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Improved H_∞ control for networked control systems with network-induced delay and packet dropout

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Abstract: The H_∞ performance analysis and controller design for linear networked control systems (NCSs) are presented. The NCSs are considered a linear continuous system with time-varying interval input delay by assuming that the sensor is time-driven and the logic Zero-order-holder (ZOH) and controller are event-driven. Based on this model, the delay interval is divided into two equal subintervals for H_∞ performance analysis. An improved H_∞ stabilization condition is obtained in linear matrix inequalities (LMIs) framework by adequately considering the information about the bounds of the input delay to construct novel Lyapunov–Krasovskii functionals (LKFs). For the purpose of reducing the conservatism of the proposed results, the bounds of the LKFs differential cross terms are properly estimated without introducing any slack matrix variables. Moreover, the H_∞ controller is reasonably designed to guarantee the robust asymptotic stability for the linear NCSs with an H_∞ performance level γ. Numerical simulation examples are included to validate the reduced conservatism and effectiveness of our proposed method.

Key words: H_∞ performance; networked control systems; packet dropout; network-induced delay

1 Introduction

NCSs have received significant attention for their successful applications in remote surgery, unmanned aerial vehicles, and automated highway systems in recent years. As mentioned in Refs. [1−2], NCSs have many benefits, such as high reliability, simple installation, lower cost, and reduced weight compared with the conventional control system. However, the introduction of NCSs often results in some new problems, among which data packet dropout and network-induced delay are two main issues. They can reduce the performance of the system and even make it unstable. Therefore, it is important to investigate NCSs with data packet dropouts or network induced delay. Several research papers on this topic have been published [3−8], and their number is growing. In existing researches, various approaches have been applied to the analysis of NCSs: the stochastic system approach [5], the sampled-data system approach [6], the switched-system approach [7], and the hybrid system approach [8].

In addition, a new approach has been employed to explore the control problem of NCSs recently by modeling the NCSs with packet dropouts and network-induced delay as a system with input delay. Based on this approach, stabilization analysis and designed controller for NCSs were proposed via LMIs [9−14]. In Ref. [9], a new robust stability criterion was derived for uncertain NCSs with packet dropouts and network-induced delay through employing a new delay decomposition approach. In Ref. [10], an improved stability criterion for Lurie NCSs was proposed and the state feedback controller was designed by assuming the network induced delays to be time varying. However, the study in Ref. [10] focused on the controller design and some novel Lyapunov–Krasovskii functional were not utilized. In Ref. [11], a stability condition was proposed by modeling NCSs as a linear discrete system with input delay from the viewpoint of ZOH. In Ref. [12], through distinguishing the impacts of delay and of packet dropout, both stability and stabilization criteria that are separately related to data packet dropouts and network induced delays were derived.

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Recently, there have been some results about the H_∞ control problem of NCSs in the reported papers [15−18]. In Ref. [15], an H_∞ controller based on observer for linear discrete NCSs with random network-induced delays was designed. HUA et al [16] reported a study of the robust H_∞ control problem that introduced a new uncertain linear switched-system method to investigate NCSs with sampling jitter and packet dropout. ZHU et al [19] considered the H_∞ control problem of multi-rate NCSs and derived a stochastic mean-square sate feedback controller based on Markov theory. Within a framework of continuous-time systems, the H_∞ control problem was studied by considering both the data packet dropouts and the network induced delay simultaneously, where the input delay belongs to a given time interval [20−22]. The robust H_∞ controller for uncertain NCSs was designed by taking full account of the massage about the lower bounds of network-induced delays in Refs. [20−21]. By introducing some slack variables, a novel LKFs was considered to obtain sufficient conditions which guarantee the robust exponential stabilization of the NCSs with an H_∞ performance level [22]. However, the results in the literature cited above can be overly conservative because of their use of over bounding techniques and introduction of slack variables. As mentioned above, the results still leave much room for improvement on the reduction of conservatism in stability analysis and H_∞ control for NCSs, which motivates the present work.

Based on the analysis mentioned above, the H_∞ control problem of NCSs is investigated in this paper by modeling them as a linear continuous system with interval input delay, in which packet dropouts and network-induced delay are considered simultaneously. The main purpose of our work is to reduce the conservatism of the proposed results. Therefore, novel LKFs are constructed by using a new delay partition method and by making use of information about the bounds of the input delay. The input delay interval is evenly divided into two equal subintervals for H_∞ stability analysis. Via applying tighter integral inequalities and convex combination to bound cross items arisen from the time derivative of LKFs, an improved sufficient condition for H_∞ performance analysis can be obtained in terms of LMIs. This allows the H_∞ controller for the NCSs to be given by solving a group of LMIs. The proposed method turn out to be effective by three numerical examples.

Notation: Throughout this work, the notation is standard. $\boldsymbol{R}^n$ denotes Euclidean space with n dimensions. $\boldsymbol{I}$ refers to an identity matrix of appropriate dimension. $\boldsymbol{P}>0$, where $\boldsymbol{P}$ is a real symmetric matrix, denotes that it is positive definite. $R(Z)$ denotes the set of real numbers (integers). $\|\cdot\|$ denotes the induced matrix 2-norm and the Euclidean vector norm. * stands for the symmetric elements of symmetric matrix. The term diag $\{\cdots\}$ stands for a diagonal matrix.

2 Problem formulation and preliminaries

Consider a class of NCSs model having a structure as shown in Fig. 1, in which the linear system is described as

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\omega(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{F}\omega(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the system state vector; $\boldsymbol{\omega}(t) \in \boldsymbol{R}^p$ denotes the external perturbation belonging to $L_2[0, \infty) \in \boldsymbol{R}^q$; $\boldsymbol{u}(\boldsymbol{t}) \in \boldsymbol{R}^k$ denotes the system control input; $\boldsymbol{y}(\boldsymbol{t}) \in \boldsymbol{R}^h$ denotes the system output; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{F}$ are real constant system matrices; and $x(t_0)=x_0$ is the initial state.

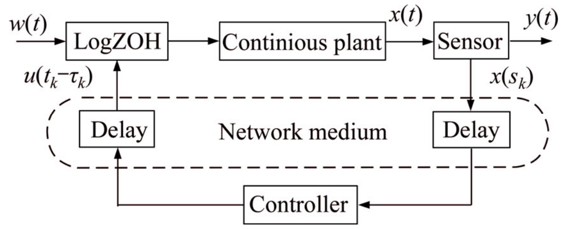

Fig. 1 Typical networked control systems

In this note, the NCSs are characterized through the following assumptions.

Assumption 1: The sensor is supposed to be time-driven, both the logic ZOH and the controller are supposed to be event-driven.

Assumption 2: Each sampled state vector is transmitted in a single data packet, each packet has a unique timestamp during its transmission in the network.

Assumption 3: The logic ZOH is designed to accept the most recently arrived packet, that is to say, the new packet's timestamp is larger than that of the data packet in the logic ZOH currently. The mechanism of the logic ZOH is similar to that in Ref. [11].

Now, define the sensor sampling period as h, sampling times of the sensor as s_k where k=1, 2, ⋯, and updating times of the logic ZOH as t_k, where k=1, 2, ⋯. When there are dropouts and disordering of packets, not all of the state packets can be accepted in the logic ZOH. Therefore, a linear controller is presented as follows:

$$\boldsymbol{u}(t) = \boldsymbol{u}(i_k h) = \boldsymbol{K}\boldsymbol{x}(i_k h) = \boldsymbol{K}\boldsymbol{x}(t_k - \tau_k),\ t \in [t_k, t_{k+1}) \tag{2}$$

where i_k (k=1, 2, ⋯) are positive integers with

$\{i_1, i_2, \cdots\} \subset \{1, 2, \cdots\}$, and τ_k is the network induced delay that means the duration from time $i_k h$ when sample data is captured from a physical plant, to time t_k, when the logic ZOH transfers data to the plant.

We further define $\rho(t) = t - t_k + \tau_k$, $t \in [t_k, t_{k+1})$, obviously, $\rho(t) \in [\tau_k, t_{k+1} - t_k + \tau_k)$. There exist two positive constants τ_m, τ_M such that

$$\tau_m \le \tau_k \le t_{k+1} - t_k + \tau_k \le \tau_M \tag{3}$$

Therefore, applying Eqs. (2) and (3), we can rewrite Eq. (1) as

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}(t-\rho(t)) + \boldsymbol{E}\boldsymbol{\omega}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{K}\boldsymbol{x}(t-\rho(t)) + \boldsymbol{F}\boldsymbol{\omega}(t), \quad t \in [t_k, t_{k+1}) \\ \boldsymbol{x}(t) = \phi(t), \quad t \in [t_o - \tau_M, t_o] \end{cases} \tag{4}$$

where $\boldsymbol{\rho}(t)$ is a piecewise linear function, the initial state $\boldsymbol{\phi}(t)$ is a differentiable function. It is supposed that $\boldsymbol{u}(t)$=0 until the first control data arrives at the physical plant.

Remark 1: The logic ZOH is supplied with a scheduling scheme that can compare the data packets. Hence, an older data packet will be abandoned even when the newer data packet arrives at the plant before the old packet. In Ref. [11], by contrast, the logic ZOH was supposed to be time-driven and had the same sampling period as the sensor, so the latest control information could not be used before the following sampling period.

Remark 2: It is noteworthy that i_k is the time stamp of the state vector packet successfully transferred from the sensor to the logic ZOH. Therefore, $\{i_1, i_2, \cdots\} \neq \{1, 2, \cdots\}$. In Eq. (2), $u(i_k h)$ is the newest available control information arriving at the logic ZOH during $t \in [t_k, t_{k+1})$. Therefore, we have $i_k < i_{k+1}$. When $i_k = i_{k+1} - 1$, it shows that no data packet dropouts occurred in transmission. If $i_k < i_{k+1} - 1$, it means the existence of packet dropouts in the transmission, then the number of packet dropouts is $i_{k+1} - i_k - 1$.

Remark 3: It is worth mentioned that the network-induced delay τ_k may be a constant delay, a short delay ($\tau_k < h$), a long delay ($\tau_k > h$), or a varying time delay. Both the packet dropouts and the network-induced delay are represented as an input time varying delay in Eq. (4). Therefore, the upper bounds of the network-induced delay can be calculated from the values of the input delay.

The following lemmas and definition are useful throughout this paper.

Lemma 1 [23]: For any given matrix $\boldsymbol{M} \in \boldsymbol{R}^{\boldsymbol{n}}$, $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, any differentiable vector function $x(t)$: $[u\ v] \rightarrow \boldsymbol{R}^{\mathrm{n}}$ and two scalars $v > u \ge 0$, the following integral inequality holds

$$\left(\int_u^v \boldsymbol{x}(s)\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{M} \left(\int_u^v \boldsymbol{x}(s)\mathrm{d}s\right) \le (v-u)\int_u^v \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{x}(s)\mathrm{d}s \tag{5}$$

Lemma 2 [24]: Suppose $\sigma_1 \le \boldsymbol{\sigma}(t) \le \sigma_2$, where $\sigma(\cdot): \boldsymbol{R}_+(or\boldsymbol{Z}_+) \rightarrow \boldsymbol{R}_+(or\boldsymbol{Z}_+)$, then, for any constant matrices $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$ and L of appropriate dimensions, $\boldsymbol{L} + (\boldsymbol{\sigma}(t) - \sigma_1)\boldsymbol{Z}_1 + (\sigma_2 - \boldsymbol{\sigma}(t))\boldsymbol{Z}_2 < 0$ holds, if and only if the inequalities (6) and (7) hold:

$$\boldsymbol{L} + (\sigma_2 - \sigma_1)\boldsymbol{Z}_1 < 0 \tag{6}$$

$$\boldsymbol{L} + (\sigma_2 - \sigma_1)\boldsymbol{Z}_2 < 0 \tag{7}$$

Definition 1: For any positive real constant γ, if there exists a controller $\boldsymbol{K}$ that enables system (4) to be asymptotically stable, and $\|\boldsymbol{y}(t)\|_2 < \gamma \|\boldsymbol{\omega}(t)\|_2$ for non-zero $L_2[0, \infty) \in \boldsymbol{R}^q$ with zero initial condition, then the H_∞ stabilization controller exists for the closed-loop system and the disturbance attenuation level is γ.

3 Main results

3.1 H_∞ performance analysis

An improved sufficient condition, which guarantee the asymptotic stability of the system Eq. (4) with an H_∞ performance level γ, is derived in this subsection.

Theorem 1: Given a scalar γ>0, positive constants τ_m, τ_M satisfying (3), and the controller $\boldsymbol{K}$, the system (Eq. (4)) is guaranteed to be asymptotically stable with an H_∞ performance level γ if there exist matrices $\boldsymbol{P}$>0, $\boldsymbol{R}$>0, $\boldsymbol{Q}_i$>0, $\boldsymbol{X}_i$>0 (i=1, ⋯, N), and $\boldsymbol{W}$>0, satisfying

$$\begin{bmatrix} \boldsymbol{\Xi} + \boldsymbol{\Gamma}_1 & \boldsymbol{\Pi\Theta} & \boldsymbol{M} \\ * & -\boldsymbol{\Theta} & 0 \\ * & * & -\boldsymbol{I} \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} \boldsymbol{\Xi} + \boldsymbol{\Gamma}_2 & \boldsymbol{\Pi\Theta} & \boldsymbol{M} \\ * & -\boldsymbol{\Theta} & 0 \\ * & * & -\boldsymbol{I} \end{bmatrix} < 0 \tag{9}$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{X}_1 & 0 & \cdots & 0 & \boldsymbol{PBK} & 0 & \boldsymbol{PE} \\ * & \boldsymbol{\Xi}_{22} & \vdots & \vdots & 0 & 0 & 0 & 0 \\ * & * & \vdots & \boldsymbol{X}_{N-1} & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \boldsymbol{\Xi}_{NN} & \boldsymbol{X}_N & 0 & 0 & 0 \\ * & * & * & * & \boldsymbol{\Xi}_{N+1N+1} & \boldsymbol{W} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Xi}_{N+2N+2} & \boldsymbol{W} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Xi}_{N+3N+3} & 0 \\ * & * & * & * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{\Xi}_{11} = \boldsymbol{PA} + \boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{Q}_1 + \boldsymbol{R} - \boldsymbol{X}_1$$

$$\boldsymbol{\Xi}_{ii} = \boldsymbol{Q}_i - \boldsymbol{Q}_{i-1} - \boldsymbol{X}_i - \boldsymbol{X}_{i-1} \quad (i = 2,\ 3,\ \cdots,\ N)$$

$$\boldsymbol{\Xi}_{N+1N+1} = -\boldsymbol{Q}_N - \boldsymbol{X}_N - \boldsymbol{W}$$

$$\boldsymbol{\Xi}_{N+2N+2} = -2\boldsymbol{W}$$

$$\boldsymbol{\Xi}_{N+3N+3} = -\boldsymbol{R} - \boldsymbol{W}$$

$$\boldsymbol{\Pi}^T = \begin{bmatrix} \boldsymbol{A} & 0 & \cdots & 0 & 0 & \boldsymbol{BK} & 0 & \boldsymbol{E} \end{bmatrix}$$

$$\boldsymbol{M}^T = \begin{bmatrix} \boldsymbol{C} & 0 & \cdots & 0 & 0 & \boldsymbol{DK} & 0 & \boldsymbol{F} \end{bmatrix}$$

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\boldsymbol{W} & \boldsymbol{W} & 0 & 0 \\ * & * & * & * & * & -\boldsymbol{W} & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\boldsymbol{W} & \boldsymbol{W} & 0 \\ * & * & * & * & * & * & -\boldsymbol{W} & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix}$$

$$\boldsymbol{\Theta} = \alpha^2 \sum_{i=1}^{N} \boldsymbol{X}_i + \beta^2 \boldsymbol{W}$$

$\alpha = \delta / N \quad \beta = \tau_M - \delta = \delta - \tau_m = (\tau_M - \tau_m)/2.$

Proof: By defining $\delta = (\tau_m + \tau_M)/2$, **Theorem 1** is proved to be hold for two cases, as follows.

1) When $\boldsymbol{\rho}(t) \in [\delta, \tau_M]$, construct a novel LKFs as

$$\begin{aligned} V(t) = {} & \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t) + \int_{t-\tau_M}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{x}(s)\mathrm{d}s + \\ & \sum_{i=1}^{N}\int_{t-\tau_M}^{t-(i-1)\frac{\delta}{N}} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_i\boldsymbol{x}(s)\mathrm{d}s + \\ & \frac{\delta}{N}\sum_{i=1}^{N}\int_{t-i\frac{\delta}{N}}^{-(i-1)\frac{\delta}{N}}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{X}_i\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta + \\ & (\tau_M - \delta)\int_{-\tau_M}^{-\delta}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{10}$$

For $t \in [t_k, t_{k+1})$, k=1, 2, $\cdots$, $\dot{V}(t)$ can be obtained as

$$\begin{aligned} \dot{V}(t) = {} & 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}[\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{BK}\boldsymbol{x}(t-\boldsymbol{\rho}(t)) + \boldsymbol{E}\boldsymbol{\omega}(t)] + \\ & \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_M)\boldsymbol{R}\boldsymbol{x}(t-\tau_M) + \\ & \sum_{i=1}^{N} x^{\mathrm{T}}(t-(i-1)\delta/N)\boldsymbol{Q}_i\boldsymbol{x}(t-(i-1)\delta/N) - \\ & \sum_{i=1}^{N} \boldsymbol{x}^{\mathrm{T}}(t-i\delta/N)\boldsymbol{Q}_i\boldsymbol{x}(t-i\delta/N) + \\ & \left(\frac{\delta}{N}\right)\sum_{i=1}^{N}\int_{t-i\frac{\delta}{N}}^{t-(i-1)\frac{\delta}{N}} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{X}_i\dot{\boldsymbol{x}}(s)\mathrm{d}s + \\ & (\tau_M - \delta)^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{W}\dot{\boldsymbol{x}}(t) - \\ & (\tau_M - \delta)\int_{t-\tau_M}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \end{aligned} \tag{11}$$

The following inequality holds from **Lemma 1:**

$$-\frac{\delta}{N}\sum_{i=1}^{N}\int_{t-i\frac{\delta}{N}}^{t-(i-1)\frac{\delta}{N}} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{X}_i\dot{\boldsymbol{x}}(s)\mathrm{d}s \le \sum_{i=1}^{N}\begin{bmatrix} \boldsymbol{x}(t-(i-1)\frac{\delta}{N}) \\ \boldsymbol{x}(t-i\frac{\delta}{N}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{X}_i & \boldsymbol{X}_i \\ * & -\boldsymbol{X}_i \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-(i-1)\frac{\delta}{N}) \\ \boldsymbol{x}(t-i\frac{\delta}{N}) \end{bmatrix} \tag{12}$$

Using a method similar to that in Ref. [24] yields

$$\begin{aligned} -(\tau_M - \delta)\int_{t-\tau_M}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s = {} & -(\tau_M - \boldsymbol{\rho}(t))\int_{t-\tau_M}^{t-\rho(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s - \\ & (\tau_M - \boldsymbol{\rho}(t))\int_{t-\rho(t)}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s - \\ & (\boldsymbol{\rho}(t) - \delta)\int_{t-\rho(t)}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s - \\ & (\boldsymbol{\rho}(t) - \delta)\int_{t-\tau_M}^{t-\rho(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \end{aligned} \tag{13}$$

The following integral inequalities are true based on **Lemma 1:**

$$-(\tau_M - \boldsymbol{\rho}(t))\int_{t-\tau_M}^{t-\rho(t)} \dot{\boldsymbol{x}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \le \begin{bmatrix} \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{W} & \boldsymbol{W} \\ * & -\boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix} \tag{14}$$

$$-(\rho(t) - \delta)\int_{t-\rho(t)}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \le \begin{bmatrix} \boldsymbol{x}(t-\delta) \\ \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{W} & \boldsymbol{W} \\ * & -\boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\delta) \\ \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \end{bmatrix} \tag{15}$$

Since $\boldsymbol{\rho}(t) - \delta \le \tau_M - \delta$ and $\tau_M - \boldsymbol{\rho}(t) \le \tau_M - \delta$, applying **Lemma 1**, it can be obtained that

$$\begin{aligned} -(\tau_M - \rho(t))\int_{t-\rho(t)}^{t-\delta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s = {} & -\frac{\tau_M - \boldsymbol{\rho}(t)}{\tau_M - \delta}\int_{t-\rho(t)}^{t-\delta}(\tau_M - \delta)\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \\ & \le -\frac{\tau_M - \rho(t)}{\tau_M - \delta}\int_{t-\rho(t)}^{t-\delta}(\boldsymbol{\rho}(t) - \delta)\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \\ & \le \frac{\tau_M - \rho(t)}{\tau_M - \delta}\begin{bmatrix} \boldsymbol{x}(t-\delta) \\ \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{W} & \boldsymbol{W} \\ * & -\boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\delta) \\ \boldsymbol{x}(t-\boldsymbol{\rho}(t)) \end{bmatrix} \end{aligned} \tag{16}$$

Similarly, the upper bound of the last integral term in Eq. (13) is obtained as

$$-(\rho(t) - \delta)\int_{t-\tau_M}^{t-\rho(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \le$$

$$\frac{\rho(t)-\delta}{\tau_M-\delta}\begin{bmatrix} \boldsymbol{x}(t-\rho(t)) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{W} & \boldsymbol{W} \\ * & -\boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\rho(t)) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix} \tag{17}$$

From Eqs. (12)–(17), $\dot{V}(t)$ can be represented as

$$\dot{V}(t) \le \boldsymbol{\zeta}_1^{\mathrm{T}}(t)(\hat{\boldsymbol{\Xi}} + \boldsymbol{\Pi\Theta\Pi}^{\mathrm{T}} + \frac{\tau_M-\rho(t)}{\tau_M-\delta}\boldsymbol{\Gamma}_1 + \frac{\rho(t)-\delta}{\tau_M-\delta}\boldsymbol{\Gamma}_2)\zeta_1(t), \quad t\in[t_k, t_{k+1}),\ k=1, 2, \cdots \tag{18}$$

where

$$\zeta_1^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t) \ \ x^{\mathrm{T}}\left(t-\frac{\delta}{N}\right) \ \ x^{\mathrm{T}}\left(t-2\frac{\delta}{N}\right) \ \cdots \ x^{\mathrm{T}}\left(t-(N-1)\frac{\delta}{N}\right) \ \ x^{\mathrm{T}}(t-\delta) \ \ x^{\mathrm{T}}(t-\rho(t)) \ \ x^{\mathrm{T}}(t-\tau_M)\omega^{\mathrm{T}}(t) \right]$$

$$\hat{\boldsymbol{\Xi}} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{X}_1 & 0 & \cdots & 0 & \boldsymbol{PBK} & 0 & \boldsymbol{PE} \\ * & \boldsymbol{\Xi}_{22} & \vdots & \vdots & 0 & 0 & 0 & 0 \\ * & * & \vdots & \boldsymbol{X}_{N-1} & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \boldsymbol{\Xi}_{NN} & \boldsymbol{X}_N & 0 & 0 & 0 \\ * & * & * & * & \boldsymbol{\Xi}_{N+1N+1} & \boldsymbol{W} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Xi}_{N+2N+2} & \boldsymbol{W} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Xi}_{N+3N+3} & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix}$$

Thus, it can be get that

$$\dot{V}(t) + \boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{y}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) \ \le \ \boldsymbol{\zeta}_1^{\mathrm{T}}(t)[\hat{\boldsymbol{\Xi}} + \boldsymbol{\Pi\Theta\Pi}^{\mathrm{T}} + \frac{\tau_M-\rho(t)}{\tau_M-\delta}\boldsymbol{\Gamma}_1 + \frac{\rho(t)-\delta}{\tau_M-\delta}\boldsymbol{\Gamma}_2 + \boldsymbol{MM}^{\mathrm{T}} + \boldsymbol{NN}^{\mathrm{T}}]\boldsymbol{\zeta}_1(t) \tag{19}$$

for $t\in[t_k, t_{k+1})$, where

$$\boldsymbol{N}^{\mathrm{T}} = [0 \ \ 0 \ \ \cdots \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \gamma\boldsymbol{I}].$$

Using **Lemma 2**, the following inequality holds:

$$\boldsymbol{\zeta}_1^{\mathrm{T}}(t)[\hat{\boldsymbol{\Xi}} + \boldsymbol{\Pi\Theta\Pi}^{\mathrm{T}} + \frac{\tau_M-\rho(t)}{\tau_M-\delta}\boldsymbol{\Gamma}_1 + \frac{\rho(t)-\delta}{\tau_M-\delta}\boldsymbol{\Gamma}_2 + \boldsymbol{MM}^{\mathrm{T}} + \boldsymbol{NN}^{\mathrm{T}}]\boldsymbol{\zeta}_1(t) < 0 \tag{20}$$

When and only when matrix inequalities (21) hold.

$$\boldsymbol{\Xi} + \boldsymbol{\Pi\Theta\Pi}^{\mathrm{T}} + \boldsymbol{\Gamma}_i + \boldsymbol{MM}^{\mathrm{T}} < 0 \quad (i=1,\ 2) \tag{21}$$

By the Schur complement, the above equalities equivalent to Eqs. LMIs (8) and (9).

In order to show that the system has H_∞ performance level γ, we consider the following index:

$$J_\tau = \int_{t_0}^{\infty}[\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{y}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t$$

$$= \int_{t_0}^{\infty}[\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{y}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) + \dot{V}(t)]\mathrm{d}t - \int_{t_0}^{\infty}\dot{V}(t)\mathrm{d}t$$

$$\le \int_{t_0}^{\infty}\left[\boldsymbol{\Xi} + \boldsymbol{\Pi\Theta\Pi}^{\mathrm{T}} + \frac{\tau_M-\rho(t)}{\tau_M-\delta}\boldsymbol{\Gamma}_1 + \frac{\rho(t)-\delta}{\tau_M-\delta}\boldsymbol{\Gamma}_2 + \boldsymbol{MM}^{\mathrm{T}} \right]\zeta_1(t)\mathrm{d}t - \lim_{t\to\infty}V(t) + V(t_0)$$

Since $\bigcup_{k=1}^{\infty}[t_k, t_{k+1}) = [t_0, \infty)$, $V(t_0)=0$ with zero initial condition and $\lim_{t\to\infty}V(t)\ge 0$, $J_\tau<0$ holds, hence, $\|\boldsymbol{y}(t)\|_2 < \gamma\|\boldsymbol{\omega}(t)\|_2$, which implies that the system (4) has an H_∞ performance level γ.

2) When $\boldsymbol{\rho}(t)\in[\tau_m, \delta]$, LKFs is constructed as

$$V(t) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Px}(t) + \int_{t-\tau_m}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Rx}(s)\mathrm{d}s + \sum_{i=1}^{N}\int_{t-i\frac{\delta}{N}}^{t-(i-1)\frac{\delta}{N}}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_i\boldsymbol{x}(s)\mathrm{d}s + \frac{\delta}{N}\sum_{i=1}^{N}\int_{-i\frac{\delta}{N}}^{-(i-1)\frac{\delta}{N}}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{X}_i\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta + (\delta-\tau_m)\int_{-\delta}^{-\tau_m}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{22}$$

By using a method similar to that used for the first case and replacing $\zeta_1(t)$ with $\zeta_2(t)$ (ζ_2^{T} = $[x^{\mathrm{T}}(t)$ $x^{\mathrm{T}}\left(t-\frac{\delta}{N}\right)$ $x^{\mathrm{T}}\left(t-2\frac{\delta}{N}\right)$ $\cdots$ $x^{\mathrm{T}}\left(t-(N-1)\frac{\delta}{N}\right)$ $\boldsymbol{x}^{\mathrm{T}}(t-\delta)$ $\boldsymbol{x}^{\mathrm{T}}(t-\boldsymbol{\rho}(t))$ $\boldsymbol{x}^{\mathrm{T}}(t-\tau_m)$ $\omega^{\mathrm{T}}(t)])$, we can obtain the same result (21). This ends the proof.

Remark 4: In Ref. [22], an over bounding technique was used to estimate certain terms such as $-\int_{t-\bar{d}_1}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z_1\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha$. The same method was used in Ref. [20] to estimate the bound of the term $-\int_{t-\eta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)\boldsymbol{R}_1\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha$. In this paper, for obtaining a less conservative condition and reasonably estimating the bound of the LKFs derivative cross term, we perform a tighter estimation based on the convex combination to handle the non-linear coefficients $(\boldsymbol{\rho}(t)-\delta)/(\tau_M-\delta)$ and $(\tau_M-\boldsymbol{\rho}(t))/(\tau_M-\delta)$, which can be seen from inequalities (13)–(18).

Remark 5: In the proof of **Theorem 1**, novel LKFs are constructed, which are modifications of those introduced in Ref. [25]. The difference is that a new delay partition method is proposed in our work, where the interval [0, δ] is divided into several equal subintervals. Thus, an improved H_∞ stabilization criterion is derived with less conservatism.

Remark 6: Obviously, the sufficient condition in **Theorem 1** can be used to solve the same problem for a linear continuous time system with interval delay. However, the results derived by **Theorem 1** for continuous time system will have high conservatism because the value of time derivative $\boldsymbol{\rho}(t)$ is not considered when constructing LKFs.

Remark 7: If there is no external perturbation input, by considering system (Eq. (4)) with $\omega(t)=0$, the system reduces to

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{BK}\boldsymbol{x}(t-\rho(t))\\ \boldsymbol{x}(t)=\phi(t),\quad t\in[t_o-\tau_M,\ t_o]\end{cases} \tag{23}$$

The stability analysis of system (23) has been widely considered in Ref. [4, 12−13, 21], an improved stability criterion for this system is stated below.

Corollary 1: Given constants τ_m, τ_M satisfying Eq. (3) and the controller $\boldsymbol{K}$, the system (Eq. (23)) is guaranteed to be asymptotically stable if there exist real matrices $\boldsymbol{P}>0$, $\boldsymbol{R}>0$, $\boldsymbol{Q}_i>0$, $\boldsymbol{X}_i>0$ (i=1, ⋯, N), and $\boldsymbol{W}>0$ satisfying LMIs

$$\begin{bmatrix}\tilde{\boldsymbol{\Xi}}+\tilde{\boldsymbol{\Gamma}}_1 & \tilde{\boldsymbol{\Pi}}\boldsymbol{\Theta}\\ * & -\boldsymbol{\Theta}\end{bmatrix}<0 \tag{24}$$

$$\begin{bmatrix}\tilde{\boldsymbol{\Xi}}+\tilde{\boldsymbol{\Gamma}}_2 & \tilde{\boldsymbol{\Pi}}\boldsymbol{\Theta}\\ * & -\boldsymbol{\Theta}\end{bmatrix}<0 \tag{25}$$

where

$$\tilde{\boldsymbol{\Xi}}=\begin{bmatrix}\Xi_{11} & \boldsymbol{X}_1 & 0 & 0 & 0 & \boldsymbol{PBK} & 0\\ * & \Xi_{22} & \vdots & 0 & 0 & 0 & 0\\ * & * & \vdots & \boldsymbol{X}_{N-1} & \vdots & \vdots & \vdots\\ * & * & * & \Xi_{NN} & \boldsymbol{X}_N & 0 & 0\\ * & * & * & * & \Xi_{N+1N+1} & \boldsymbol{W} & 0\\ * & * & * & * & * & \Xi_{N+2N+2} & \boldsymbol{W}\\ * & * & * & * & * & * & \Xi_{N+3N+3}\end{bmatrix}$$

$$\tilde{\boldsymbol{\Pi}}^T=\begin{bmatrix}\boldsymbol{A} & 0 & \cdots & 0 & 0 & \boldsymbol{BK} & 0\end{bmatrix}$$

$$\tilde{\boldsymbol{\Gamma}}_1=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & 0 & 0\\ * & 0 & \cdots & 0 & 0 & 0 & 0\\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & -\boldsymbol{W} & \boldsymbol{W} & 0\\ * & * & * & * & * & -\boldsymbol{W} & 0\\ * & * & * & * & * & * & 0\end{bmatrix}$$

$$\tilde{\boldsymbol{\Gamma}}_2=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & 0 & 0\\ * & 0 & \cdots & 0 & 0 & 0 & 0\\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & -\boldsymbol{W} & \boldsymbol{W}\\ * & * & * & * & * & * & -\boldsymbol{W}\end{bmatrix}$$

Proof: Choosing $\boldsymbol{C}$=0, $\boldsymbol{D}$=0, $\boldsymbol{E}$=0, $\boldsymbol{F}$=0 and γ=0 in **Theorem 1** and using the method that we have employed in **Theorem 1**, inequalities (24) and (25) can be achieved simply; the proof is completed.

3.2 H_∞ controller design

On the basis of the H_∞ performance analysis for the NCSs, the H_∞ controller can be derived as following:

Theorem 2: Given a positive scalar $\gamma>0$ and constants τ_m, τ_M satisfying (3), the system (4) is asymptotically stable under an H_∞ performance level γ, if there are real matrices $\bar{\boldsymbol{P}}>0$, $\bar{\boldsymbol{R}}>0$, $\bar{\boldsymbol{Q}}_i>0$, $\bar{\boldsymbol{X}}_i>0$ (i=1, ⋯, N), $\bar{\boldsymbol{W}}>0$ and $\bar{\boldsymbol{K}}$ satisfying LMIs

$$\begin{bmatrix}\bar{\boldsymbol{\Xi}}+\bar{\boldsymbol{\Gamma}}_1 & \boldsymbol{\Omega}\\ * & \bar{\boldsymbol{\Lambda}}\end{bmatrix}<0 \tag{26}$$

$$\begin{bmatrix}\bar{\boldsymbol{\Xi}}+\bar{\boldsymbol{\Gamma}}_2 & \boldsymbol{\Omega}\\ * & \bar{\boldsymbol{\Lambda}}\end{bmatrix}<0 \tag{27}$$

where

$$\bar{\boldsymbol{\Xi}}=\begin{bmatrix}\bar{\Xi}_{11} & \bar{\boldsymbol{X}}_1 & 0 & \cdots & 0 & \boldsymbol{B}\bar{\boldsymbol{K}} & 0 & \boldsymbol{E}\\ * & \bar{\Xi}_{22} & \vdots & \vdots & 0 & 0 & 0 & 0\\ * & * & \vdots & \bar{\boldsymbol{X}}_{N-1} & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \bar{\Xi}_{NN} & \bar{\boldsymbol{X}}_N & 0 & 0 & 0\\ * & * & * & * & \bar{\Xi}_{N+1N+1} & \bar{\boldsymbol{W}} & 0 & 0\\ * & * & * & * & * & \bar{\Xi}_{N+2N+2} & \bar{\boldsymbol{W}} & 0\\ * & * & * & * & * & * & \bar{\Xi}_{N+3N+3} & 0\\ * & * & * & * & * & * & * & -\gamma^2\boldsymbol{I}\end{bmatrix}$$

with

$$\bar{\boldsymbol{\Xi}}_{11}=\boldsymbol{A}\bar{\boldsymbol{P}}+\bar{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}}+\bar{\boldsymbol{Q}}_1+\bar{\boldsymbol{R}}-\bar{\boldsymbol{X}}_1$$

$$\bar{\boldsymbol{\Xi}}_{ii}=\bar{\boldsymbol{Q}}_i-\bar{\boldsymbol{Q}}_{i-1}-\bar{\boldsymbol{X}}_i-\bar{\boldsymbol{X}}_{i-1}\quad (i=2, 3, \cdots, N)$$

$$\bar{\boldsymbol{\Xi}}_{N+1N+1}=-\bar{\boldsymbol{Q}}_N-\bar{\boldsymbol{X}}_N-\bar{\boldsymbol{W}}$$

$$\bar{\boldsymbol{\Xi}}_{N+2N+2}=-2\bar{\boldsymbol{W}}$$

$$\bar{\boldsymbol{\Xi}}_{N+3N+3}=-\bar{\boldsymbol{R}}-\bar{\boldsymbol{W}}$$

$$\bar{\boldsymbol{\Lambda}}=\mathrm{diag}\left[\alpha^{-2}(\bar{\boldsymbol{X}}_1-2\bar{\boldsymbol{P}})\ \cdots\ \alpha^{-2}(\bar{\boldsymbol{X}}_N-2\bar{\boldsymbol{P}})\ \ \beta^{-2}(\bar{\boldsymbol{W}}-2\bar{\boldsymbol{P}})\ \ -\boldsymbol{I}\right]$$

$$\bar{\boldsymbol{\Gamma}}_1=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & 0 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\bar{\boldsymbol{W}} & \bar{\boldsymbol{W}} & 0 & 0\\ * & * & * & * & * & -\bar{\boldsymbol{W}} & 0 & 0\\ * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & 0\end{bmatrix}$$

$$\bar{\boldsymbol{\Gamma}}_2=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & 0 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & * & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\bar{\boldsymbol{W}} & \bar{\boldsymbol{W}} & 0\\ * & * & * & * & * & * & -\bar{\boldsymbol{W}} & 0\\ * & * & * & * & * & * & * & 0\end{bmatrix}$$

$$\boldsymbol{\Omega}=\begin{bmatrix} \bar{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} & \cdots & \bar{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} & \bar{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} & \bar{\boldsymbol{P}}\boldsymbol{C}^{\mathrm{T}} \\ 0 & \cdots & 0 & 0 & 0 \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 \\ (\boldsymbol{B}\bar{\boldsymbol{K}})^{\mathrm{T}} & \cdots & (\boldsymbol{B}\bar{\boldsymbol{K}})^{\mathrm{T}} & (\boldsymbol{B}\bar{\boldsymbol{K}})^{\mathrm{T}} & (\boldsymbol{D}\bar{\boldsymbol{K}})^{\mathrm{T}} \\ 0 & \cdots & 0 & 0 & 0 \\ \boldsymbol{E}^{\mathrm{T}} & \cdots & \boldsymbol{E}^{\mathrm{T}} & \boldsymbol{E}^{\mathrm{T}} & \boldsymbol{F}^{\mathrm{T}} \end{bmatrix}$$

Moreover, the desired controller is described as

$$\boldsymbol{K}=\bar{\boldsymbol{K}}\bar{\boldsymbol{P}}^{-1} \tag{28}$$

Proof: Based on **Theorem 1**, it is obvious that system (4) will be asymptotically stable with H_∞ performance level γ, if inequalities (21) hold. Now, by defining $\bar{\boldsymbol{P}}=\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{Q}}_i=\boldsymbol{P}^{-1}\boldsymbol{Q}_i\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{X}}_i=\boldsymbol{P}^{-1}\boldsymbol{X}_i\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{R}}=\boldsymbol{P}^{-1}\boldsymbol{R}\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{W}}=\boldsymbol{P}^{-1}\boldsymbol{W}\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{K}}=\boldsymbol{K}\boldsymbol{P}^{-1}$, pre- and post-multiplying inequalities (21) by $\mathrm{diag}[\boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \cdots, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, I]$, and using the Schur complement, the following inequalities are obtained:

$$\begin{bmatrix} \bar{\boldsymbol{\Xi}}+\bar{\boldsymbol{\Gamma}}_1 & \boldsymbol{\Omega} \\ * & \boldsymbol{\Lambda} \end{bmatrix}<0 \tag{29}$$

$$\begin{bmatrix} \bar{\boldsymbol{\Xi}}+\bar{\boldsymbol{\Gamma}}_2 & \boldsymbol{\Omega} \\ * & \boldsymbol{\Lambda} \end{bmatrix}<0 \tag{30}$$

where

$$\boldsymbol{\Lambda}=\mathrm{diag}\left[-\alpha^{-2}\boldsymbol{X}_1^{-1} \quad \cdots \quad -\alpha^{-2}\boldsymbol{X}_N^{-1} \quad -\beta^{-2}\boldsymbol{W}^{-1} \quad -\boldsymbol{I}\right].$$

The above inequalities cannot be implemented using LMIs because of the existence of the nonlinear terms $\boldsymbol{X}_1^{-1}$, $\cdots$, $\boldsymbol{X}_N^{-1}$, and $\boldsymbol{W}^{-1}$. Noting that $(\bar{\boldsymbol{W}}-\bar{\boldsymbol{P}})\bar{\boldsymbol{W}}^{-1}(\bar{\boldsymbol{W}}-\bar{\boldsymbol{P}})\geq 0$ and $(\bar{\boldsymbol{X}}_i-\bar{\boldsymbol{P}})\bar{\boldsymbol{X}}_i^{-1}(\bar{\boldsymbol{X}}_i-\bar{\boldsymbol{P}})\geq 0$, we have $-\bar{\boldsymbol{P}}\bar{\boldsymbol{W}}^{-1}\bar{\boldsymbol{P}}\leq\bar{\boldsymbol{W}}-2\bar{\boldsymbol{P}}$ and $-\bar{\boldsymbol{P}}\bar{\boldsymbol{X}}_i^{-1}\bar{\boldsymbol{P}}\leq\bar{\boldsymbol{X}}_i-2\bar{\boldsymbol{P}}$. Therefore,

$$-\boldsymbol{X}_i^{-1}\leq\bar{\boldsymbol{X}}_i-2\bar{\boldsymbol{P}} \tag{31}$$

$$-\boldsymbol{W}^{-1}\leq\bar{\boldsymbol{W}}-2\bar{\boldsymbol{P}} \tag{32}$$

From inequalities (29)−(32), the result is derived in **Theorem 2**.

4 Numerical examples

In this section, three standard numerical examples are introduced to validated the improvement of the proposed method.

Example 1: Consider a closed-loop system (4) with

$$\boldsymbol{A}=\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix} 0 \\ 1 \end{bmatrix},\quad \boldsymbol{C}=[0 \quad 1],\ \ \boldsymbol{D}=0.3,$$

$$\boldsymbol{E}=\begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, \boldsymbol{F}=0.5, \text{ and } \ \boldsymbol{K}=[-1 \quad 1] \tag{33}$$

This example has been considered in Ref. [21]. With the given bounds of input delay τ_m and τ_M, the values of $\gamma_{\min}$ guaranteed H_∞ performance are obtained. These results are listed in Table 1. For comparison, results from Ref. [21] are listed in this table, from which, we can seen that our results are better than those in Ref. [21].

Table 1 Minimum $\gamma_{\min}$ for different cases

τ_M	τ_m	Ref. [21]	**Theorem 1** (N=2)
0.4	0	3.1207	1.6264
0.4	0.1	2.8113	1.6011
0.6	0.05	—	2.2934
0.6	0.2	11.2314	2.1401
0.6	0.3	6.0780	2.0777

Example 2 [26]: Consider a closed-loop system (4) with

$$\boldsymbol{A}=\begin{bmatrix} -1.84 & 0.33 \\ 7.18 & -1.14 \end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix} 2.43 \\ -0.42 \end{bmatrix},\quad \boldsymbol{E}=\begin{bmatrix} 1.86 \\ -0.76 \end{bmatrix},$$

$$\boldsymbol{C}=[0.57 \quad 0.78],\ \ \boldsymbol{D}=0,\ \ \boldsymbol{F}=-0.56 \tag{34}$$

According to **Theorem 2**, the H_∞ controller is designed for system (34) with a minimum H_∞ disturbance level γ. By assuming that τ_M=0.25, τ_m=0.1, N=2, and solving LMIs (26) and (27), the H_∞ controller gain matrix is derived as $\boldsymbol{K}$=[−1.0624 −0.3455]. In addition, the H_∞ performance level is obtained as $\gamma_{\min}$=1.9571. For illustrating the H_∞ performance of system (34), the initial condition is given by $\boldsymbol{x}_0$=[0.5 −0.5]$^{\mathrm{T}}$ and the external perturbation input signals are given by

$$\omega(t)=\begin{cases} 0.2\sin t, & 5\ \mathrm{s}\leq t\leq 15\ \mathrm{s} \\ 0, & \text{otherwise} \end{cases} \tag{35}$$

The system state responses are displayed in Fig. 2,

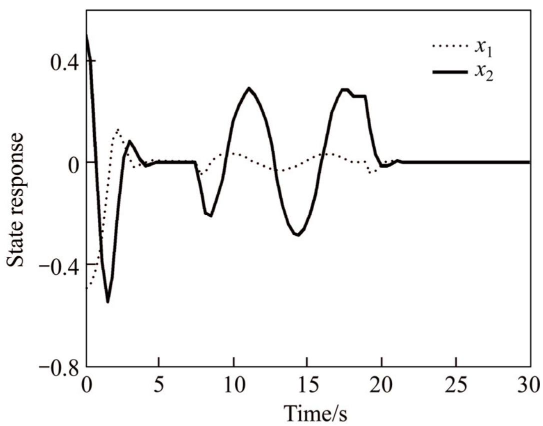

Fig. 2 Curves of state response

from which it can be seen that two states of the continuous plant converge to zero. It implies that the H_∞ controller design is effective.

Example 3: Consider the stability of system (23) with

$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}0\\ 0.1\end{bmatrix} \tag{36}$$

and the controller is given by $\boldsymbol{K}$=[−3.75 −11.5]. The example has been discussed in Refs. [4, 13, 20−21]. In the paper, the maximum upper bounds τ_M for different values of τ_m are calculated through **Corollary 1**. The results are listed in Table 2, and it is obvious that the results derived by **Corollary 1** are less conservative than those produced by other existing ones.

Table 2 Maximum upper bounds τ_M for different τ_m

τ_m	Ref. [21]	Ref. [4]	Ref. [13]	**Corollary 1** (N=2)	**Corollary 1** (N=3)
0	1.0081	1.0239	1.0240	1.0440	1.1786
0.05	1.0105	1.0274	1.0314	1.0442	1.1864
0.1	1.0132	1.0274	1.0378	1.0469	1.1939
0.15	1.0161	1.0292	1.0431	1.0485	1.2004
0.2	1.0193	1.0310	1.0475	1.0504	1.2062

Moreover, considering the existence of the external disturbance, system (36) can be expressed as system (4) with

$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}0\\ 0.1\end{bmatrix},\ \boldsymbol{C}=[0\ 1],\ \boldsymbol{E}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},$$
$$\boldsymbol{D}=0.1,\ \text{and}\ \boldsymbol{F}=0 \tag{37}$$

The H_∞ performance of system (37) under the given controller can be studied. For the case of τ_m=0 and τ_M=0.8695, the minimum allowable value of $\gamma_{\min}$ was found as 6.82 and 1.0005 in Ref. [20] and Ref. [22], respectively, while the value $\gamma_{\min}$=0.9221 is obtained from **Theorem 1** by solving LMIs (8) and (9) with N=2. Then, H_∞ controller is designed in the form of (28). Applying **Theorem 2** with the given delay bounds τ_m=0, τ_M=0.52 and N=2, the following matrices are obtained:

$$\bar{\boldsymbol{P}}=\begin{bmatrix}0.0164 & -0.0170\\ -0.0170 & 0.1046\end{bmatrix},\quad \bar{\boldsymbol{K}}=[-0.0360\quad -0.3366].$$

Thus, the minimum allowable value $\gamma_{\min}$=3.9395 is obtained and the H_∞ controller is derived as $\boldsymbol{K}=\bar{\boldsymbol{K}}\bar{\boldsymbol{P}}^{-1}=[-6.6514\quad -4.2990]$.

5 Conclusions

1) By modeling the NCSs with packet dropouts and network-induced delay as a linear continuous system with time-varying interval input delay, an improved stability criterion which guarantees that NCSs will be asymptotically stable with H_∞ performance level γ has been derived. Based on the condition, an H_∞ controller has been designed via solving a set of LMIs.

2) On the basis of a new delay partition method, novel LKFs have been constructed. Tighter integral inequalities have been employed to bound Lyapunov functional differential cross terms for the purpose of reducing the conservatism of the proposed results.

3) The feasibility and the improvement of the proposed method have been validated by three numerical examples.

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