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Thermal damage constitutive model for rock considering damage threshold and residual strength

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Abstract: With the gradual depletion of mineral resources in the shallow part of the earth, resource exploitation continues to move deeper into the earth, it becomes a hot topic to simulate the whole process of rock strain softening, deformation and failure in deep environment, especially under high temperature and high pressure. On the basis of Lemaitre's strain-equivalent principle, combined with statistics and damage theory, a statistical constitutive model of rock thermal damage under triaxial compression condition is established. At the same time, taking into account the existing damage model is difficult to reflect residual strength after rock failure, the residual strength is considered in this paper by introducing correction factor of damage variable, the model rationality is also verified by experiments. Analysis of results indicates that the damage evolution curve reflects the whole process of rock micro-cracks enclosure, initiation, expansion, penetration, and the formation of macro-cracks under coupled effect of temperature and confining pressure. Rock thermal damage shows logistic growth function with the increase of temperature. Under the same strain condition, rock total damage decreases with the rise of confining pressure. By studying the electron microscope images (SEM) of rock fracture, it is inferred that 35.40 MPa is the critical confining pressure of brittle to plastic transition for this granite. The model parameter *F* reflects the average strength of rock, and *M* reflects the morphological characteristics of rock stress–strain curves. The physical meanings of model parameters are clear and the model is suitable for complex stress states, which provides valuable references for the study of rock deformation and stability in deep engineering.

Key words: rock damage model; temperature effect; confining pressure; damage threshold; residual strength

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1 Introduction

In the deep mining of mineral resources, nuclear waste storage and some other underground engineering activities, more and more rocks are in the coupled environment of high temperature and pressure, and the rock deformation and damage mechanism are quite different from normal temperature [1–8]. Since we know, rock contains many micro-cracks and micro-voids, and these original defects will propagate and form

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macroscopic cracks under the effect of temperature and loading, resulting in the instability and destruction of engineering, so it is necessary to study constitutive model of rock materials under the complex geological conditions for the long-term safety and stability.

Since DOUGILL et al [9] introduced damage mechanics into the field of rock materials, many scholars focused on the rock damage constitutive theory [10–14]. However, numerous micro-cracks in rock materials, ranging from 0.01 to 1.0 mm in length, are statistically distributed, hence statistical approaches along with damage mechanics have been used in establishing rock constitutive model by many researchers [15–19]. DENG et al [20] derived a new constitutive model by using the theory of continuous damage mechanics together with statistical mesoscopic strength theory, based on maximum entropy distribution. It was found that the entropy-distribution-based constitutive model was considerably flexible and was better than the conventional Weibull-distribution-based model if appropriate parameters in the entropy model were chosen. However, there was no direct relationship between model parameters and rock statistical characteristics in this entropy models, such as mean and variance. LIU et al [21] put forward the definition of thermal damage and mechanical damage for marble after exposure to high temperature, deduced overall damage evolution equation and established thermal damage constitutive model on the basis of macroscopic phenomenological damage mechanics and non-equilibrium statistical theory. The results indicated that the effect of temperature on the mechanical properties of marble can be accurately described through the definition of thermal damage using elastic modulus. However, the experimental results were obtained only in uniaxial compression tests, not in triaxial compression tests. ZHAO et al [22] proposed an extended definition of damage, and developed a modified statistical damage constitutive model to reflect strain-softening and residual strength behavior for rocks loaded in conventional triaxial compression test. The model parameters were estimated based on the extremum method, and the validation indicated that the calculated results had good agreement with experimental observation, but the temperature effect had not been considered.

Despite the fact that above researches have greatly enriched the development of rock constitutive model, but study is still insufficient, especially on the coupled temperature and confining pressure. Obviously, rocks in deep underground is always subjected to the coupled temperature and triaxial stress state, therefore, the role of temperature and confining pressure should be taken into account simultaneously to realistically simulate rock behavior.

The purpose of this paper is to put forward a new statistical damage constitutive model, which can reflect the coupled effect of temperature and confining pressure. By considering rock damage threshold and residual strength, the simulation method of rock deformation process is further improved.

2 Thermo-mechanical coupled damage constitutive model

2.1 Thermal damage evolution equation

Due to the random distribution of micro-cracks and micro-voids in rock materials, it is assumed that rock strength obeys the Weibull statistical distribution, which is expressed as:

$$f(k)=\frac{m}{F}\left(\frac{k}{F}\right)^{m-1}\exp\left[-\left(\frac{k}{F}\right)^{m}\right] \tag{1}$$

where $f(k)$ is the distribution function of micro-intensity, k is the random distribution variable, m and F are Weibull distribution parameters. Under the action of loading, the original micro-cracks inside the rock will expand and evolve, leading to the continuous damage of the rock, so the continuous damage variable (D) can be defined as:

$$\begin{aligned}D&=\frac{V_{\mathrm{P}}}{V}=\frac{\iiint_V\int_0^k f(k)\mathrm{d}k\mathrm{d}x\mathrm{d}y\mathrm{d}z}{V}\\&=\frac{V\int_0^k f(k)\mathrm{d}k}{V}=\int_0^k f(k)\mathrm{d}k\end{aligned} \tag{2}$$

where V_{P} is the rock damaged volume, V is the rock total volume. Substituting Eq. (1) into Eq. (2), the damage variable can be described as:

$$D=1-\exp\left[-\left(\frac{k}{F}\right)^{m}\right] \tag{3}$$

For the plastic materials, the Drucker-Prager

(D-P) criterion is only a yield criterion, whereas for a brittle or quasi-brittle rock material, it can also be called strength criterion or failure criterion. The D-P strength criterion takes into account the effects of intermediate principal stress and hydrostatic pressure, overcoming the main weaknesses of Mohr-Coulomb criterion [23]. Therefore, in this paper, it is assumed that the rock micro-element strength satisfies the D-P criterion, which can be expressed by the principal stress as:

$$f([\sigma]) = k = \alpha_0 I_1 + \sqrt{J_2} = \frac{\sin\varphi}{\sqrt{9+3\sin^2\varphi}} I_1 + \sqrt{J_2} \tag{4}$$

where I_1 is the first invariant of stress tensor, J_2 is the second invariant of stress deviator tensor and φ is the internal friction angle. Substituting Eq. (4) into Eq. (3), D can be represented as:

$$D = 1 - \exp\left[-\left(\frac{\alpha_0 I_1 + \sqrt{J_2}}{F}\right)^m\right] \tag{5}$$

The Weibull distribution parameters m and F have influence on the shape of rock damage curves, and they are directly affected by temperature [15]. Therefore, the influence of temperature on the statistical constitutive model of rock damage can be considered by introducing Eqs. (6) and (7), as below:

$$m(T) = m_0(1 - D_T) \tag{6}$$

$$F(T) = F_0(1 - D_T) \tag{7}$$

where m_0 and F_0 are the Weibull distribution parameters at room temperature, and D_{T} is the thermal damage caused by temperature. Substituting Eqs. (6) and (7) into Eq. (5), the damage evolution equation with the temperature can be got as:

$$D = 1 - \exp\left\{-\left[\frac{\alpha_0 I_1 + \sqrt{J_2}}{F_0(1 - D_T)}\right]^{m_0(1-D_T)}\right\} \tag{8}$$

As we know, there is a threshold stress in the damage evolution of rock material under loading. When the stress state is below the threshold point, the damage caused by loading inside the rock is so small that can be considered as zero, so the total damage is only the thermal damage caused by temperature at this situation. When the stress state exceeds the threshold point, the damage value can be calculated according to Eq. (8). For the whole stress state, the damage evolution equation of rock material can be expressed as:

$$D(\sigma, T) = \begin{cases} D_T \quad (\sigma_1 < \sigma_{\mathrm{D}}) \\ 1 - \exp\left\{-\left[\dfrac{\alpha_0 I_1 + \sqrt{J_2}}{F_0(1 - D_T)}\right]^{m_0(1-D_T)}\right\} \quad (\sigma_1 \geq \sigma_{\mathrm{D}}) \end{cases} \tag{9}$$

where σ_{D} is the threshold stress of rock damage. Since the elastic modulus of rock material is related to the temperature, which can be used to define rock thermal damage. Setting the thermal damage is zero at room temperature, then the thermal damage can be defined as:

$$D_T = 1 - \frac{E_T}{E_0} \tag{10}$$

where E_{T} is the elastic modulus at the temperature of T and E_0 is the elastic modulus at room temperature. Substituting Eq. (10) into Eq. (9), the damage evolution equation of rock material at different temperatures expressed by elastic modulus is obtained as:

$$D(\sigma, T) = \begin{cases} D_T \quad (\sigma_1 < \sigma_D) \\ 1 - \exp\left[-\left(\dfrac{\alpha_0 I_1 + \sqrt{J_2}}{F_0 \dfrac{E_T}{E_0}}\right)^{m_0 \frac{E_T}{E_0}}\right] \quad (\sigma_1 \geq \sigma_D) \end{cases} \tag{11}$$

2.2 Thermo-mechanical coupled damage constitutive model

According to the theory of strain equivalent principle proposed by LEMAITRE [24], the damage constitutive model of rock can be obtained as:

$$\left[\sigma^*\right] = \frac{[\sigma]}{1-[D]} = \frac{[E][\varepsilon]}{1-[D]} \tag{12}$$

where $[\sigma^*]$ is the effective stress tensor, $[\sigma]$ is the nominal stress tensor, $[D]$ is the damage matrix, $[E]$ is the elastic modulus matrix and $[\varepsilon]$ is the strain matrix.

Due to the influence of friction and confining pressure, rock materials still have certain post-peak residual strength, which is characterized by the pure friction with the cohesive force of zero. In most cases, the test curve of residual strength is similar to the horizontal straight line. In most cases, the characteristics of compressive stress and shear stress that can still continue to be transmitted after the rock micro-element destruction has not been taken into account in Eq. (12), therefore, the damage variable correction factor (δ) is introduced in this paper to consider the residual strength. δ is defined as below [24]:

$$\delta=\sqrt{\frac{\sigma_{\rm r}}{\sigma_{\rm C}}} \tag{13}$$

where $\sigma_{\rm r}$ is the residual stress and $\sigma_{\rm C}$ is peak stress.

Then, Eq. (12) can be changed as:

$$\left[\sigma^*\right]=\frac{[\sigma]}{1-\delta[D]}=\frac{[E][\varepsilon]}{1-\delta[D]} \tag{14}$$

Assuming that the rock damage is isotropic, then the relationship between nominal stress (σ_i) and effective stress (σ_i^*) can be got as:

$$\sigma_i^*=\frac{\sigma_i}{1-\delta D} \quad (i=1,\ 2,\ 3) \tag{15}$$

Combined with the Hooke's law for linear elasticity, the strain is expressed as:

$$\varepsilon_i=\frac{1}{E}\left[\sigma_i^*-\mu\left(\sigma_j^*+\sigma_k^*\right)\right] \quad (i, j, k=1,\ 2,\ 3) \tag{16}$$

Substituting Eq. (15) into Eq. (16), then Eq. (16) can be rewritten as:

$$\varepsilon_i=\frac{1}{E(1-\delta D)}\left[\sigma_i-\mu\left(\sigma_j+\sigma_k\right)\right] \ (i, j, k=1, 2, 3) \tag{17}$$

Substituting Eq. (8) into Eq. (17), then:

$$\sigma_i=E\varepsilon_i\left\{1-\delta+\delta\exp\left[-\left(\frac{\alpha_0 I_1+\sqrt{J_2}}{F_0(1-D_T)}\right)^{m_0(1-D_T)}\right]\right\}+ \mu\left(\sigma_j+\sigma_k\right) \tag{18}$$

The elastic modulus of the rock at different temperatures from Eq. (10) is:

$$E=E(T)=E_T=E_0(1-D_T) \tag{19}$$

Substituting Eq. (19) into Eq. (18), the constitutive model (after damage threshold) of the rock material subjected to the temperature effect is obtained as:

$$\sigma_i=E_0(1-D_T)\varepsilon_i\left\{1-\delta+ \delta\exp\left[-\left(\frac{\alpha_0 I_1+\sqrt{J_2}}{F_0(1-D_T)}\right)^{m_0(1-D_T)}\right]\right\}+\mu(\sigma_j+\sigma_k) \tag{20}$$

Considering that the stress–strain curve of the rock passes through the coordinate origin, the constitutive model can be fitted using the polynomial function before the damage threshold, the function relation can be set as:

$$\sigma_i=A(T)\varepsilon_i\left[\varepsilon_i+B(T)\right] \tag{21}$$

where $A(T)$ and $B(T)$ are the temperature dependent coefficients. Combining Eq. (20) with (21), the complete damage constitutive model under the temperature effect is rewritten as:

$$\sigma_i=\begin{cases} A(T)\varepsilon_i\left[\varepsilon_i+B(T)\right] \quad (0\le\varepsilon\le\varepsilon_D) \\ E_0(1-D_T)\varepsilon_i\left\{1-\delta+ \delta\exp\left[-\left(\frac{\alpha_0 I_1+\sqrt{J_2}}{F_0(1-D_T)}\right)^{m_0(1-D_T)}\right]\right\}+ \mu(\sigma_j+\sigma_k) \quad (\varepsilon>\varepsilon_D) \end{cases} \tag{22}$$

2.3 Parameters solution

The key factor to establish the statistical constitutive model of rock material is to determine the Weibull distribution parameters properly. The traditional solution method is to fit the data of the triaxial compression test linearly, but if the experimental data of the stress–strain curve are not successfully obtained, the Weibull distribution parameter can not be determined [14]. Therefore, the traditional method of linear fitting needs to be improved.

In this work, the expressions of two distribution parameters (m and F) are determined by introducing the characteristic parameters of rock peak strength (σ_C) and peak strain (ε_C), taking into account the peak conditions and geometric conditions of the full stress–strain curve of the rock. The expressions of two fitting parameters (A and B) are obtained by introducing the stress (σ_D) and strain (ε_D) at the damage threshold point. The progress of solution method is as follows.

In the conventional triaxial compression test, $\sigma_1>\sigma_2=\sigma_3$, when $\varepsilon_1>\varepsilon_D$, Eq. (22) can be simplified as:

$$\sigma_1=E\varepsilon_1\left\{1-\delta+\delta\exp\left[-\left(\frac{\alpha_0 I_1+\sqrt{J_2}}{F}\right)^m\right]\right\}+2\mu\sigma_3 \tag{23}$$

where

$$I_1=\sigma_{ii}^*=\frac{E\varepsilon_1(\sigma_1+2\sigma_3)}{\sigma_1-2\mu\sigma_3} \tag{24}$$

$$\sqrt{J_2}=\sqrt{\frac{1}{2}s_{ij}^*s_{ij}^*}=\frac{E\varepsilon_1(\sigma_1-\sigma_3)}{\sqrt{3}(\sigma_1-2\mu\sigma_3)} \tag{25}$$

where s_{ij}^* is the effective stress deviator tensor in Eq. (25). The stress–strain curve satisfies the constitutive equation at the peak stress point, so there has:

$$\sigma_1\big|_{\varepsilon_1=\varepsilon_c}=\sigma_c \tag{26}$$

The first derivative of the stress–strain curve at the peak stress is zero, that is:

$$\frac{\partial\sigma_1}{\partial\varepsilon_1}\bigg|_{\varepsilon_1=\varepsilon_c}=0 \tag{27}$$

Combined Eq. (26) with (27), the expressions of m and F are:

$$m=-\left\{(\sigma_c-2\mu\sigma_3)/\left\{[\sigma_c-2\mu\sigma_3+E\varepsilon_c(\delta-1)]\cdot \ln\frac{\sigma_c-2\mu\sigma_3+(\delta-1)E\varepsilon_c}{E\varepsilon_c\delta}\right\}\right\} \tag{28}$$

$$F=\frac{f_c}{\left[-\ln\frac{\sigma_c-2\mu\sigma_3+(\delta-1)E\varepsilon_c}{E\varepsilon_c\delta}\right]^{\frac{1}{m}}} \tag{29}$$

where

$$f_c=E\varepsilon_c\frac{\alpha_0(\sigma_c+2\sigma_3)+\frac{\sqrt{3}}{3}(\sigma_c-\sigma_3)}{\sigma_c-2\mu\sigma_3} \tag{30}$$

when $0\le\varepsilon_1\le\varepsilon_D$,

$$\sigma_1=A\varepsilon_1(\varepsilon_1+B)=A\varepsilon_1^2+AB\varepsilon_1 \tag{31}$$

Firstly, the rock stress–strain curve is continuous in the compression section and the damage evolution section, so:

$$A\varepsilon_1^2+AB\varepsilon_1=E\varepsilon_1\left\{1-\delta+\delta\exp\left[-\left(\frac{f}{F}\right)^m\right]\right\}+2\mu\sigma_3 \tag{32}$$

Secondly, the stress–strain curve of rock is continuous with the first derivative of the compression section and the damage evolution section, so:

$$\begin{aligned}\frac{\partial\sigma_1}{\partial\varepsilon_1}\bigg|_{\varepsilon_1=\varepsilon_D}&=E\left\{1-\delta+\delta\exp\left[-\left(\frac{f}{F}\right)^m\right]\right\}+ E\varepsilon_1\delta\exp\left[-\left(\frac{f}{F}\right)^m\right]\left(-\frac{mf^{m-1}}{F^m}\right)\cdot\\ &\left\{\alpha_0[[\left[E(\sigma_1+2\sigma_3)+E\varepsilon_1\frac{\partial\sigma_1}{\partial\varepsilon_1}\right]-\frac{(\sigma_1+2\sigma_3)E\varepsilon_1\frac{\partial\sigma_1}{\partial\varepsilon_1}}{\sigma_1-2\mu\sigma_3}]/(\sigma_1-2\mu\sigma_3)]+\right.\\ &\left.\frac{E(\sigma_1-\sigma_3)+E\varepsilon_1\frac{\partial\sigma_1}{\partial\varepsilon_1}}{\sqrt{3}(\sigma_1-2\mu\sigma_3)}-\frac{\sqrt{3}E\varepsilon_1(\sigma_1-\sigma_3)}{3(\sigma_1-2\mu\sigma_3)^2}\cdot\frac{\partial\sigma_1}{\partial\varepsilon_1}\right\}\\ &=2A\varepsilon_1+AB\end{aligned} \tag{33}$$

Combined Eq. (32) with (33), then:

$$A=\frac{d\varepsilon_D-2\mu\sigma_3}{\varepsilon_D^2} \tag{34}$$

$$B=\frac{c\varepsilon_D+d\varepsilon_D^2}{d\varepsilon_D-2\mu\sigma_3}-2\varepsilon_D \tag{35}$$

where

$$c=E\varepsilon_D\left\{1-\delta+\delta\exp\left[-\left(\frac{f_D}{F}\right)^m\right]\right\} \tag{36}$$

$$\begin{aligned}d=&E\varepsilon_D\delta\exp\left[-\left(\frac{f_D}{F}\right)^m\right]\cdot\left(-\frac{mf_D^{m-1}}{F^m}\right)\cdot\\ &\left\{\alpha_0\left[\frac{(2E^2\varepsilon_D+2\sigma_3)}{E\varepsilon_D-2\mu\sigma_3}-\frac{(E\varepsilon_D+2\sigma_3)E^2\varepsilon_D}{(E\varepsilon_D-2\mu\sigma_3)^2}\right]+\right.\\ &\left.\frac{2E^2\varepsilon_D-E\sigma_3}{\sqrt{3}(E\varepsilon_D-2\mu\sigma_3)}-\frac{\sqrt{3}E^2\varepsilon_D(E\varepsilon_D-\sigma_3)}{3(E\varepsilon_D-2\mu\sigma_3)^2}\right\}\end{aligned} \tag{37}$$

$$f_D=E\varepsilon_D\frac{\alpha_0(\sigma_D+2\sigma_3)+\frac{\sqrt{3}}{3}(\sigma_D-\sigma_3)}{\sigma_D-2\mu\sigma_3} \tag{38}$$

Thus, all parameters in the constitutive model (22) are determined.

One thing should be pointed out that it is only applicable for specific confining pressure conditions to determine m and F using Eqs. (28) and (29). For the practical use, the widely used Mohr-Coulomb strength criterion may be adopted to estimate rock peak stress σ_c, such as:

$$\sigma_c=\frac{2c_f\cos\varphi_f}{1-\sin\varphi_f}+\frac{1+\sin\varphi_f}{1-\sin\varphi_f}\sigma_3 \tag{39}$$

where c_f represents rock cohesion at peak stress, and φ_f represents the internal friction angle at peak stress. Using Eq.(39), the value of σ_c can be calculated under different confining pressures. It was verified that the peak strain ε_c has a remarkable linear relevance to σ_3 through the experimental data obtained from different rocks under triaxial compression tests [17]. Therefore, it is suggested that:

$$\varepsilon_c = b + a\sigma_3 \tag{40}$$

where a and b can be obtained via a linear regression based on a series of test data.

From the above analysis, the model parameters m and F can be determined for different confining pressure conditions. In this case, m and F are only related to rock conventional mechanical parameters (such as E, μ, c_f, φ_f, etc.).

3 Analysis of rock damage evolution characteristics

3.1 Thermal damage

In order to describe the effect of temperature on rock damage, the elastic modulus is calculated by using uniaxial compression test curves of granite under different temperatures [26]. Table 1 gives the experimental data, combined with Eq. (10), thermal damage values are calculated.

Table 1 Average mechanical values of rock and thermal damage under different temperatures

Temperature/°C	Peak stress/MPa	Peak strain/10^{-3}	Elastic modulus/GPa	Thermal damage
25	120.370	4.160	31.310	0.000
200	121.768	4.831	28.566	0.088
400	97.943	4.231	27.548	0.120
600	54.624	5.862	10.785	0.656
800	41.766	5.777	8.742	0.721
1000	19.183	6.339	3.219	0.897

Both the elastic modulus and thermal damage show logistic increase curves with the rise of temperature (See Figure 1).

The thermal damage curve can be divided into three stages. I) From 25 °C to 400 °C, the thermal damage rises slowly, which is 0.12 at 400 °C, with an average increase of 0.03% per 1 °C; II) From 400 °C to 600 °C, it increases rapidly, which increased from 0.12 of 400 °C to 0.656 of 600 °C, with an increase of 0.286% per 1 °C, this is likely due to the reversible reaction of α quartz to β quartz occurred at 573 °C; III) From 600 °C to 1000 °C, it

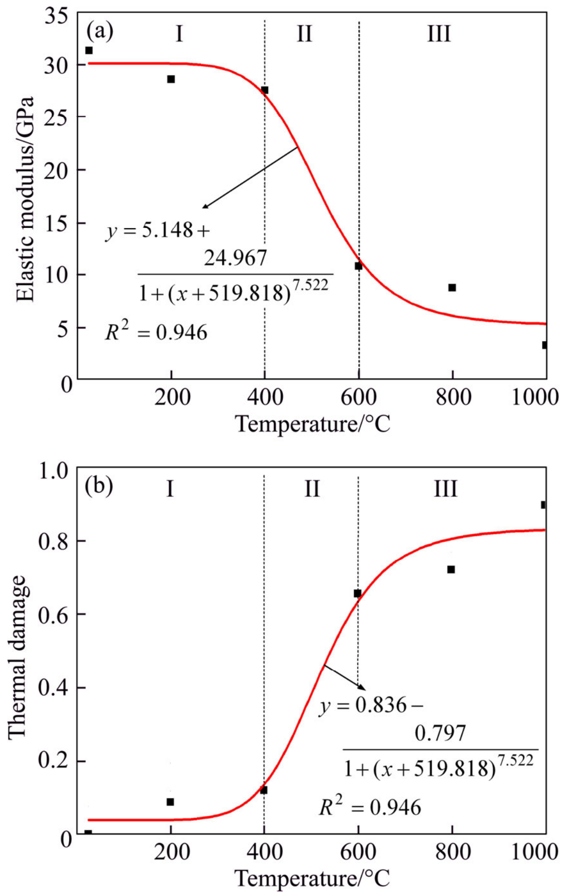

Figure 1 Variation of elastic modulus (a) and thermal damage variable (b) with temperature

rises slowly again, which is 0.897 at 1000 °C, increased 0.06% per 1 °C. At this temperature stage, the feldspar appears an endothermic valley at 700–900 °C, and the crystal lattice of mica is destroyed at 997 °C, leading to escapement of hydroxyl and formation of sodium feldspar, all of these reactions lead to a fundamental deterioration of rock mechanical properties, and the thermal damage increases gradually to 1. The fitting function of thermal damage (D_T) and temperature (T) is as follows:

$$D_T = 0.836 - \frac{0.797}{1+\left(T/519.818\right)^{7.522}},\ R^2=0.946 \tag{41}$$

3.2 Damage evolution characteristics

Rock damage is the essential reason for the deterioration of microstructure and macroscopic physical properties of the material. If the damage variable is defined from the view of microscopic point, the rock damage evolution law can be revealed through the combination of macroscopic and mesoscopic methods. Combined with Eq. (22) and the conventional triaxial compression test of granite at different temperatures by XU et al [26] (the experimental data are given in Table 2), the damage evolution curves of granite under different temperatures and confining pressures are plotted in Figure 2.

Table 2 Peak stress and strain of rock under different temperatures and confining pressures

Temperature/°C	Confining pressure/MPa	Peak stress/MPa	Peak strain/10^{-3}
25	10	209.160	6.740
	20	305.694	7.515
	30	370.037	9.489
	40	367.265	11.363
200	10	199.670	6.076
	20	278.430	9.565
	30	429.040	10.309
	40	450.190	15.555
400	10	232.380	6.076
	20	339.340	9.565
	30	377.950	10.309
	40	451.780	15.555
600	10	180.921	8.035
	20	291.990	9.100
	30	336.442	9.866
	40	381.113	11.840
800	10	139.361	6.608
	20	272.342	9.193
	30	293.590	10.749
	40	360.651	9.912
1000	10	175.561	10.283
	20	154.422	8.479
	30	76.310	9.063
	40	147.391	7.816

The total damage variable rises with the increase of the strain, which can reflect the linear elastic deformation of rock material when the stress level is low. In the early stage of loading, the micro-cracks in the rock are gradually closed. With the increase of loading, the closed micro-cracks are further compacted and have a relatively sliding trend. However, the stress–strain curve is still in the elastic state, and the stress level is not enough to make the micro-cracks propagate, so the new loading damage is not produced at this stage. The initial horizontal section of the damage evolution curve is the thermal damage caused by the temperature (See stage I in Figure 2). When the stress of the rock material exceeds yield point, the plastic deformation occurs, the new micro-cracks begin to expand between the relatively weak grain boundaries, and the rock damage begins to evolve and stable expand. With the increase of the stress level, the micro-cracks inside the rock are densely concentrated, overlapped and connected, forming the macroscopic cracks and accelerating the rock damage. At last, the macroscopic cracks are connected to form the main rupture surface, which leads to the sudden release of the stress. The rock strength decreases rapidly and the damage tends to 1.

With the increase of confining pressure, the damage of the rock under the same strain condition is decreased, indicating that the confining pressure improves the stress state of the rock, restrains the development of the damage and increases the macroscopic average intensity of the rock. As the confining pressure rises, the slope of the damage curve decreases with the increase of strain, which indicates that the confining pressure increases the dislocation of the rock particles, weakens the strain recovery ability and enhances rock plasticity.

4 Verification of theoretical model

In order to verify the nationality and accuracy of established model, the theoretical constitutive model fitted according to Eq. (22) is compared with the stress–strain curves obtained by the test. Because the stress–strain curves under the action of each temperature reveal similar shapes, only the theoretical model and experimental curves at 600 °C in different confining pressures are discussed here. When the stress exceeds the yield point, the rock enters the nonlinear deformation stage and begins to produce damage, so it can be considered that the damage threshold stress (σ_D) is the yield point of the rock, here we take σ_D=0.6σ_C, and ε_D is the damage threshold strain corresponding to σ_D in test strain–stress curves. The theoretical fitting parameters after 600 °C are given in Table 3.

As can be seen from Figure 3, the theoretical damage constitutive models established in this paper are generally consistent with test curves,

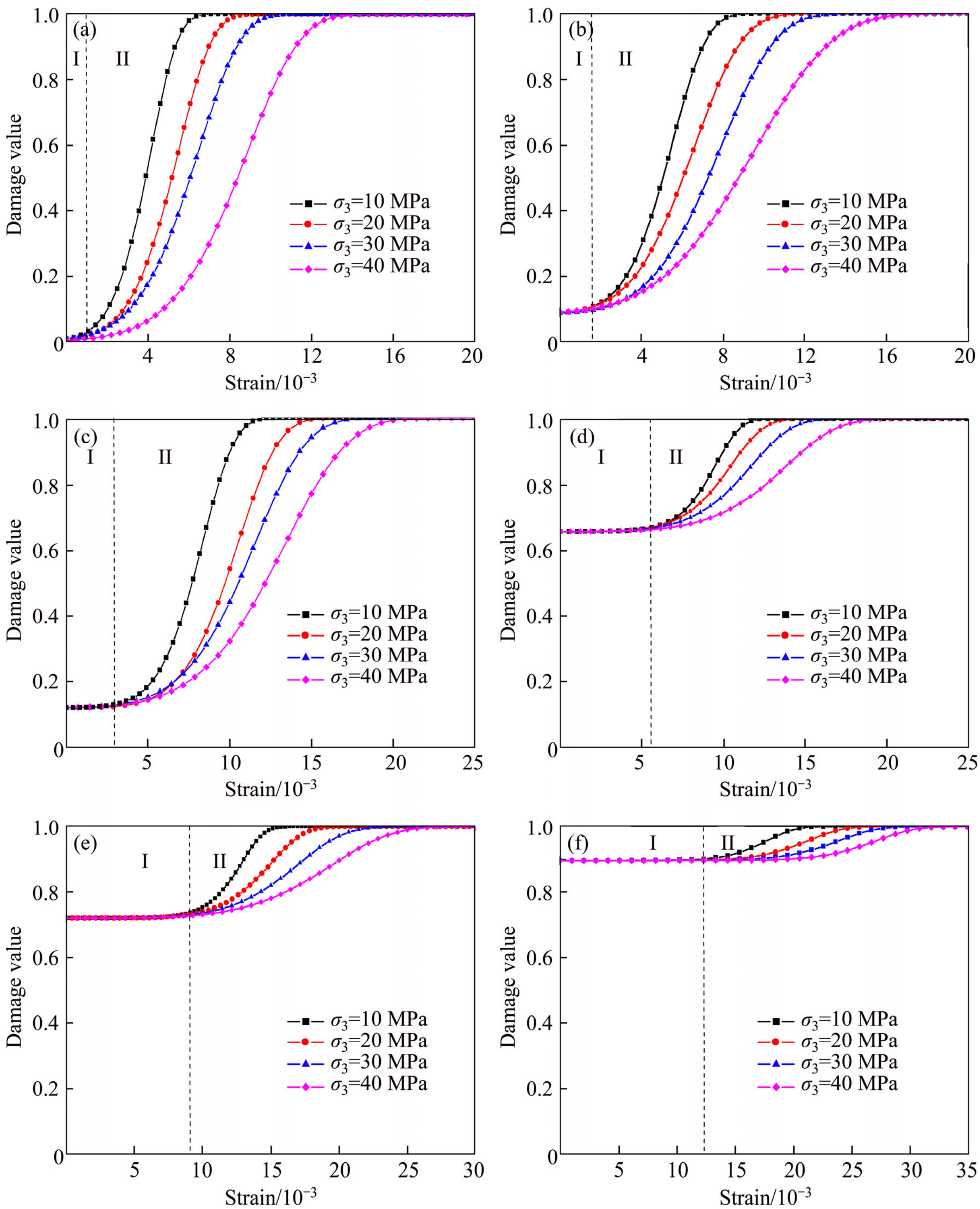

Figure 2 Relationship between damage value and strain at different temperatures and confining pressures: (a) 25 °C; (b) 200 °C; (c) 400 °C; (d) 600 °C; (e) 800 °C; (f) 1000 °C

Table 3 Theoretical fitting parameters at various confining pressures after 600 °C

Confining pressure/MPa	Peak stress, σ_C/MPa	Peak strain, $\varepsilon_C/10^{-3}$	Damage threshold stress, σ_D/MPa	Damage threshold strain, $\varepsilon_D/10^{-3}$	Residual stress, σ_r/MPa	Correction factor, δ
10	180.921	8.035	108.552	5.210	58.396	0.568
20	291.990	9.100	175.194	5.631	81.748	0.529
30	336.442	9.866	201.864	6.091	111.259	0.575
40	381.113	11.840	228.666	7.311	161.298	0.650

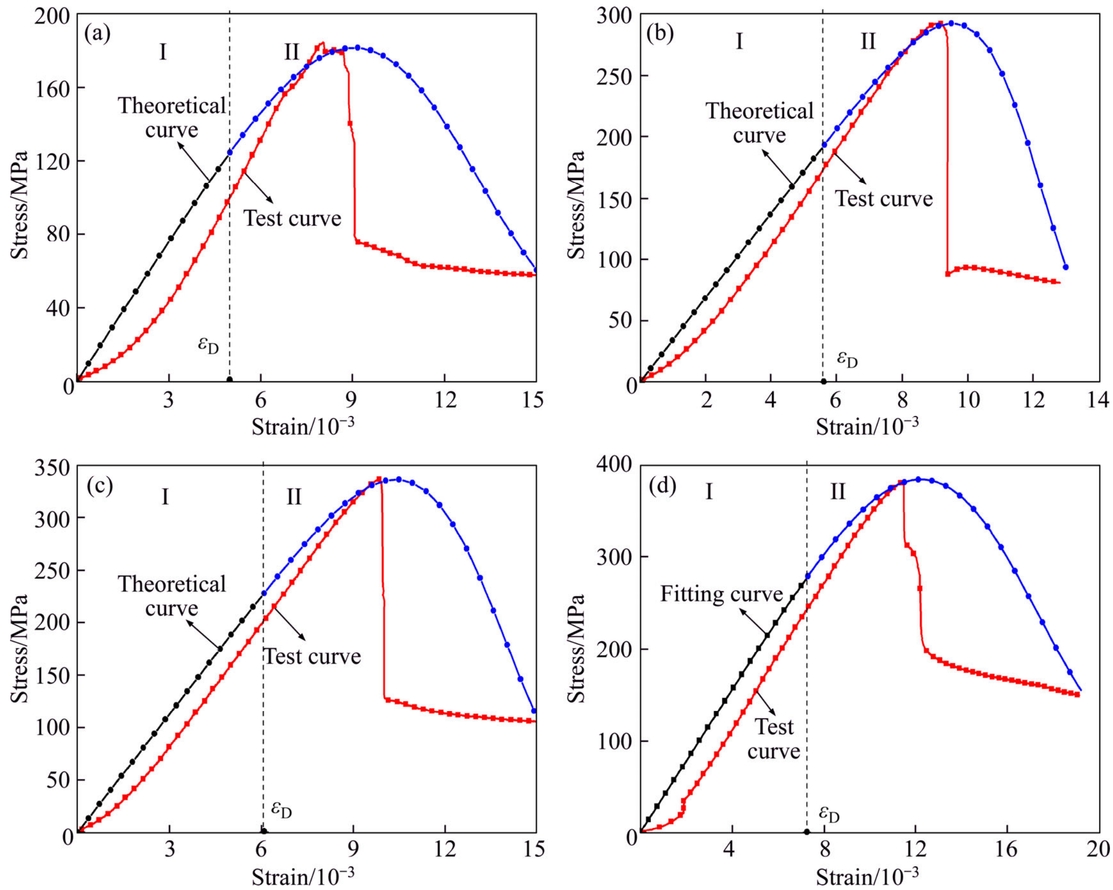

Figure 3 Theoretical stress–strain curves and test curves at 600 °C in different confining pressures: (a) 10 MPa; (b) 20 MPa; (c) 30 MPa; (d) 40 MPa

which can reflect rock post-peak softening process by introducing the damage correction coefficient to consider the residual strength, it proves the rationality of the model.

However, there are also some deviations between theoretical stress–strain curves and test curves. Such as, the test curve reflects the four stages of rock compaction, linear elasticity, yield and post-peak strength well, while theoretical curve before damage threshold is considered as quadratic polynomial function, which cannot reflect the initial compaction stage of rock, resulting the value of theoretical stress is larger than the test at the same strain before peak stress.

Therefore, in order to establish a constitutive model that is more suitable for rock actual deformation, it is necessary to describe the compaction stage of the initial stress–strain curve. According to author's previous study [27, 28], the compaction coefficient K may be introduced into the damage constitutive equation, and K is defined as the ratio of the slope of rock stress–strain curve, this part will be studied in the future.

5 Physical meaning of distribution parameters

Through the mathematical calculation, the curves of Weibull distribution parameters (F and m) with the change of temperature and confining pressure are given in Figures 4–7. The physical meaning of the parameters is discussed as following.

As can be seen from Figures 4 and 5, the distribution parameter F increases linearly with the rise of confining pressure before 600 °C, but when temperature exceeds 600 °C, F no longer continues to increase with the confining pressure, which is consistent with the variation of the rock triaxial peak strength with the confining pressure (See Figure 6), indicating that the parameter F can represent the average strength of the rock.

It can be seen from Figure 7 that the variation of the distribution parameter m with the confining

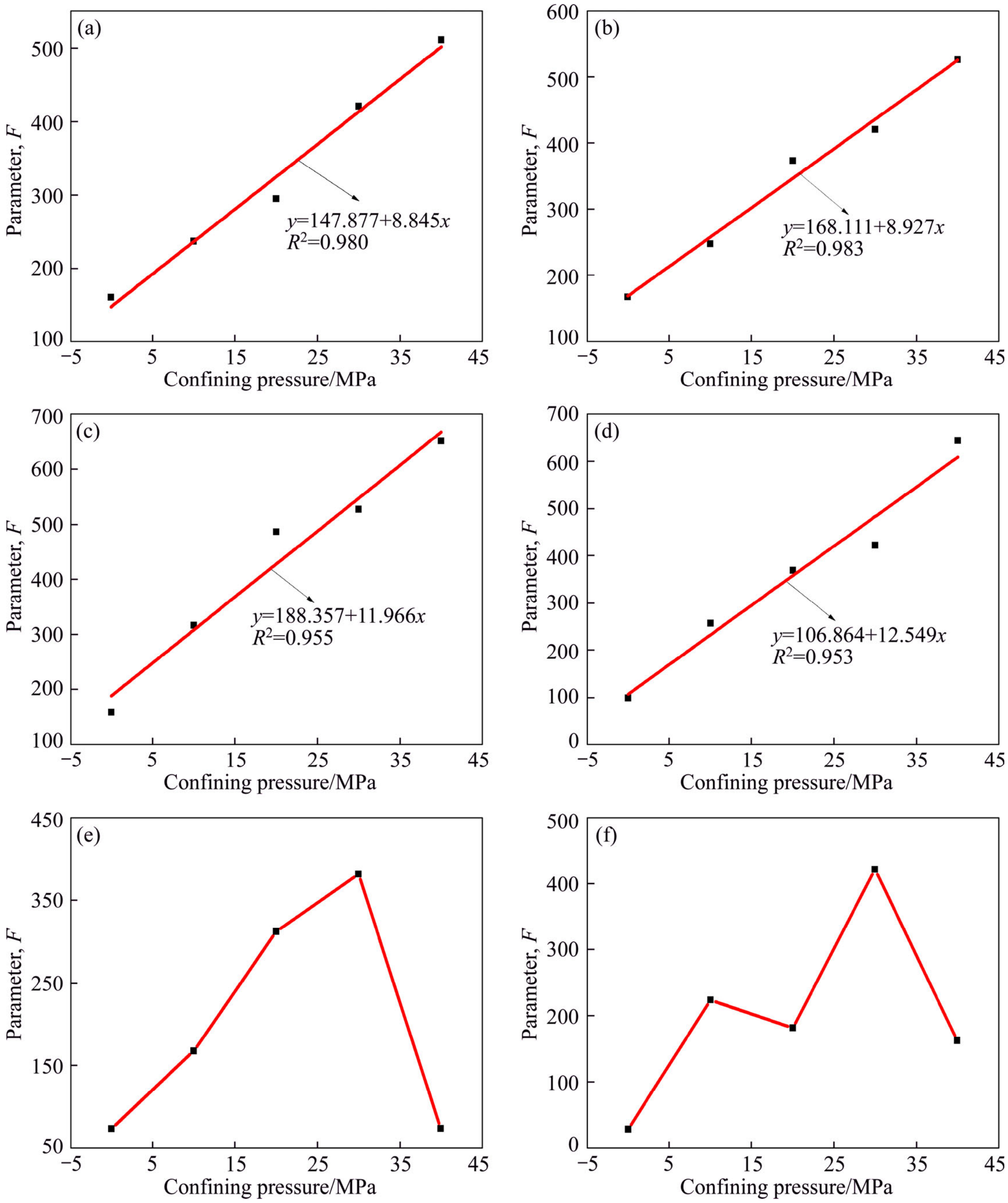

Figure 4 Fitting curve of distribution parameter F with confining pressure: (a) 250 °C; (b) 200 °C; (c) 400 °C; (d) 600 °C; (e) 800 °C; (f) 1000 °C

pressure can be divided into three stages. The parameter m firstly increases when the confining pressure increases from uniaxial to 10 MPa, then fluctuates with the confining pressure rises from 10 to 30 MPa, and finally shows downward trend as the confining pressure increases from 30 to 40 MPa under different temperatures. According to the analysis of a large number of experimental data, MOGI [29] found that the critical confining pressure of brittle to plastic transition is σ_1/σ_3=3.4. In this experiment, the uniaxial compressive strength of granite is 120.37 MPa, which can be inferred that the rock critical confining pressure of brittle to plastic transition is around 35.40 MPa.

In order to verify the critical confining pressure of granite from brittle to plastic transition, Figure 8 shows the scanning electron microscopy (SEM) image of rock fracture with the confining pressure of 40 MPa under different temperatures.

It can be seen that when the confining pressure reaches 40 MPa, there are many parallel slip textures and multiple dimples on the fracture

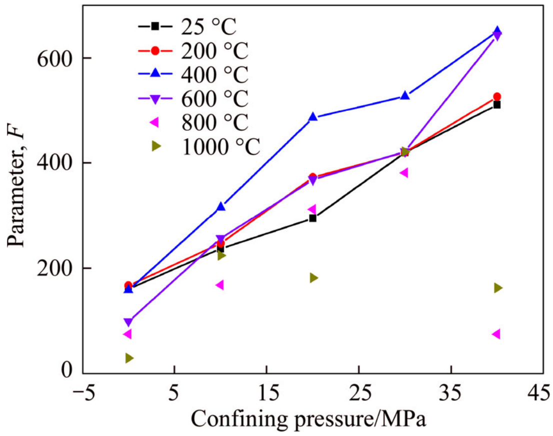

Figure 5 Parameter F with confining pressure

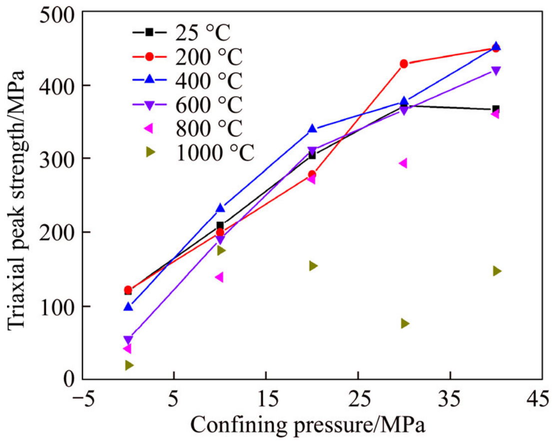

Figure 6 Triaxial peak strength with confining pressure

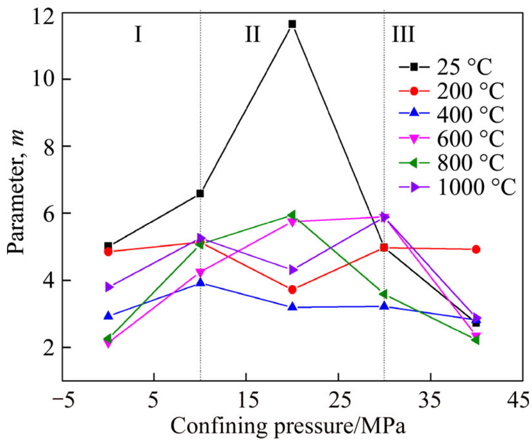

Figure 7 Distribution parameter m with confining pressure

images of the rock at different temperatures. As the temperature increases, the dimples become more and more, indicating that the rock had a plastic fracture pattern.

When the rock shows plastic deformation, the internal slip will be produced along the crystal interface. Due to the mutual restraint between different grains, the slippage within the rock must be carried out in multiple slip zones, resulting in multiple parallel slip textures on the fracture image. In rock, the slip separation feature often appears as curved stripes, and hence it is called a serpentine slip pattern. During the deformation process, the slip deformation is first carried out along a set of surfaces parallel to the maximum shear stress plane, and a new surface is formed due to the separation of the slip surface. In the multiple slip, these new surfaces are curved and staggered to form a meandering slip, forming a dense serpentine slip pattern. The dimple is the most obvious meso-characteristic morphology of the plastic fracture surface of the rock, and it is mainly composed of small pits in the fracture images. The formation of the dimple is the result of the accumulation of voids inside the rock. There are a lot of voids in the rock, and a certain slip surface will be formed between the voids under the action of external stress, then the voids gradually grow and eventually penetrate each other in the role of slip, resulting in dimples.

According to the analysis of above rock fracture image, it is reasonable to conclude that the brittle-plastic critical confining pressure of this granite is about 35.40 MPa. The decrease of the parameter m when the confining pressure increases from 30 to 40 MPa means that rock has a brittle to plastic transformation, so m can represent the morphological characteristics of the rock stress–strain curve.

6 Conclusions

The outcomes of this research indicate that the established thermal damage model is in agreement with the experimental phenomena under the action of temperature and confining pressure, which provides a new way from the point of micromechanical damage response to estimate the deformation process and reveal the failure mechanism of deep rocks. The results from the findings can be summarized as follows.

1) The stress–strain curve of the rock is essentially a crack-dominated deformation failure process. With the increase of strain, the micro-cracks are generated, accumulated, and then gradually connected as macroscopic cracks until completely destroyed, corresponding to the rock development process of initial damage, stable

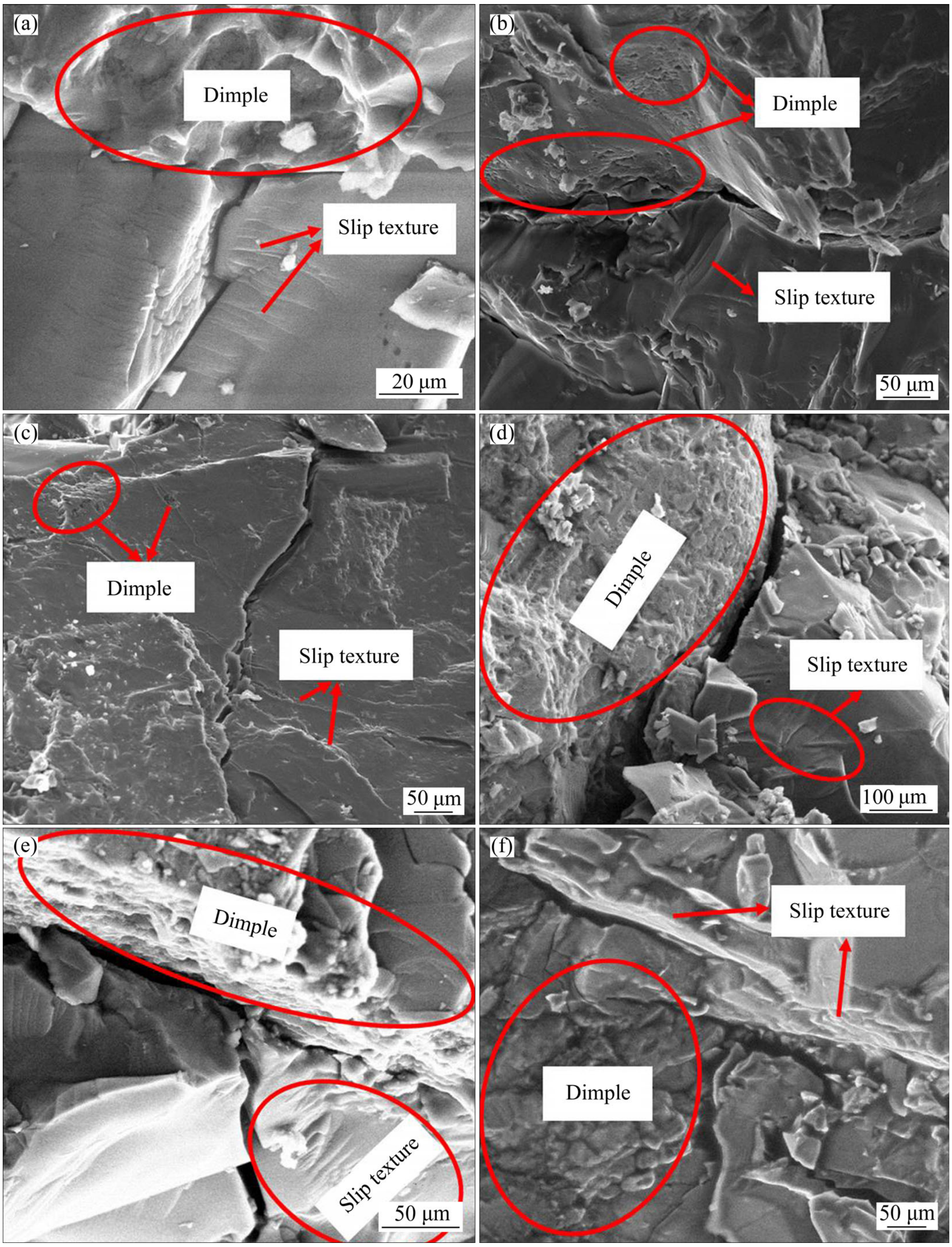

Figure 8 SEM image of rock fracture at confining pressure of 40 MPa: (a) 25 °C; (b) 200 °C; (c) 400 °C; (d) 600 °C; (e) 800 °C; (f) 1000 °C

expansion, acceleration, until the damage variable tends to 1. The micromechanical response is consistent with the macroscopic mechanical properties of the rock.

2) The thermal damage increases logistically with the rise of temperature, demonstrating the deterioration of the mechanical properties of rock. Under the action of high temperature, the thermal motion of the rock molecules intensifies and the thermal expansion of the various minerals crossing the grain boundary is uncoordinated, resulting in a large number of thermal cracks, which are extended and penetrated with the increase of the temperature.

3) The main role of confining pressure is to restrain the expansion of micro-cracks in the rock. The degree of total damage is reduced as the confining pressure increases, indicating that the confining pressure enhances the rock resistance and

plastic deformation. According to the analysis of rock fracture image, it is reasonable to conclude that the brittle-plastic critical confining pressure of this granite is about 35.40 MPa.

4) The distribution parameters (F and m) have good physical meanings. The parameter F represents the average strength of the rock, and m represents the morphological characteristics of the rock stress–strain curve.

Although the theoretical models established in this paper are generally consistent with the test curves, there are also some deviations. Therefore, to establish a constitutive model that is more suitable with the rock actual deformation, more work is required on the basis of this article. Such as not only the other statistical distribution function can be re-selected, but also different rock strength criteria can be re-chosen according to the practical engineering.

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中文导读

考虑残余强度的岩石热损伤统计本构模型研究

摘要：随着地球浅部矿物资源逐渐枯竭，资源开采不断走向地球深部，模拟深部高温高压条件下岩石应变软化变形破坏行为是岩石力学研究的重要内容。本文基于 Lemaitre 应变等价性理论，结合统计学和损伤力学，同时引入损伤变量修正系数考虑岩石残余强度对峰后曲线的影响，建立了三轴压缩条件下岩石热力耦合统计损伤本构方程，并通过试验验证模型的合理性。研究结果表明：温度–荷载总损伤演化曲线反映了岩石内部微裂纹闭合、萌生、扩展、贯通、直至出现宏观裂纹的全过程；岩石的热损伤变量随温度的升高呈 logistic 函数增长；同等应变情况下，损伤变量随围压的升高而减小，通过研究岩石破裂后 SEM 电镜图片，推断出 35.40 MPa 是花岗岩的脆塑性转换临界围压；模型分布参数 F 反映了岩石的平均强度，m 反映了岩石应力–应变曲线的形态特征。该模型不仅能反映温度、围压对岩石损伤的影响，而且能较好地反映岩石峰后残余强度阶段变形特征，模型参数物理意义明确，适用于复杂应力状态情况，这对于研究深部岩石损伤软化问题具有重要的意义。

关键词：损伤本构；温度效应；围压；损伤临界值；残余强度