



# Fatigue life prediction of workpiece with 3D rough surface topography based on surface reconstruction technology

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**Abstract:** The fatigue performance of a workpiece depends on its surface quality. In traditional fatigue life prediction, the effect of surface quality is commonly accounted for by using empirical correction factors, which is imprecise when safety is of great concern. For surface quality, the surface topography is an important parameter, which introduces stress concentration that reduces the fatigue life. It is not feasible to test the stress concentration of different surface topographies. On the one hand, it is time-consuming and high-cost, and on the other hand, it cannot reflect the general statistical characteristics. With the help of surface reconstruction technology and interpolation method, a more efficient and economic approach is proposed, where FE simulation of workpiece with the reconstructed surface topography is used as a foundation for fatigue life prediction. The relationship between surface roughness ( $S_a$ ) and fatigue life of the workpiece is studied with the proposed approach.

**Key words:** fatigue life; stress concentration; surface reconstruction technology; surface topography

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## 1 Introduction

Fatigue crack failure of workpiece surface generally appears because of the stress concentration. Surface integrity affects the fatigue failure of a workpiece. In traditional prediction of fatigue life, the effect of surface quality is investigated by using empirical correction factors [1], which is imprecise when safety is of great concern. For surface quality, the surface topography is an important parameter, which introduces stress concentration that reduces the fatigue life. Besides, fatigue life differs significantly with surface topography [2, 3]. Therefore, accurate prediction of

fatigue life considering workpiece surface topography is of great importance. In traditional engineering design, the fatigue data come from the sample test, but it is not feasible to test the stress concentration of different surface topographies. On one hand, it is time-consuming and high-cost, and on the other hand, it cannot reflect the general statistical characteristics. Therefore, with the help of surface reconstruction technology, FE simulation of workpiece with the reconstructed surface topography is used as a foundation for fatigue life prediction [4, 5].

There are two ways to obtain a 3D rough surface topography: one is to obtain the surface topography height sequence matrix  $Z(M, N)$  with

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measuring instruments such as white light interferometers, in this case, the precision will be improved and the time consumed will be increased while the sampling interval is decreased. The other is to get the surface topography height sequence matrix  $Z(M, N)$  [4] through the reconstruction method based on statistical distribution parameters and exponential autocorrelation function. With this method, the surface topography with specific parameters can be obtained efficiently. In recent years, research on the effects of surface topography has been increasingly conducted based on surface reconstruction technology. YANG et al [6] and PU et al [7] studied the contact performance of surface topography generated by surface reconstruction technology. In the study of surface topography considering the effect of stress concentration on the fatigue life, the theoretical stress concentration factor is introduced:  $K_t = \sigma_{\max} / \sigma_0$ , where  $\sigma_{\max}$  is the maximum stress and  $\sigma_0$  is the nominal stress. NEUBER [8] and AROLA et al [9] treated surface topography as a series of microscopic notches and proposed expressions for the stress concentration factor imposed by statistical parameters of the surface topography. Statistical parameters can only describe the average characteristics of a surface topography so that a large volume of information about the surface topography will be lost. Therefore, geometric modeling based on statistical characterization parameters is considered inadequate, and consequently an increasing number of researchers have conducted geometric modeling analysis directly based on real surface topography [10–12]. ÅS et al [10] and SURARATCHAI et al [11] used the finite element method to calculate the stress concentration factor of 2D profiles from the measured 3D surface topography, and predicted the fatigue life of the workpiece through fatigue test. ÅS et al [12] provided a geometric modeling analysis of the measured 3D surface topography to calculate the stress concentration factor.

Compared with the aforementioned researches, effects of the reconstructed surface topography on workpiece are found from FE simulation, which is more efficient and economic. The researches about the properties of rough surface and the characterization of surface topography we have done are the basis of this work [13–15]. The layout of the paper is as follows: 1) The discrete points of the rough surface topography with specific

parameters are first obtained by the surface reconstruction technology and the Bezier interpolation method is used to produce more data points between sampling points to improve the fitting precision. The parameters of the reconstructed surface topography are calculated. 2) All the data points are then imported into CATIA through point cloud technology to simulate the 3D rough surface of the workpiece. The finite element model is established to calculate the stress concentration factor and the fatigue notch factor, and the fatigue life is finally determined by the local stress–strain method. 3) The relationship between surface roughness  $S_a$  and fatigue life of the workpiece is studied with the proposed approach.

## 2 Fatigue life prediction of workpiece with reconstructed surface topography

### 2.1 Reconstruction of 3D surface topography

Many researchers have characterized the engineering surface topography with the application of random process theory, which makes it possible to generate surface topography by random simulation. Here, the aim is to reconstruct a 3D surface topography: a  $(M, N)$  matrix of surface topography that follows a specified statistical distribution and a given  $(m, n)$  ACF can be carried out by the following transformation [4]:

$$Z_{ij} = \sum_{k=1}^m \sum_{l=1}^n \alpha_{kl} \eta_{i+k, j+l} \quad i = 1, 2, \dots, M, j = 1, 2, \dots, N \quad (1)$$

where  $\alpha_{kl}$  is a coefficient to be determined in order to produce the desired ACF. Since  $\eta_{ij}$  is independent and has unit variance:

$$E(\eta_{ij} \eta_{kl}) = \begin{cases} 1, & \text{if } i = k, j = l \\ 0, & \text{if otherwise} \end{cases} \quad (2)$$

Then

$$R_{pq} = E(Z_{ij} Z_{i+p, j+q}) = \sum_{k=1}^{m-p} \sum_{l=1}^{n-q} \alpha_{kl} \alpha_{k+p, l+q} \quad (3)$$

Equation (3) represents a non-linear system of  $nm$  equations for the determination of the coefficient  $\alpha_{kl}$ . During the solution, the NCGM was implemented to deal with the convergence and space problems [16].

### 2.2 Interpolation

There are no data points in the interval due to

the existence of sampling interval, which affects the precision of point cloud fitting. Therefore, the height sequence matrix  $Z(M, N)$  needs interpolation management. The height sequence matrix  $Z(M, N)$  is divided into  $(M-1) \times (N-1)$  regions with four adjacent points in  $X/Y$  directions taken as a region. Each region is interpolated with 12 points to generate a control matrix  $V_{4 \times 4}$  with 16 control points, and the Bezier surface matrix is used to generate the height matrix. As such, the piecewise linear interpolation with singular points can be avoided. Specific steps are as follows:

1) Determination of tangent vector:

$$k = \frac{\text{gradient}(\mathbf{p})}{\text{norm}(\text{gradient}(\mathbf{p}))} \quad (4)$$

where  $\mathbf{p}$  is the discrete data point matrix on the curve, gradient is the central difference, and norm is the vector norm.

2) Determination of control points  $b$  and  $c$  between two discrete points on the curve:

$$\begin{cases} b = p(i) + \alpha k(i) \\ c = p(i+1) - \alpha k(i+1) \end{cases} \quad (5)$$

where  $i \in \{1, 2, \dots, N-1\}$ ,  $\alpha = \mu \cdot \text{norm}(\text{gradient}(\mathbf{p}))$  is the control coefficient, and  $\mu$  is just a constant whose value usually equals  $1/2$ .

3) Four points in each region can build 6 lines, and calculation is made based on steps 1) and 2) to get 12 control points which are combined with the original 4 measuring points to constitute the control matrix  $V_{4 \times 4}$ , which provides direction information of interpolation points. With  $4t+t^2$  ( $t \in \{0, 1, 2, \dots\}$ ) points interpolated in each region, the control matrix  $V_{4 \times 4}$  and the matrix form of Bezier surface are used to calculate the new height matrix  $Z(t+2, t+2)$  in each region. The interpolation of a region is illustrated in Figure 1.

$$\begin{cases} Z(i, j) = B^T(u(i))V_{4 \times 4}B(v(j)) \\ B(x) = \begin{bmatrix} 1 & -3 & 3 & -1 \\ 0 & 0 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^3 \\ x^2 \\ x \\ 1 \end{bmatrix} \\ u(i) = (i-1)/(t+1) \\ v(j) = (j-1)/(t+1) \\ i \in \{1, 2, 3, \dots, t+2\} \\ j \in \{1, 2, 3, \dots, t+2\} \\ t \in \{0, 1, 2, \dots\} \end{cases} \quad (6)$$

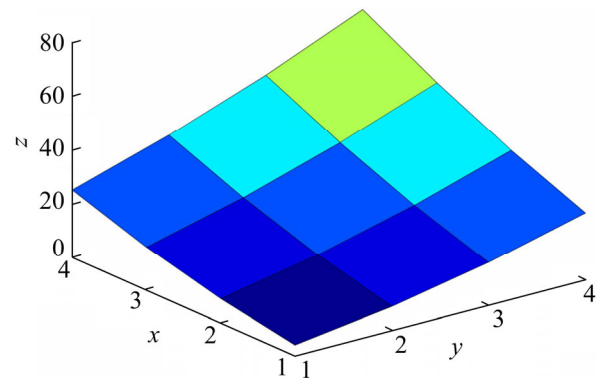


Figure 1 Schematic diagram of region with interpolation

where  $Z(i, j)$  is the height of point of the  $i$ th row and the  $j$ th column in region after interpolation, and  $B(x)$  is the Bernstein basis function.

4) Each small region after interpolation is spliced to form a new height matrix  $Z((M-1)t+M, (N-1)t+N)$ .

### 2.3 Fatigue life

The surface topography data points after interpolation are imported into 3D modeling software through point cloud technology to generate micro patch groups. 3D entities are first modeled and generated through surface joining and stretching operations, and then imported into finite element software. As the surface topography is small and complex, the tetrahedral mesh technique is adopted and the mesh density is controlled by the element size which is determined by the surface curvature. The mesh is shown in Figure 2. A small stress ( $\sigma$ ) is applied on both sides of the 3D model, fixed constraints are added at the bottom, and the calculation results are shown in Figure 3. From the finite element calculation results shown in Figure 3, it can be known that there exists obvious stress concentration in the valley location. After the stress results of the 3D surface topography are processed, the average stress of the effective valleys is calculated and taken as the maximum stress ( $\sigma_{\max}$ ). A small stress ( $\sigma$ ) is applied on both sides as the nominal stress ( $\sigma_0$ ), and the stress concentration factor can be calculated from the formula ( $K_t = \sigma_{\max} / \sigma_0$ ). The fatigue stress concentration factor is calculated based on its relationship with the stress concentration factor ( $K_f = 1 + q(K_t - 1)$ ), where Notch sensitivity ( $q$ ) is calculated by the PETERSON [3] formula according to the value of the material

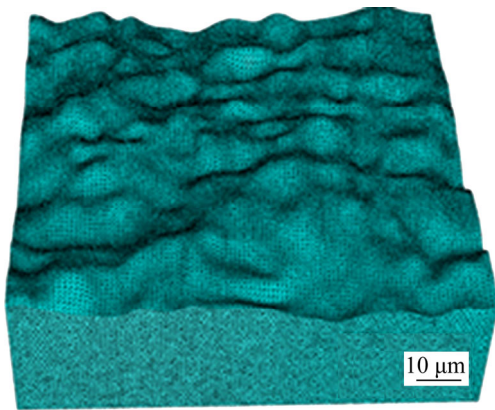


Figure 2 Finite element mesh

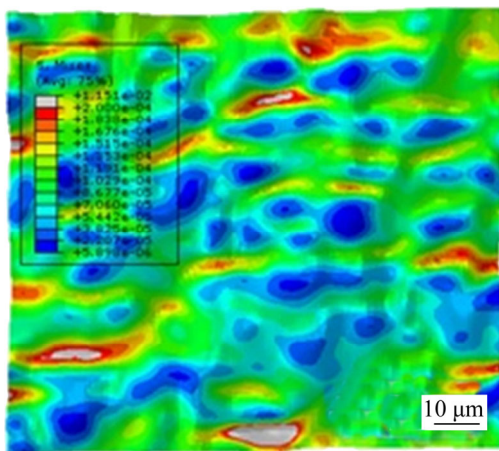


Figure 3 Stress nephogram

parameter and the effective valley root radius ( $\rho$ ).

If the strain of component in smooth condition is  $\varepsilon$ , the strain of component is calculated by the linear theory when the stress concentration of the surface topography is taken into account:  $\varepsilon_1 = K_f \varepsilon$ . According to the actual strain of the workpiece with surface topography, and assuming the load ratio is  $R = -1$ , the local stress strain method is used to predict the fatigue life of the workpiece with surface topography based on Manson-Coffin equation:

$$\Delta\varepsilon / 2 = \Delta\varepsilon_e / 2 + \Delta\varepsilon_p / 2 = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (7)$$

where  $\Delta\varepsilon$  is the total strain,  $\Delta\varepsilon_e$  is the elastic strain,  $\Delta\varepsilon_p$  is the plastic strain,  $\sigma'_f$  is the fatigue strength coefficient,  $b$  is the fatigue strength index,  $N_f$  is the fatigue life of cycle counting,  $\varepsilon'_f$  is the fatigue ductility coefficient, and  $c$  is the fatigue ductility index. The core idea of this method is that the fatigue life is identical as long as the maximum local stress and strain are equal. The fatigue life of the workpiece with concentrated stress is

considered equal to that of the smooth specimen with the same local stress and strain.

### 3 Relationship between surface roughness $S_a$ and fatigue life

Surface topography height sequence matrices in grinding with specific roughness  $S_a$  are obtained through surface reconstruction technology, and the size of the reconstructed surface is  $100 \mu\text{m} \times 100 \mu\text{m}$ , as shown in Figure 4. 17CrNiMo6 steel is selected as the element material. Our team is engaged in gear research, and 17CrNiMo6 steel is a commonly used high-performance gear material. On one hand, the material parameters are easy to obtain. On the other hand, the ultimate strength of the material is high, and the stress concentration effect of the surface topography is more obvious. The material parameters are shown in Table 1. The parameters are calculated by programming according to the reconstructed surface topography height sequence matrices, as shown in Table 2. Among those parameters, the effective valley root radius ( $\rho$ ) plays an important role in calculating the notch sensitivity ( $q$ ). In practice, it is difficult to measure the radius of valley. In Ref. [8], a graphical radius gage is used to estimate the radius, which is cumbersome and low in accuracy. In this article, the valley of the 3D surface topography is assumed to be spherical, and the three-point circle method is utilized to calculate the radius of valley. Nine points of matrix  $Z(i-1:i+1, j-1:j+1)$  including valley point  $Z(i, j)$  can construct four triangles through the valley point. As the distance between points and the height are known, three sides of the triangle can be calculated by the Pythagorean theorem, and the radius  $R$  can be calculated by the cosine theorem and triangle circumcircle radius formula, as shown in Figure 5.

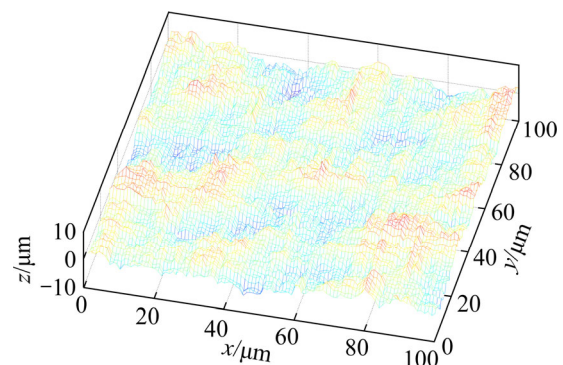


Figure 4 Reconstructed rough surface in grinding of  $2 \mu\text{m} S_a$

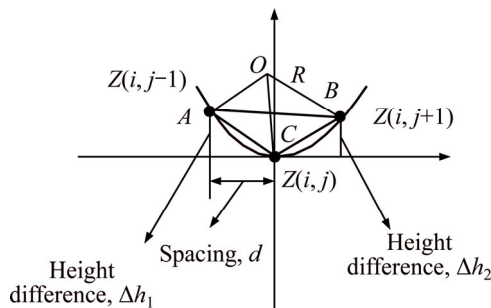


**Table 1** Material characteristic parameters of 17CrNiMo6

Parameter	Value
Elasticity modulus, $E/\text{GPa}$	206
Poisson ratio	0.3
Yield limit, $\sigma_s/\text{MPa}$	785
Ultimate strength, $\sigma_b/\text{MPa}$	1250
Fatigue strength coefficient, $\sigma'_f/\text{MPa}$	1875
Fatigue strength index, $b$	-0.087
Fatigue ductility coefficient, $\epsilon'_f$	0.364
Fatigue ductility index, $c$	-0.58

**Table 2** Characteristic parameters of reconstructed rough surface of 17CrNiMo6 ( $\mu\text{m}$ )

Reconstructed $S_a$	Actual $S_a$	$\rho$	$q$
0.3	0.3126	1.1506	0.0182
0.5	0.5155	1.2006	0.019
0.8	0.8324	1.3220	0.0209
1	0.9964	1.4145	0.0223
2	1.9735	3.0451	0.0468
4	4.0117	6.0229	0.0886
6	6.2127	10.8152	0.1486



**Figure 5** Calculation of valley root radius

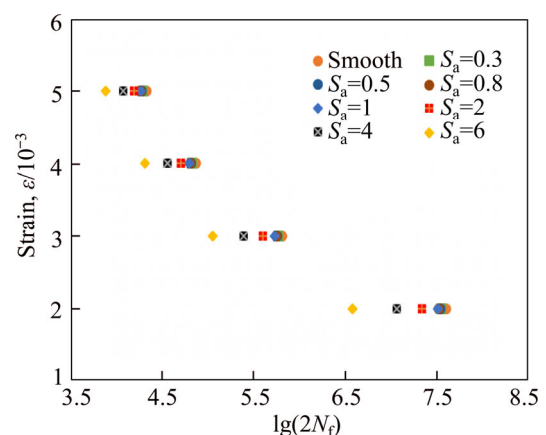
The average value of four triangular circumradii is taken as the valley root radius. The effective valley root radius ( $\rho$ ) is calculated by this method.

All the data points of the surface topography with specific roughness  $S_a$  are then imported into CATIA through point cloud technology to simulate the 3D rough surface of the workpiece, and the finite element models are established to calculate the stress concentration factor and the fatigue notch factor. The results are shown in Table 3. According to the results of the stress concentration factor and the fatigue stress concentration factor, the fatigue life of 17CrNiMo6 with different values of roughness  $S_a$  is predicted by the local stress–strain method. The fatigue life prediction results are

shown in Figure 6. It can be seen from the figure that the fatigue life of 17CrNiMo6 increases with the decrease of average roughness  $S_a$ , as the stress concentration becomes more obvious with the increase of roughness. Similar results have been reported [17]. When the strain is 0.002, the fatigue life at the roughness  $S_a$  of 0.8  $\mu\text{m}$  is decreased by 5.81% compared to the result at the roughness  $S_a$  of 0.3  $\mu\text{m}$ ; the fatigue life at the average roughness  $S_a$  of 0.5  $\mu\text{m}$  is decreased by 1.52% compared to the result at the roughness  $S_a$  of 0.3  $\mu\text{m}$ . By contrast, the decrease is not obvious because the depth, width and root radius of valley are so small that play a minor role in the effect of stress concentration when the roughness  $S_a$  is very small, so the influence on fatigue life becomes smaller when roughness  $S_a$  is less than 0.5  $\mu\text{m}$ . When the strain is 0.005, the fatigue life at the roughness  $S_a$  of 6  $\mu\text{m}$  is decreased by 62.44% compared to the result at the roughness  $S_a$  of 0.3  $\mu\text{m}$ ; when the strain is 0.002, the fatigue life at the roughness  $S_a$  of 6  $\mu\text{m}$  is decreased by 89.45% compared to the result at the roughness  $S_a$  of 0.3  $\mu\text{m}$ . By contrast, the fatigue life drops more significantly in the high cycle fatigue range, and the

**Table 3** Results of stress concentration factor and fatigue stress concentration factor

$S_a/\mu\text{m}$	$K_t$	$K_f$
0.3126	1.57	1.0104
0.5155	1.62	1.0118
0.8324	1.76	1.0158
0.9964	1.85	1.0189
1.9735	2.18	1.0552
4.0117	2.36	1.1205
6.2127	2.65	1.2452



**Figure 6** Fatigue life prediction of workpiece made of 17CrNiMo6 with different values of roughness  $S_a$

effect of stress concentration is obvious in low stress and strain level. The influence of roughness  $S_a$  on the fatigue life in the high cycle fatigue range is greater than that in the low cycle fatigue range.

## 4 Conclusions

1) The surface topography with specific parameters is obtained through surface reconstruction technology. Interpolation is performed before point cloud fitting to make the surface smoother and improve the fitting accuracy. The valley bottom of the 3D surface topography is assumed to be spherical, and the three-point circle method is used to calculate the radius of curvature.

2) An efficient and economic fatigue life prediction method of workpiece considering 3D rough surface topography is proposed based on surface reconstruction technology. The relationship between surface roughness  $S_a$  and fatigue life of the workpiece is studied with the proposed approach. For the workpiece made of 17CrNiMo6 material, numerical calculation shows that the fatigue life increases with the decrease of roughness  $S_a$ , but the influence on fatigue life becomes smaller when the roughness  $S_a$  is less than 0.5  $\mu\text{m}$ . The influence of roughness  $S_a$  on the fatigue life in the high cycle fatigue range is greater than that in the low cycle fatigue range.

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## 中文导读

### 基于表面重构技术预测考虑三维表面形貌的工件的疲劳寿命

**摘要：**本文研究工件三维表面形貌的应力集中效应对工件疲劳寿命的影响。通过实验测试不同表面三维微观形貌的应力集中影响是不合适的，一方面需要大量的时间和成本，另外一方面单个工件的表面形貌不能反应统计特征。因此，本文提出一种更经济、更有效的方法基于表面重构技术和插值技术：利用重建后插值得到的工件三维表面形貌采用有限元方法进行受力分析，将应力云图结果作为疲劳寿命预测的基础，研究了工件表面粗糙度  $S_a$  与工件疲劳寿命之间的关系。

**关键词：**疲劳寿命；应力集中；表面重构技术；表面形貌