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Robust adaptive control for a class of uncertain non-affine nonlinear systems using neural state feedback compensation

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Abstract: A robust adaptive control is proposed for a class of uncertain nonlinear non-affine SISO systems. In order to approximate the unknown nonlinear function, an affine type neural network (ATNN) and neural state feedback compensation are used, and then to compensate the approximation error and external disturbance, a robust control term is employed. By Lyapunov stability analysis for the closed-loop system, it is proven that tracking errors asymptotically converge to zero. Moreover, an observer is designed to estimate the system states because all the states may not be available for measurements. Furthermore, the adaptation laws of neural networks and the robust controller are given based on the Lyapunov stability theory. Finally, two simulation examples are presented to demonstrate the effectiveness of the proposed control method. Finally, two simulation examples show that the proposed method exhibits strong robustness, fast response and small tracking error, even for the non-affine nonlinear system with external disturbance, which confirms the effectiveness of the proposed approach.

Key words: adaptive control; neural networks; uncertain non-affine systems; state feedback; Lyapunov stability

1 Introduction

In the past decades, the control problem of the uncertain nonlinear systems has been given a lot of attention in the control field. Many powerful methodologies for designing the controller are proposed for uncertain nonlinear systems owing to advances in nonlinear control theory.

Neural network (NN) approaches and fuzzy logic (FL) have been widely used in modelling and controlling of nonlinear systems because of their capabilities of nonlinear function approximation, learning, adaption and generalization. Based on this universal approximation property, many adaptive neural network control schemes have been developed to solve control problem of the uncertain nonlinear systems. The adaptive neural network control (ANNC) schemes were proposed by using NNs for uncertain nonlinear systems [1–3]. In such schemes, an NN was employed to approximate the uncertain nonlinear functions. There are parameter variations, unavoidable model uncertainties and unknown external disturbances for any practical system. These uncertainties will degrade the controller performance. In these cases, the conventional control approaches are not applicable, and usually the robust control approaches are suggested to address this issue. Therefore, the analytical study of robust adaptive control of uncertain nonlinear systems using NN has received much attention during last decade. The robust adaptive neural network control (RANNC) problem for a class of nonlinear systems with external disturbance was investigated [4–5].

It is difficult to describe dynamic models by using accurate math models in many nonlinear systems. Fuzzy logic control (FLC) is a rule-based type of control that uses fuzzy set concepts and fuzzy logic. It can deal with complex and ill-defined systems for which the application of conventional control techniques is not straightforward or feasible. The problem of adaptive fuzzy tracking control was presented for the external disturbances of a class of uncertain nonlinear systems [6–8].

In the last decade, many researchers proposed that the leaning abilities of NNs can be used as a powerful tool in fuzzy system design. The adaptive algorithms based on fuzzy neural networks (FNNs) were developed for the tracking control of the nonlinear system [9–11]. The proposed algorithm was based on the FNNs controllers. The FNN was used to online control the nonlinear system.

Variable structure control (VSC) which plays an important role in the nonlinear control theory is a type of robust control design. The VSC is insensitive to parameter uncertainties, modelling error and external disturbances. As a result, there is high motivation to variable structure control in the control of these uncertain nonlinear systems [12–13].

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However, the states of the system were assumed to be available in most of the previous works, while this assumption may not be satisfied in practical applications. In these cases, observer-based or output feedback controllers might be used.

Varieties of hybrid control systems have been designed by using different combinations of above methods [14–16]. However, the above mentioned studies had the condition that the systems or subsystems should be affine. In practice, there are many nonlinear systems with non-affine structure.

Recently, several approaches have been developed for nonlinear non-affine systems [17–19]. The tracking control problem was studied for a class of uncertain non-affine systems by SHEN and ZHANG [17]. Based on the principle of sliding mode control (SMC), using the neural networks (NNs) and the property of the basis function, a novel adaptive design schemes were proposed. BAHRAM and MOHAMMAD [18] introduced a new decentralized adaptive neural network controller for a class of large-scale nonlinear systems with unknown non-affine subsystems and unknown interconnections represented by nonlinear functions. A radial basis function neural network was used to represent the controller's structure. A novel fuzzy adaptive controller was investigated for a class of non-affine systems with unknown control direction by BOULKROUNE et al [19]. An equivalent model in affine-like form was first derived for the original non-affine system by using a Taylor series expansion. Then, a fuzzy adaptive control was designed based on the affine-like equivalent model. The stability of the closed loop system was guaranteed through Lyapunov stability analysis.

In this work, a robust adaptive control scheme is proposed for a class of the uncertain nonlinear non-affine SISO systems. The mean value theorem is used to transform unknown non-affine system into a similar affine system. Moreover, an affine type neural network and neural state feedback compensation are employed to approximate the unknown nonlinear function, and observer is used to estimate unmeasured states. The effects of neural network approximation error and external disturbance are compensated using a robust term in the control signal. Furthermore, in the proposed approach, the controller singularity issue is avoided, and this approach can also be applied to affine systems.

2 Problem formulation

Consider a SISO uncertain nonlinear system described by

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x}_n = f(\boldsymbol{X},u) + d(\boldsymbol{X},t) \quad (i=1,\ 2,\ \cdots,\ n-1) \\ y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{X} = [x,\ \dot{x},\ \cdots,\ x^{(n-1)}]^{\mathrm{T}} = [x_1,\ x_2,\ \cdots,\ x_n]^{\mathrm{T}} = [y,\ \cdots,\ y^{(n-1)}]^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of the system, $y \in \mathbf{R}$ is the system output, $u \in \mathbf{R}$ is the control input, $f(\boldsymbol{X},u)$ is smooth unknown non-affine function in control input and the state vector, and $d(\boldsymbol{X},t) \in \mathbf{R}$ is a unknown external disturbance. The control goal is to design a controller that the system output $y(t)$ tracks the desired trajectory $y_{\mathrm{d}}(t)$.

Assumption 1: The desired trajectory $y_{\mathrm{d}}(t)$ and its time derivatives $y_{\mathrm{d}}^{(i)}(t),\ i=1,\ \cdots,\ n$, are smooth and bounded.

Assumption 2: The external disturbance $d(\boldsymbol{X},\ t)$ is bounded:

$$|d(\boldsymbol{X},t)| \le \varepsilon_{\mathrm{d}},\ \ \forall \boldsymbol{X},\ t$$

Assumption 3: For all $(\boldsymbol{X},u) \in \Omega_x \times \mathbf{R}$ with a controllability region Ω_x, the function $g(\boldsymbol{X},u)=\dfrac{\partial f(\boldsymbol{X},u)}{\partial u}$ is nonzero.

Lemma 1 (Mean value theorem): Suppose that $f(x,y): \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$ is continuous at both endpoints $y=a$ and $y=b$, and has a derivative at each point of an open set $\mathbf{R}^n\times(a,\ b)$. Therefore, there is a point $\pi \in (a,b)$:

$$f(x,b) - f(x,a) = f'(x,\pi)(b-a) \tag{2}$$

The tracking error vector is defined as

$$\boldsymbol{E}(t) = [e_1,\ \dot{e}_1,\ \cdots,\ e_1^{(n-1)}]^{\mathrm{T}} \tag{3}$$

where e_1 is the output error:

$$e_1 = y - y_{\mathrm{d}} \tag{4}$$

Using Eqs. (1) and (3), the tracking error system can be written as

$$\begin{cases} \dot{\boldsymbol{E}} = \boldsymbol{AE} + \boldsymbol{B}[f(\boldsymbol{X},u) - y_{\mathrm{d}}^{(n)} + d(\boldsymbol{X},t)] \\ e_1 = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{E} \end{cases} \tag{5}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},\ \boldsymbol{C}^{\mathrm{T}} = [1\ \ 0\ \ \cdots\ \ 0\ \ 0].$$

Considering the mean value theorem, the non-affine function $f(\boldsymbol{X},\ u)$ can be written as

$$f(\boldsymbol{X},u) = f(\boldsymbol{X}) + g(\boldsymbol{X},u_\lambda)u \tag{6}$$

where u_λ is a point between zero and u. Then, Eq. (5) can be rewritten as

$$\begin{cases} \dot{\boldsymbol{E}} = \boldsymbol{AE} + \boldsymbol{B}[f(\boldsymbol{X}) + g(\boldsymbol{X},u_\lambda)u - y_{\mathrm{d}}^{(n)} + d(\boldsymbol{X},t)] \\ e_1 = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{E} \end{cases}$$

If the functions $f(\boldsymbol{X})$ and $g(\boldsymbol{X},\ u_\lambda)$ are known and there is no external disturbance $d(\boldsymbol{X}, t)$, and the state $\boldsymbol{X}$ is available, then we can choose

$$u_0 = g(\boldsymbol{X}, u_\lambda)^{-1}[-f(\boldsymbol{X}) + y_{\rm d}^{(n)} - \boldsymbol{\Lambda}^{\rm T}\boldsymbol{E}] \tag{7}$$

where $\boldsymbol{\Lambda} = [\lambda_n, \lambda_{n-1}, \cdots, \lambda_1]^{\rm T}$ should be chosen such that the corresponding characteristic polynomial becomes Hurwitz, and then the closed loop error dynamics is stable. Consequently, $\lim\limits_{t\to\infty}\boldsymbol{E}(t) = 0$.

However, if $f(\boldsymbol{X})$ and $g(\boldsymbol{X}, u_\lambda)$ are unknown, controller design is difficult. Therefore, in this work, a robust adaptive controller is developed.

3 Controller design and stability analysis

The uncertain nonlinear non-affine SISO system (1) can be formulated as

$$\begin{aligned} y^{(n)} &= f(\boldsymbol{X}, u) + d(\boldsymbol{X}, t) = u + (f(\boldsymbol{X}, u) - u) + d(\boldsymbol{X}, t) \\ &= u + \varDelta(\boldsymbol{X}, u) + d(\boldsymbol{X}, t) \end{aligned} \tag{8}$$

where

$$\varDelta(\boldsymbol{X}, u) = f(\boldsymbol{X}, u) - u \tag{9}$$

Using Eqs. (9) and (6), $\varDelta(\boldsymbol{X}, u)$ can be rewritten as

$$\begin{aligned} \varDelta(\boldsymbol{X}, u) &= f(\boldsymbol{X}, u) - u = f(\boldsymbol{X}) + g(\boldsymbol{X}, u_\lambda)u - u \\ &= f(\boldsymbol{X}) + [g(\boldsymbol{X}, u_\lambda) - 1]u = f(\boldsymbol{X}) + g(\bar{\boldsymbol{X}}) \end{aligned} \tag{10}$$

where $g(\bar{\boldsymbol{X}}) = [g(\boldsymbol{X}, u_\lambda) - 1]u$, $\bar{\boldsymbol{X}} = [\boldsymbol{X}^{\rm T}, u]^{\rm T}$.

Using Eqs. (8) and (10), the tracking error system can be written as

$$\begin{cases} \dot{\boldsymbol{E}} = \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[u + \varDelta(\boldsymbol{X}, u) - y_{\rm d}^{(n)} + d(\boldsymbol{X}, t)] \\ e_1 = \boldsymbol{C}^{\rm T}\boldsymbol{E} \end{cases} \tag{11}$$

or

$$\begin{cases} \dot{\boldsymbol{E}} = \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[u + f(\boldsymbol{X}) + g(\bar{\boldsymbol{X}}) - y_{\rm d}^{(n)} + d(\boldsymbol{X}, t)] \\ e_1 = \boldsymbol{C}^{\rm T}\boldsymbol{E} \end{cases} \tag{12}$$

In order to approximate unknown function $\varDelta(\boldsymbol{X}, u)$, an affine type neural network (ATNN) and neural state feedback compensator are proposed.

The output of the neural compensator can be described as

$$\hat{h}(\boldsymbol{X}) = \hat{h}_{\rm nn}(\boldsymbol{X}) + \hat{h}_{\rm sfc}(\boldsymbol{X}) = \hat{\boldsymbol{W}}^{\rm T}\boldsymbol{\phi}(\boldsymbol{X}) + \hat{\boldsymbol{K}}^{\rm T}(\boldsymbol{X})\boldsymbol{X} \tag{13}$$

where $\hat{h}_{\rm nn}(\boldsymbol{X}) = \hat{\boldsymbol{W}}^{\rm T}\boldsymbol{\phi}(\boldsymbol{X})$ is the neural network compensator, and $\hat{h}_{\rm sfc}(\boldsymbol{X}) = \hat{\boldsymbol{K}}^{\rm T}(\boldsymbol{X})\boldsymbol{X}$ is the neural state feedback compensator. $\hat{\boldsymbol{K}}(\boldsymbol{X}) = \hat{\boldsymbol{P}}^{\rm T}\boldsymbol{\psi}(\boldsymbol{X})$, $\boldsymbol{W} = [\boldsymbol{W}_1, \boldsymbol{W}_2, \cdots, \boldsymbol{W}_l]^{\rm T}$ and $\boldsymbol{P} = [P_{11}, P_{12}, \cdots, P_{1m}, \cdots, P_{n1}, P_{n2}, \cdots, P_{nm}]^{\rm T}$ are the adjustable parameter vectors, and l, m are node numbers of neural network. $\boldsymbol{\phi}(\boldsymbol{X}) = [\phi_1(\boldsymbol{X}), \phi_2(\boldsymbol{X}), \cdots, \phi_l(\boldsymbol{X})]^{\rm T}$ and $\boldsymbol{\psi}(\boldsymbol{X}) = [\psi_1(\boldsymbol{X}), \psi_2(\boldsymbol{X}), \cdots, \psi_m(\boldsymbol{X})]^{\rm T}$ are basis function vectors.

The commonly used Gaussian function is used as basis function:

$$\phi_i(\boldsymbol{X}) = \exp\left[\frac{-(\boldsymbol{X} - \boldsymbol{\mu}_{\phi i})^{\rm T}(\boldsymbol{X} - \boldsymbol{\mu}_{\phi i})}{\eta_{\phi i}^2}\right], i=1, 2, \cdots, l \tag{14}$$

$$\psi_i(\boldsymbol{X}) = \exp\left[\frac{-(\boldsymbol{X} - \boldsymbol{\mu}_{\psi i})^{\rm T}(\boldsymbol{X} - \boldsymbol{\mu}_{\psi i})}{\eta_{\psi i}^2}\right], \quad i=1, 2, \cdots, m \tag{15}$$

where $\boldsymbol{\mu}_{\phi i}$ and $\boldsymbol{\mu}_{\psi i}$ are the center vectors of the receptive field; $\boldsymbol{\eta}_{\phi i}$ and $\boldsymbol{\eta}_{\psi i}$ are the widths of the Gaussian function, respectively.

It is supposed that $\boldsymbol{X}$ and $\boldsymbol{W}$ belong to compact sets U and $\varOmega$, respectively, defined as

$$\begin{cases} U = \{\boldsymbol{X} \in \mathbf{R}^n : \|\boldsymbol{X}\| \le M_1\} \\ \varOmega = \{\boldsymbol{W} \in \mathbf{R}^l : \|\boldsymbol{W}\| \le M_2\} \end{cases} \tag{16}$$

where M_1 and M_2 are the designed parameters.

Define the optimal parameter vector $\boldsymbol{W}$:

$$\boldsymbol{W} = \arg\min_{\hat{\boldsymbol{W}}\in\varOmega}\left\{\sup_{\boldsymbol{X}\in U}\left|\varDelta(\boldsymbol{X}, u) - \hat{h}(\boldsymbol{X}, u, \boldsymbol{W})\right|\right\} \tag{17}$$

and approximation error ε:

$$\varepsilon = \varDelta(\boldsymbol{X}, u) - \hat{h}(\boldsymbol{X}) \tag{18}$$

where $\hat{\boldsymbol{W}}$ denotes the estimation of $\boldsymbol{W}$ and $\tilde{\boldsymbol{W}} = \boldsymbol{W} - \hat{\boldsymbol{W}}$.

The robust adaptive controller comprises of two controllers and can be defined as

$$u = u_{\rm f} + u_{\rm r} \tag{19}$$

where $u_{\rm f}$ is the feedback controller as

$$u_{\rm f} = y_{\rm d}^{(n)} - \boldsymbol{\Lambda}^{\rm T}\boldsymbol{E} \tag{20}$$

and $u_{\rm r}$ is the adaptive neural network robust controller, which is defined as

$$u_{\rm r} = u_{\rm n} + u_{\rm vs} + u_{\rm rc} \tag{21}$$

where $u_{\rm n}$ is the neural compensator to approximate the unknown uncertainties, $u_{\rm sm}$ is VSC to eliminate the effects of the neural network approximation errors and the external disturbances, and $u_{\rm rc}$ is the robust compensation controller.

Substituting Eq. (19) into Eq. (12) and using Eq. (20), the error dynamic equation can be obtained as

$$\dot{\boldsymbol{E}} = \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[\varDelta(\boldsymbol{X}, u) + d(\boldsymbol{X}, t) - \boldsymbol{\Lambda}^{\rm T}\boldsymbol{E} + u_{\rm r}] \tag{22}$$

By using Eq. (13), the function $\varDelta(\boldsymbol{X}, u)$ can be represented as

$$\begin{aligned} \varDelta(\boldsymbol{X}, u) &= f(\boldsymbol{X}) + g(\bar{\boldsymbol{X}}) = h(\boldsymbol{X}) + \varepsilon(\boldsymbol{X}, u) \\ &= \boldsymbol{W}^{\rm T}\boldsymbol{\phi}(\boldsymbol{X}) + \boldsymbol{K}^{\rm T}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X}, u) \end{aligned} \tag{23}$$

where $\|\varepsilon(\boldsymbol{X}, u)\| \le \varepsilon_{\rm m}$.

The output of the neural network compensator is given as

$$u_{\rm n} = -\hat{\boldsymbol{W}}^{\rm T}\boldsymbol{\phi}(\boldsymbol{X}) - \hat{\boldsymbol{K}}^{\rm T}(\boldsymbol{X})\boldsymbol{X} = -\hat{\boldsymbol{W}}^{\rm T}\boldsymbol{\phi}(\boldsymbol{X}) - \hat{\boldsymbol{P}}^{\rm T}\boldsymbol{\psi}(\boldsymbol{X})\boldsymbol{X} \tag{24}$$

$\hat{\boldsymbol{W}}$ and $\dot{\hat{\boldsymbol{P}}}$ are updated using the update laws:

$$\dot{\hat{\boldsymbol{W}}} = \gamma_1 \boldsymbol{\phi}^{\mathrm{T}} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{P}_1 \boldsymbol{E} \tag{25}$$

$$\dot{\hat{\boldsymbol{P}}} = \gamma_2 \boldsymbol{\psi}^{\mathrm{T}} \boldsymbol{X} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{P}_1 \boldsymbol{E} \tag{26}$$

where γ_1 and γ_2 are positive constants.

The VSC and robust compensation controller can be defined as

$$u_{\mathrm{vs}} = -k_{\mathrm{s}} \operatorname{sgn}(\boldsymbol{B}^{\mathrm{T}} \boldsymbol{P}_1 \boldsymbol{E}) \tag{27}$$

$$u_{\mathrm{rc}} = -\eta \boldsymbol{B}^{\mathrm{T}} \boldsymbol{P}_1 \boldsymbol{E} \tag{28}$$

where $k_{\mathrm{s}} \ge \varepsilon_{\mathrm{m}} + \varepsilon_{\mathrm{d}}$, η is a small positive constant and $\boldsymbol{P}_1$ is a symmetric positive definite matrix that satisfies the Lyapunov equation:

$$(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}})^{\mathrm{T}} \boldsymbol{P}_1 + \boldsymbol{P}_1 (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}}) = -\boldsymbol{Q}_1 \tag{29}$$

and $\boldsymbol{Q}_1$ is a positive definite matrix.

Theorem 1: For the system defined by Eq. (1), the robust adaptive controller is considered as Eqs. (19)–(21), (24), (27), (28) with the adaption laws Eqs. (25), (26). If Assumptions 1–3 are satisfied, the closed-loop system is stable in the sense of Lyapunov.

Proof: Introducing Eq. (24), the error dynamic equation (22) can be rewritten as

$$\begin{aligned}
\dot{\boldsymbol{E}} &= \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[h(\boldsymbol{X}) + \varepsilon(\boldsymbol{X},u) + d(\boldsymbol{X},t) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{r}}] \\
&= \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \boldsymbol{K}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + \\
&\quad d(\boldsymbol{X},t) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{r}}] \\
&= \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \boldsymbol{K}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + \\
&\quad d(\boldsymbol{X},t) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} - \hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) - \hat{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + u_{\mathrm{vs}} + u_{rc}] \\
&= \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) - \hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \boldsymbol{K}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} - \\
&\quad \hat{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + d(\boldsymbol{X},t) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{vs}} + u_{\mathrm{rc}}] \\
&= \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}[\tilde{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \tilde{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + d(\boldsymbol{X},t) - \\
&\quad \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{vs}} + u_{\mathrm{rc}}]
\end{aligned} \tag{30}$$

where $\tilde{\boldsymbol{W}} = \boldsymbol{W} - \hat{\boldsymbol{W}}$ and $\tilde{\boldsymbol{K}} = \boldsymbol{K} - \hat{\boldsymbol{K}}$.

Consider a Lyapunov function candidate as

$$V_1 = \frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{2\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\tilde{\boldsymbol{W}} + \frac{1}{2\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\tilde{\boldsymbol{P}} \tag{31}$$

where $\tilde{\boldsymbol{P}} = \boldsymbol{P} - \hat{\boldsymbol{P}}$.

Differentiating Eq. (31) yields

$$\dot{V}_1 = \frac{1}{2}\dot{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\dot{\boldsymbol{E}} + \frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}} + \frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}} \tag{32}$$

Substituting Eq. (30) into Eq. (32) yields

$$\begin{aligned}
\dot{V}_1 &= \frac{1}{2}(\boldsymbol{A}\boldsymbol{E}(t) + \boldsymbol{B}[\tilde{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \tilde{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + \\
&\quad d(\boldsymbol{X},t) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{vs}} + u_{\mathrm{rc}}])\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1(\boldsymbol{A}\boldsymbol{E}(t) + \\
&\quad \boldsymbol{B}[\tilde{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{X}) + \tilde{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{X})\boldsymbol{X} + \varepsilon(\boldsymbol{X},u) + d(\boldsymbol{X},t) - \\
&\quad \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{vs}} + u_{\mathrm{rc}}]) + \frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}} + \frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}} \\
&= \frac{1}{2}\boldsymbol{E}^{\mathrm{T}}[(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}})^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_1(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}})]\boldsymbol{E} + \\
&\quad (\varepsilon + d + u_{\mathrm{vs}})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E}u_{\mathrm{rc}} + \boldsymbol{\phi}^{\mathrm{T}}\tilde{\boldsymbol{W}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \\
&\quad \frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}} + \boldsymbol{\psi}^{\mathrm{T}}\tilde{\boldsymbol{P}}\boldsymbol{X}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}}
\end{aligned} \tag{33}$$

We can write $\dot{\tilde{\boldsymbol{W}}} = -\dot{\hat{\boldsymbol{W}}}$ and $\dot{\tilde{\boldsymbol{P}}} = -\dot{\hat{\boldsymbol{P}}}$. Using update laws Eqs. (25) and (26), thus

$$\begin{aligned}
&\boldsymbol{\phi}^{\mathrm{T}}\tilde{\boldsymbol{W}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}} + \boldsymbol{\psi}^{\mathrm{T}}\tilde{\boldsymbol{P}}\boldsymbol{X}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}} \\
&= \tilde{\boldsymbol{W}}\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} - \tilde{\boldsymbol{W}}\frac{1}{\gamma_1}\gamma_1\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \\
&\quad \tilde{\boldsymbol{P}}\boldsymbol{\psi}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} - \tilde{\boldsymbol{P}}\frac{1}{\gamma_2}\gamma_2\boldsymbol{\psi}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} \\
&= 0
\end{aligned} \tag{34}$$

Using Eqs. (34) and (29), Eq. (33) can be rewritten as

$$\dot{V}_1 = -\frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{E} + (\varepsilon + d + u_{\mathrm{vs}})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E}u_{\mathrm{rc}} \tag{35}$$

Substituting u_{vs} and u_{rc} into Eq. (35) yields

$$\begin{aligned}
\dot{V}_1 &= -\frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{E} + (\varepsilon + d + u_{\mathrm{vs}})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} + u_{\mathrm{rc}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} \le \\
&\quad -\frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{E} + (\varepsilon_{\mathrm{m}} + \varepsilon_{\mathrm{d}} - k_{\mathrm{s}})\operatorname{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} - \\
&\quad \eta\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} \le -\frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{E} - \eta\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} \le \\
&\quad -\frac{1}{2}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{E} \le 0
\end{aligned} \tag{36}$$

Since V>0 and $\dot{V} \le 0$, this shows stability in the sense of Lyapunov. As a result, the stability of the proposed robust adaptive controller can be guaranteed.

Since the state vector is unmeasurable in practice, the unmeasurable states are estimated. An observer is considered as

$$\begin{cases}
\dot{\hat{\boldsymbol{E}}} = \boldsymbol{A}\hat{\boldsymbol{E}} - \boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}}\hat{\boldsymbol{E}} + \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{E} - \hat{\boldsymbol{E}}) \\
\hat{e}_1 = \boldsymbol{C}^{\mathrm{T}}\hat{\boldsymbol{E}}
\end{cases} \tag{37}$$

where $\boldsymbol{K}_0$ is the observer gain vector to be designed so that the matrix $\boldsymbol{A}$–$\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}$ is Hurwitz, and $\hat{\boldsymbol{E}}$ is the estimation of $\boldsymbol{E}$. Define the observation error as $\tilde{\boldsymbol{E}} = \boldsymbol{E} - \hat{\boldsymbol{E}}$. Then, using Eqs. (11) and (37), we can write

$$\begin{cases}
\dot{\tilde{\boldsymbol{E}}} = (\boldsymbol{A} - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{E}} + \boldsymbol{B}[u + \Delta(\boldsymbol{X},u) - y_{\mathrm{d}}^{(n)} + d(\boldsymbol{X},t)] \\
\tilde{e}_1 = \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{E}}
\end{cases} \tag{38}$$

or

$$\begin{cases}
\dot{\tilde{\boldsymbol{E}}} = (\boldsymbol{A} - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{E}} + \boldsymbol{B}[\Delta(\boldsymbol{X},u) + d(\boldsymbol{X},t) - \Lambda^{\mathrm{T}}\boldsymbol{E} + u_{\mathrm{r}}] \\
\tilde{e}_1 = \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{E}}
\end{cases} \tag{39}$$

Equation (39) can be rewritten as

$$\begin{cases}\dot{\tilde{\boldsymbol{E}}}=(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{E}}+\boldsymbol{B}[\tilde{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\hat{\boldsymbol{X}})+\tilde{\boldsymbol{K}}^{\mathrm{T}}(\hat{\boldsymbol{X}})\hat{\boldsymbol{X}}+\\ \quad\varepsilon(\boldsymbol{X},u)+d(\boldsymbol{X},t)-\boldsymbol{\Lambda}^{\mathrm{T}}\hat{\boldsymbol{E}}+u_{\mathrm{vs}}+u_{\mathrm{rc}}]\\ \tilde{e}_1=\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{E}}\end{cases}\tag{40}$$

Assumption 4: For the given positive-definite matrix $\boldsymbol{Q}_2$, there exists a positive-definite solution $\boldsymbol{P}_2$ for the matrix equations:

$$\begin{cases}(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}\boldsymbol{P}_2+\boldsymbol{P}_2(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})=-\boldsymbol{Q}_2\\ \boldsymbol{P}_2\boldsymbol{B}=\boldsymbol{C}\end{cases}\tag{41}$$

The output of the neural network compensator is given as

$$u_{\mathrm{n}}=-\hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\hat{\boldsymbol{X}})-\hat{\boldsymbol{K}}^{\mathrm{T}}(\hat{\boldsymbol{X}})\hat{\boldsymbol{X}}=-\hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\phi}(\hat{\boldsymbol{X}})-\hat{\boldsymbol{P}}^{\mathrm{T}}\boldsymbol{\psi}(\hat{\boldsymbol{X}})\hat{\boldsymbol{X}}\tag{42}$$

$\hat{\boldsymbol{W}}$ is updated using the update law:

$$\dot{\hat{\boldsymbol{W}}}=\gamma_1\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\hat{\boldsymbol{E}}\tag{43}$$

$$\dot{\hat{\boldsymbol{P}}}=\gamma_2\boldsymbol{\psi}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\hat{\boldsymbol{E}}\tag{44}$$

The VSC and robust compensation controller can be defined as

$$u_{\mathrm{vs}}=-k_{\mathrm{s}}\,\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}})\tag{45}$$

where $k_{\mathrm{s}}\ge\varepsilon_{\mathrm{m}}+\varepsilon_{\mathrm{d}}$.

$$u_{\mathrm{rc}}=-\boldsymbol{K}_0{}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{E}}\tag{46}$$

Theorem 2: For the system defined by Eq. (1), the robust adaptive controller is considered as Eqs. (19)–(21), (42), (45), (46) with the adaption laws Eqs. (43) and (44). If Assumptions 1–4 are satisfied, the closed-loop system is stable in the sense of Lyapunov.

Proof: Consider a Lyapunov function candidate as

$$\boldsymbol{V}_2=\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{E}}+\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}+\frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}+\frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}}\tag{47}$$

Differentiating Eq. (47) yields

$$\dot{\boldsymbol{V}}_2=\frac{1}{2}\dot{\hat{\boldsymbol{E}}}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{E}}+\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_1\dot{\hat{\boldsymbol{E}}}+\frac{1}{2}\dot{\tilde{\boldsymbol{E}}}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}+\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_2\dot{\tilde{\boldsymbol{E}}}+\frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}+\frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}}\tag{48}$$

Substituting Eqs. (37) and (40) into Eq. (48) yields

$$\begin{aligned}\dot{\boldsymbol{V}}=&\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}[(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}})^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{P}_1(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{\Lambda}^{\mathrm{T}})]\hat{\boldsymbol{E}}+\\ &\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}[(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}\boldsymbol{P}_2+\boldsymbol{P}_2(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})]\tilde{\boldsymbol{E}}+\\ &\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{E}}+\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{B}[\tilde{\boldsymbol{W}}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\phi}_{\mathrm{f}}(\hat{\boldsymbol{X}})+\tilde{\boldsymbol{W}}_g^{\mathrm{T}}\boldsymbol{\phi}_g(\hat{\boldsymbol{X}})+\\ &\varepsilon(\boldsymbol{X},u)+d(\boldsymbol{X},t)-\boldsymbol{\Lambda}^{\mathrm{T}}\hat{\boldsymbol{E}}+u_{\mathrm{vs}}+u_{\mathrm{rc}}]+\frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}+\\ &\frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}}\end{aligned}\tag{49}$$

Then, using Eqs. (29) and (41), Eq. (49) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{V}}=&-\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_1\hat{\boldsymbol{E}}+\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{E}}-\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_2\tilde{\boldsymbol{E}}+\\ &\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{B}[\tilde{\boldsymbol{W}}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\phi}_{\mathrm{f}}(\hat{\boldsymbol{X}})+\tilde{\boldsymbol{W}}_g^{\mathrm{T}}\boldsymbol{\phi}_g(\hat{\boldsymbol{X}})+\varepsilon(\boldsymbol{X},u)+d(\boldsymbol{X},t)-\\ &\boldsymbol{\Lambda}^{\mathrm{T}}\hat{\boldsymbol{E}}+u_{\mathrm{vs}}+u_{\mathrm{rc}}]+\frac{1}{\gamma_1}\tilde{\boldsymbol{W}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}+\frac{1}{\gamma_2}\tilde{\boldsymbol{P}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{P}}}\end{aligned}\tag{50}$$

Substituting Eqs. (45) and (46) and adaptation laws Eqs. (43), (44) in Eq. (50), we can write

$$\begin{aligned}\dot{\boldsymbol{V}}=&-\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_1\hat{\boldsymbol{E}}-\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_2\tilde{\boldsymbol{E}}+(\varepsilon+d+u_{\mathrm{vs}})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\le\\ &-\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_1\hat{\boldsymbol{E}}-\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_2\tilde{\boldsymbol{E}}+(\varepsilon_{\mathrm{m}}+\varepsilon_{\mathrm{d}}-k_{\mathrm{s}})\cdot\\ &\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E})\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\le-\frac{1}{2}\hat{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_1\hat{\boldsymbol{E}}-\frac{1}{2}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{Q}_2\tilde{\boldsymbol{E}}\le 0\end{aligned}\tag{51}$$

As a result, the closed-loop system is stable, and the stability of the proposed robust adaptive controller can be guaranteed.

Remark 1: In the variable structure control, high frequency oscillation (termed as ‘chattering’) exists in the control input. To solve this problem, a modification of VSC with a saturation function has been proposed to reduce the control chattering:

$$u_{\mathrm{vs}}=\begin{cases}-k_{\mathrm{s}}\,\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}),\ \left|\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}\right|>\varphi\\ -k_{\mathrm{s}}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}),\ \left|\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_2\tilde{\boldsymbol{E}}\right|\le\varphi\end{cases}\tag{52}$$

Remark 2: The proposed approach can be applied to affine systems and avoid the controller singularity issue.

4 Simulation results

Two case studies are given to demonstrate the effectiveness of the proposed control method. The first simulation is nonlinear non-affine system and the second is affine system.

4.1 Example 1

Consider the following non-affine SISO nonlinear system [17]:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=\dfrac{1-\exp(-x_1)}{1+\exp(-x_1)}-(x_2^2+2x_1)\sin x_2+(1-0.5\sin x_2)u+\\ \quad 0.4\sin u+d(\boldsymbol{X},t)\\ y=x_1\end{cases}\tag{53}$$

where $d(\boldsymbol{X},t)=0.5\sin(x_1+5t)$, $y=x_1$ and $\dot{y}=x_2$.

The desired trajectory is $y_{\mathrm{d}}(t)=\sin t+\sin(0.5t)$. According to the design procedures in Section 3, the design parameters are selected as $k_{\mathrm{s}}=1$, $\gamma_{\mathrm{f}}=0.2$, $\gamma_{\mathrm{g}}=0.3$ and

φ=0.001.

The feedback and observer gain matrices are given by

$$\boldsymbol{\Lambda}=[100\ \ 20]^{\mathrm{T}},\ \boldsymbol{K}_0=[30\ \ 200]^{\mathrm{T}}$$

Besides, the positive-definite matrices $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are chosen as

$$\boldsymbol{Q}_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \boldsymbol{Q}_2=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$$

The simulations are carried out with initial conditions $\boldsymbol{X}_0=[-0.2,\ 0.1]^{\mathrm{T}}$, and $\boldsymbol{E}_0=[0.1,\ 0.1]^{\mathrm{T}}$.

The simulation results are shown in Figs. 1 and 2.

Figure 1 shows the tracking responses of system outputs and reference signals, control inputs and tracking errors with Theorem 1. Figure 2 shows the tracking responses of system outputs, control input and tracking errors with Theorem 2.

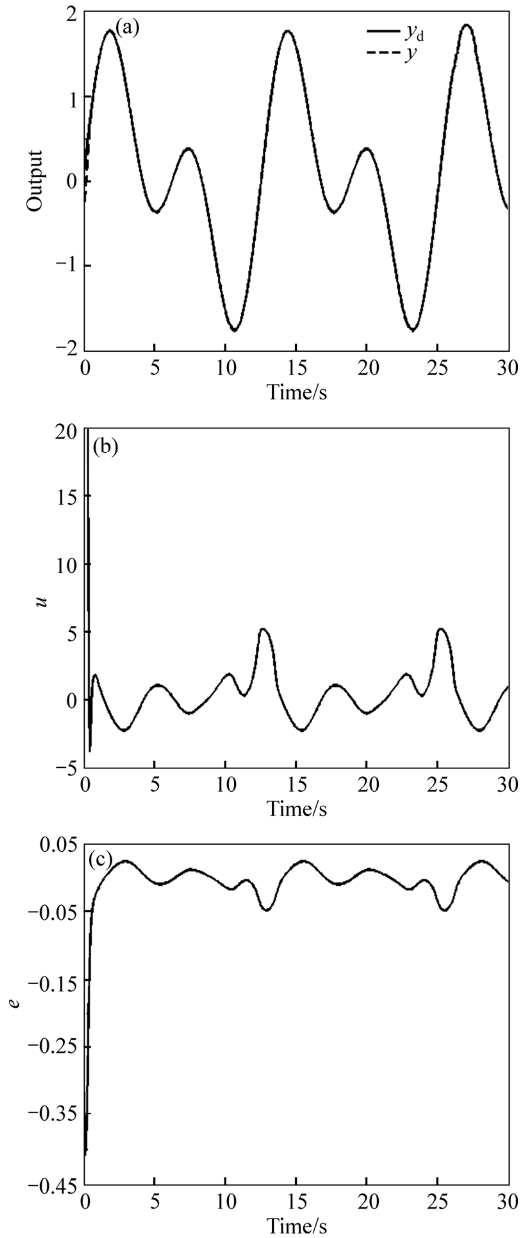

Fig. 1 Simulation results with Theorem 1 in Example 1: (a) Output of system; (b) Input of system; (c) Tracking error

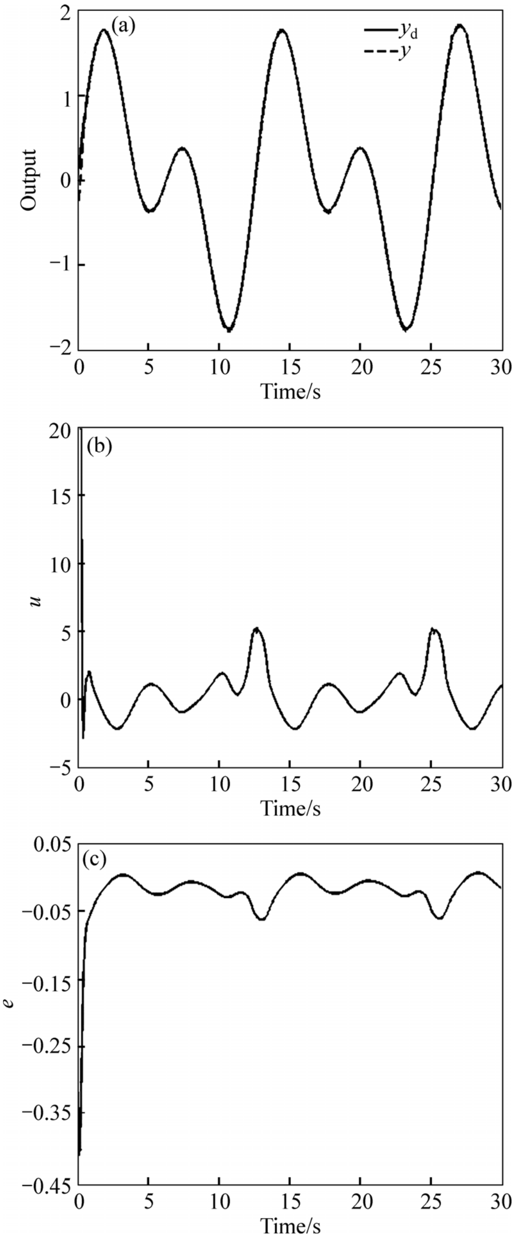

Fig. 2 Simulation results with Theorem 2 in Example 1: (a) Output of system; (b) Input of system; (c) Tracking error

From Figs. 1 and 2, it is shown that the proposed scheme is a good method to improve the tracking error and the control performance of the uncertain nonlinear non-affine systems.

4.2 Example 2

Consider a inverted pendulum system. The dynamic equations of such system are given by [6, 16]

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=\dfrac{g\sin x_1-(mLx_2\sin x_1\cos x_1)/(m+M)}{L[4/3-m\cos^2 x_1/(m+M)]}+\\ \qquad\dfrac{\cos x_1/(m+M)}{L[4/3-m\cos^2 x_1/(m+M)]}u+d\end{cases}\tag{54}$$

where $x_1=\theta$ and $x_2=\dot{\theta}$ are the angular position and velocity of the pole, respectively. g=9.8 m/s^2 denotes the acceleration due to gravity. M=1 kg and m=0.1 kg are the masses of cart and the mass of pole, respectively. L= 0.5 m is the half length of pole and d is the external disturbance. The desired trajectory is $y_\mathrm{d}(t)=\sin t$. The disturbance is $d=1+\sin(\pi t/2)$. The controller parameters are chosen as k_s=2, φ=0.001, γ_f=0.1 and γ_g=0.2. The feedback and observer gain matrices, and matrices $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are chosen as the same as the first example. The initial conditions are chosen as zeros. The simulation results of the inverted pendulum system are shown in Figs. 3 and 4.

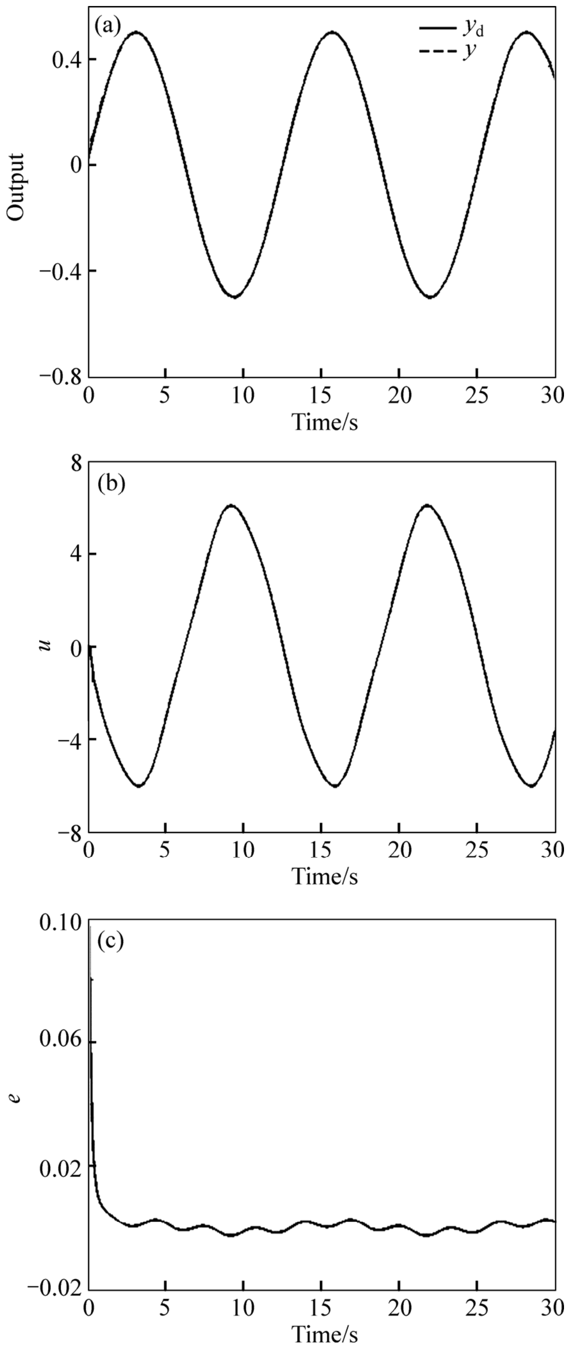

Fig. 3 Simulation results with Theorem 1 in Example 2: (a) Output of system; (b) Input of system; (c) Tracking error

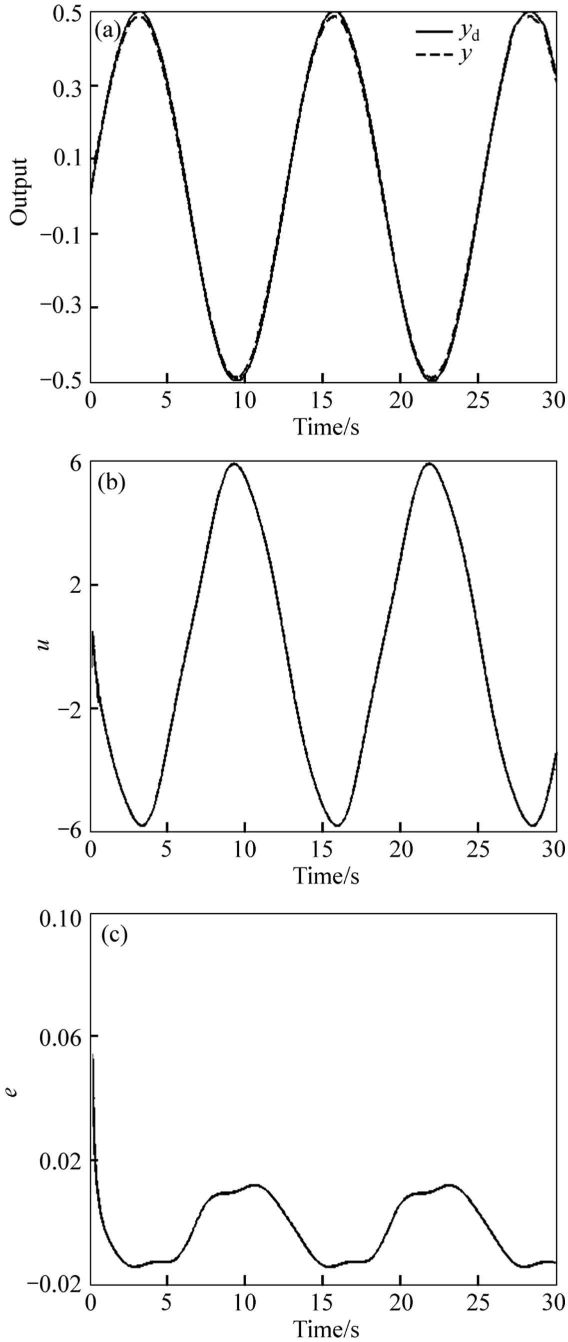

Fig. 4 Simulation results with Theorem 2 in Example 2: (a) Output of system; (b) Input of system; (c) Tracking error

Figures 3 and 4 show that the desired performance is successfully achieved by the designed robust adaptive controller.

5 Conclusions

The robust adaptive control is proposed for a class of the uncertain non-affine nonlinear SISO systems. The neural state feedback compensator is employed to approximate the unknown nonlinear function. The adaptive laws of the neural networks and robust controller are derived based on the stability of the closed loop error dynamics. The mean value theorem is used to transform non-affine nonlinear system into affine nonlinear system, and the state observer is employed to estimate unmeasured states. Furthermore, the singularity problem of the proposed approach is avoided, and this approach can also be applied to affine systems. Finally, two simulation examples are applied to nonlinear systems. Simulation results confirm the effectiveness of the proposed control method for non-affine systems and affine systems.

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