_{n+3}) + \cdots \\ &\quad + d(x_{n+v-2}, x_{n+v-1}) + d(x_{n+v-1}, x_{n+v}) + d(Tx_{n+v-1}, Tx^*)] \\ &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_{n+3}) + \cdots \\ &\quad + d(x_{n+v-2}, x_{n+v-1}) + d(x_{n+v-1}, x_{n+v}) + \lambda d(x_{n+v-1}, x^*)] \end{aligned}$$

Since, $\lim_{n\to\infty} d(x^*, x_n) = 0$ and $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$, we have $d(x^*, Tx^*) = 0$ i. e., $Tx^* = x^*$.

For uniqueness, let y^* be another fixed point of T. Then it follows from (2.1) that $d(x^*, y^*) = d(Tx^*, Ty^*) \leq \lambda d(x^*, y^*) < d(x^*, y^*)$, is a contradiction. Therefore, we must have $d(x^*, y^*) = 0$, i.e., $x^* = y^*$. □

Remark 2.2. If $v = 1$ from Theorem 2.1 we obtain a Banach fixed point theorem in b-metric spaces (see Theorem 2. 1. [5]).

Remark 2.3. If $v = 2$ from Theorem 2.1 we obtain a Banach fixed point theorem in rectangular b-metric spaces (see Theorem 2. 1 in [9]) and solution of Open Problem 1 in [6].

The following theorem is the analogue of the Reich contraction principle in $b_v(s)$-metric space.

Theorem 2.4. *Let (X, d) be a complete $b_v(s)$-metric space and $T : X \to X$ be a mapping satisfying:*

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) \tag{2.3}$$

for all $x, y \in X$, where α, β, γ are nonnegative constants with $\alpha + \beta + \gamma < 1$ and $\min\{\beta, \gamma\} < \frac{1}{s}$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$ be arbitrary. Define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \geq 0$. From condition (2.3) we have that

$$d(x_{n+1}, x_n) \leq \alpha d(x_n, x_{n-1}) + \beta d(x_n, x_{n+1}) + \gamma d(x_{n-1}, x_n).$$

Therefore,

$$d(x_{n+1}, x_n) \leq \frac{\alpha + \gamma}{1 - \beta} d(x_n, x_{n-1}). \tag{2.4}$$

Put $r = \frac{\alpha+\gamma}{1-\beta}$. We have that $r \in [0, 1)$. It follows from (2.4) that

$$d(x_{n+1}, x_n) \leq r^n d(x_1, x_0) \quad \text{for all} \;\; n \geq 1. \tag{2.5}$$

If $x_n = x_{n+1}$ then x_n is fixed point of T. So, suppose that $x_n \neq x_{n+1}$ for some $n \geq 0$. Then from Lemma 1.11 we obtain $x_n \neq x_m$ for all distinct $n, m \in \mathbb{N}$. From conditions (2.3) and (2.5) we obtain

$$\begin{aligned} d(x_m, x_n) &\leq \alpha d(x_{m-1}, x_{n-1}) + \beta d(x_{m-1}, x_m) + \gamma d(x_{n-1}, x_n) \\ &\leq \alpha d(x_{m-1}, x_{n-1}) + \beta r^{m-1} d(x_0, x_1) + \gamma r^{n-1} d(x_0, x_1) \\ &= \alpha d(x_{m-1}, x_{n-1}) + (\beta r^{m-1} + \gamma r^{n-1}) d(x_0, x_1) \end{aligned}$$

From this, together with Lemma 1.12 (we can put

$$\lambda = \max\{\alpha, r\}, c_1 = \beta r^{-1} d(x_0, x_1), c_2 = \gamma r^{-1} d(x_0, x_1)$$

for all $m, n \in \mathbb{N}$, note that if $r = 0$ then proof is trivial) we conclude that $\{x_n\}$ is Cauchy. By completeness of (X, d) there exists $x^* \in X$ such that

$$\lim_{n\to\infty} x_n = x^*. \tag{2.6}$$

Now we obtain that x^* is the unique fixed point of T. Namely, we have

$$\begin{aligned} d(x^*, Tx^*) &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-1}, x_{n+v}) \\ &\quad + d(x_{n+v}, Tx^*)] \\ &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-1}, x_{n+v}) \\ &\quad + d(Tx_{n+v-1}, Tx^*)] \\ &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-1}, x_{n+v}) \\ &\quad + \alpha d(x_{n+v-1}, x^*) + \beta d(x_{n+v-1}, x_{n+v}) + \gamma d(x^*, Tx^*)]. \end{aligned}$$

and

$$\begin{aligned} d(Tx^*, x^*) &\leq s[d(Tx^*, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-1}, x_{n+v}) \\ &\quad + d(x_{n+v}, x^*)] \\ &\leq s[\alpha d(x^*, x_n) + \beta d(x^*, Tx^*) + \gamma d(x_n, x_{n+1}) \\ &\quad + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+v-1}, x_{n+v}) + d(x_{n+v}, x^*)]. \end{aligned}$$

Since $\lim_{n\to\infty} d(x^*, x_n) = 0$, $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ and $\min\{\beta, \gamma\} < \frac{1}{s}$, we have $d(x^*, Tx^*) = 0$ i. e., $Tx^* = x^*$.

For uniqueness, let y^* be another fixed point of T. Then it follows from (2.3) that

$$\begin{aligned} d(x^*, y^*) &= d(Tx^*, Ty^*) \leq \alpha d(x^*, y^*) + \beta d(x^*, Tx^*) + \gamma d(y^*, Ty^*) \\ &= \alpha d(x^*, y^*) < d(x^*, y^*) \end{aligned}$$

is a contradiction. Therefore, we must have $d(x^*, y^*) = 0$, i.e., $x^* = y^*$. □

From Theorem 2.4 we obtain the following variant of Kannan theorem [7] in b-rectangular metric spaces.

Theorem 2.5. *Let (X, d) be a complete $b_v(s)$-metric space and $T : X \to X$ be a mapping satisfying:*

$$d(Tx, Ty) \leq \beta d(x, Tx) + \gamma d(y, Ty) \tag{2.7}$$

for all $x, y \in X$, where β, γ nonnegative constants with $\beta + \gamma < 1$ and $\min\{\beta, \gamma\} < \frac{1}{s}$. Then T has a unique fixed point.

Remark 2.6. If $v = 2$ from Theorem 2.4 we obtain a Reich [10] fixed point theorem in rectangular b-metric spaces and partial solutions of Open Problem 2 in [6].

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