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Robust fault estimation for uncertain switched linear systems with time-varying delay

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Abstract: The problem of fault estimation is investigated for a class of uncertain switched systems with time-varying delay. A robust observer-based fault estimator is designed such that the augment error system is exponentially stable and the H_∞ performance index meets the predefined requirements. Based on the multiple Lyapunov-Krasovskii functions and the average dwell-time method, the delay dependent sufficient conditions on the existence of desired fault estimator are established. However, since these conditions are not linear matrix inequalities (LMIS), they can not be solved by MATLAB. By using a novel method, these conditions are presented in terms of LMIS. Finally, a numerical example is carried out. The designed fault estimator could tract the fault signal timely. Besides, the error between estimation and fault is very small. Therefore, the validity of the obtained results is illustrated.

Key words: fault estimation; switched linear system; time-varying delay; model uncertainty; average dwell-time

1 Introduction

Switched system belongs to hybrid systems. It is composed of several subsystems and a switching law [1]. The switching law supervises the switches among the subsystems. In practice, switched systems are widely applied in many fields, such as communication system [2], formation flying [3], networks control [4], and power systems [5]. Recently, the switched system has attracted great attention of researchers. Most of the obtained results focus on several basic problems, such as stability [6], and stabilization [7−9].

For control system, security and reliability are very important. The fault of control system usually leads to the degradation of performances and security incidents. Therefore, for improving the system reliability, fault detection and isolation (FDI) is a very significant problem in engineering. Many effective approaches have been proposed for FDI. Recently, H_∞ filtering formulation method has been widely used to study FDI. In Ref. [10], FDI was studied based on robust sliding mode observers. The problem of robust fault detection was considered for switched systems in Ref. [11]. In Ref. [12], fault detection of uncertain discrete switched system was converted into a H_∞ filtering problem by constructing a robust fault filter. The fault detection of switched system was investigated via building an observer [13−15]. In Ref. [14], by using a new Lyapunov function, the sufficient condition was established for the existence of fault detection filters.

On the other hand, fault estimation is also very meaningful, especially for fault tolerant control (FTC). Therefore, many researchers devote to investigating this problem. During the past decades, several effective methods have been developed, such as sliding mode observer approach [16], adaptive technique [17−18] and learning method based on neural network [19−20]. The observer approach possesses an advantage that the state estimation and fault estimation could be obtained simultaneously. WANG and ZHANG [21] used a class of adaptive observers to estimate both system state and fault. In Ref. [22], based on adaptive observer technique, the parameter fault detection and estimation were studied for nonlinear systems with time delay.

However, for switched system, just several works on fault estimation were published. The fault of switched linear systems was estimated via building a hybrid controller which is composed of a fault estimator and an impulsive controller [23]. In Ref. [24], the problem of fault estimation and accommodation was investigated for switched linear system. Furthermore, an observer-based fault tolerant controller was obtained on the basis of fault estimation. Based on a novel switched descriptor observer, the sensor fault estimation was investigated in Ref. [25]. XIANG et al [26] detected the fault of a class of uncertain switched nonlinear systems via state updating approach. In Ref. [27], the problem of fault

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detection was investigated for switched nonlinear systems under asynchronous switching. An estimator was designed for discrete-time switched positive linear systems in Ref. [28], and the developed method is useful for fault estimation. Based on fault estimation, the problem of fault tolerant control was studied for switched discrete-time systems [29]. In above mentioned literatures, some effective approaches were proposed for fault estimation. However, the model uncertainty usually exists in dynamical systems. It has not been taken into account in most of published papers.

In practice, unknown disturbance, model uncertainty and fault signal are coupled together in control system. It should be noted that unknown disturbance and model uncertainty may interfere with fault estimation. Therefore, the desired fault estimator should not only be sensitive to fault but also be robust to disturbance and model uncertainty. In this work, by taking time-varying delay and model uncertainty into account, the problem of robust fault estimation is investigated for a class of uncertain switched linear systems. A robust fault estimator is designed based on building state observer. The error between the fault and the fault estimation satisfies the predefined performance index. Based on the multiple Lyapunov-Krasovskii functions, the sufficient conditions on the existence of desired fault estimator are established in terms of linear matrix inequalities (LMIS). Finally, a numerical example is given to illustrate the validity of obtained results.

Notations: $\mathbf{R}^n$ stands for n-dimensional real vector space; $\mathbf{R}^{n\times n}$ denotes the space of $n\times n$ matrices with real entries; let $\|\boldsymbol{x}\| = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{x} = \{\boldsymbol{x}_1^2 + \cdots + \boldsymbol{x}_n^2\}$, where $\boldsymbol{x}_i$ is the ith element of vector, $\boldsymbol{x} \in \mathbf{R}^n$; let $\underline{m} = \{1, \cdots, m\}$ and $\underline{n} = \{1, \cdots, n\}$, where m and n are arbitrary positive integers; l_2 stands for 2-norm; $\boldsymbol{I}$ represents the identity matrix.

2 Problem and preliminaries

Consider the following uncertain switched linear system with time-varying delay:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_{\sigma(t)} + \Delta\boldsymbol{A}_{\sigma(t)})\boldsymbol{x}(t) + \\ \qquad (\boldsymbol{A}_{\mathrm{d}\sigma(t)} + \Delta\boldsymbol{A}_{\mathrm{d}\sigma(t)})\boldsymbol{x}(t-\tau(t)) + \\ \qquad \boldsymbol{B}_{\sigma(t)}\boldsymbol{d}(t) + \boldsymbol{G}_{\sigma(t)}\boldsymbol{f}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_{\sigma(t)}\boldsymbol{x}(t) + \boldsymbol{C}_{\mathrm{d}\sigma(t)}\boldsymbol{x}(t-\tau(t)) + \\ \qquad \boldsymbol{D}_{\sigma(t)}\boldsymbol{d}(t) + \boldsymbol{J}_{\sigma(t)}\boldsymbol{f}(t) \\ \boldsymbol{x}(t) = \boldsymbol{\phi}(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the system state vector; $\tau(t)$ stands for time-varying delay; $\boldsymbol{y}(t)$ is the system output; $\boldsymbol{d}(t)$ stands for the external disturbance; $\boldsymbol{f}(t)$ is the fault which belongs to $l_2[0, +\infty)$; $\sigma(t) \in \underline{m}$ denotes the switching law which is a piecewise continuous function; $\boldsymbol{\phi}(t)$ denotes the continuous vector-valued initial function; the model uncertainties $\Delta\boldsymbol{A}_{\sigma(t)}$ and $\Delta\boldsymbol{A}_{\mathrm{d}\sigma(t)}$ are norm bounded, described by the following equality: $\left[\Delta\boldsymbol{A}_{\sigma(t)} \;\; \Delta\boldsymbol{A}_{\mathrm{d}\sigma(t)}\right] = \boldsymbol{H}_{\sigma(t)}\boldsymbol{F}_{\sigma(t)}\left[\boldsymbol{E}_{1\sigma(t)} \quad \boldsymbol{E}_{2\sigma(t)}\right]$; $\boldsymbol{F}_{\sigma(t)}$ is unknown matrix satisfying $\boldsymbol{F}_{\sigma(t)}^{\mathrm{T}}\boldsymbol{F}_{\sigma(t)} \le \boldsymbol{I}$; $\boldsymbol{H}_{\sigma(t)}$, $\boldsymbol{E}_{1\sigma(t)}$ and $\boldsymbol{E}_{2\sigma(t)}$ are known matrices; $\boldsymbol{A}_{\sigma(t)}$, $\boldsymbol{A}_{\mathrm{d}\sigma(t)}$, $\boldsymbol{B}_{\sigma(t)}$, $\boldsymbol{G}_{\sigma(t)}$, $\boldsymbol{C}_{\sigma(t)}$, $\boldsymbol{C}_{\mathrm{d}\sigma(t)}$, $\boldsymbol{D}_{\sigma(t)}$ and $\boldsymbol{J}_{\sigma(t)}$ are known system matrices with appropriate dimensions; besides, $0 \le \tau(t) \le \tau$ and $\dot{\tau}(t) \le d < 1$, where τ and d are known positive constants.

First, some definitions and lemmas are introduced.

Definition 1: If there are two positive constants α and β such that

$$\|\boldsymbol{x}(t)\| \le \alpha \exp\{-\beta(t-t_0)\} \sup_{t_0\in[-\tau,0]} \|\boldsymbol{x}(t_0)\|, \;\forall t \ge t_0 \tag{2}$$

then system (1) is globally uniformly exponentially stable (GUES) under the switching law $\sigma(t)$ [6].

Definition 2: For $T \ge t \ge 0$, let $N_{\sigma(t)}(t, T)$ denote the switching number of $\sigma(t)$ over $(t, T]$. If

$$N_{\sigma(t)}(t,T) \le N_0 + \frac{T-t}{\tau_{\mathrm{a}}} \tag{3}$$

then τ_{a} is called the average dwell-time (ADT). $\tau_{\mathrm{a}} \ge 0$ and N_0 is a non-negative integer [7].

Assumption 1: System (1) is a strict continuous system, which implies that system state trajectory is continuous everywhere. In other words, state variable does not jump at any switching instant [30].

Lemma 1 [8]: For matrices $\boldsymbol{D}$, $\boldsymbol{E}$ and symmetric matrix $\boldsymbol{Y}$,

$$\boldsymbol{Y} + \boldsymbol{DFE} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}} < 0 \tag{4}$$

holds for $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} \le \boldsymbol{I}$ if and only if there exists a positive constant ε such that

$$\boldsymbol{Y} + \varepsilon\boldsymbol{DD}^{\mathrm{T}} + \varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} < 0 \tag{5}$$

Next, the problem of robust fault estimation of system (1) would be expounded in detail. In this work, the considered fault belongs to a frequency range located at low frequencies, and its minimal state space description is given as

$$\begin{cases} \dot{\boldsymbol{x}}_w(t) = \boldsymbol{A}_w\boldsymbol{x}_w(t) + \boldsymbol{B}_w\boldsymbol{f}_0(t) \\ \boldsymbol{f}(t) = \boldsymbol{C}_w\boldsymbol{x}_w(t) + \boldsymbol{D}_w\boldsymbol{f}_0(t) \end{cases} \tag{6}$$

where $\boldsymbol{x}_w(t)$ is the fault state vector; $\boldsymbol{f}_0(t)$ is the fictitious signal; $\boldsymbol{f}(t)$ denotes the weighed fault; $\boldsymbol{A}_w$, $\boldsymbol{B}_w$, $\boldsymbol{C}_w$ and $\boldsymbol{D}_w$ are known matrices obtained from the prior knowledge of fault.

Define the following matrices:

$$\bar{\boldsymbol{x}}^{\mathrm{T}}(t) = \left[\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{x}_w^{\mathrm{T}}(t)\right],$$

$$\boldsymbol{w}^{\mathrm{T}}(t) = \left[\boldsymbol{d}^{\mathrm{T}}(t) \quad \boldsymbol{f}_0^{\mathrm{T}}(t)\right],$$

$$\bar{A}_{\sigma(t)}=\begin{bmatrix} A_{\sigma(t)} & G_{\sigma(t)}C_w \\ 0 & A_w \end{bmatrix},$$
$$\Delta\bar{A}_{\sigma(t)}=\begin{bmatrix} \Delta A_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{A}_{\mathrm{d}\sigma(t)}=\begin{bmatrix} A_{\mathrm{d}\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\Delta\bar{A}_{\mathrm{d}\sigma(t)}=\begin{bmatrix} \Delta A_{\mathrm{d}\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{B}_{\sigma(t)}=\begin{bmatrix} B_{\sigma(t)} & G_{\sigma(t)}D_w \\ 0 & B_w \end{bmatrix},$$
$$\bar{C}_{\sigma(t)}=\begin{bmatrix} C_{\sigma(t)} & J_{\sigma(t)}C_w \end{bmatrix},$$
$$\bar{C}_{\mathrm{d}\sigma(t)}=\begin{bmatrix} C_{\mathrm{d}\sigma(t)} & 0 \end{bmatrix},$$
$$\bar{D}_{\sigma(t)}=\begin{bmatrix} D_{\sigma(t)} & J_{\sigma(t)}D_w \end{bmatrix},$$
$$\bar{C}_w=\begin{bmatrix} 0 & C_w \end{bmatrix},$$
$$\bar{D}_w=\begin{bmatrix} 0 & D_w \end{bmatrix},$$
$$\bar{H}_{\sigma(t)}=\begin{bmatrix} H_{\sigma(t)} \\ 0 \end{bmatrix},$$
$$\bar{F}_{\sigma(t)}=F_{\sigma(t)},$$
$$\bar{E}_{1\sigma(t)}=\begin{bmatrix} E_{1\sigma(t)} & 0 \end{bmatrix},$$
$$\bar{E}_{2\sigma(t)}=\begin{bmatrix} E_{2\sigma(t)} & 0 \end{bmatrix} \tag{7}$$

The augment system (8) could be obtained from system (1) and (6), written as

$$\begin{cases} \dot{\bar{x}}(t)=(\bar{A}_{\sigma(t)}+\Delta\bar{A}_{\sigma(t)})\bar{x}(t)+ \\ \qquad (\bar{A}_{\mathrm{d}\sigma(t)}+\Delta\bar{A}_{\mathrm{d}\sigma(t)})\bar{x}(t-\tau(t))+\bar{B}_{\sigma(t)}w(t) \\ y(t)=\bar{C}_{\sigma(t)}\bar{x}(t)+\bar{C}_{\mathrm{d}\sigma(t)}\bar{x}(t-\tau(t))+\bar{D}_{\sigma(t)}w(t) \\ f(t)=\bar{C}_w\bar{x}(t)+\bar{D}_w w(t) \\ \bar{x}^{\mathrm{T}}(t)=\begin{bmatrix} \phi^{\mathrm{T}}(t) & 0 \end{bmatrix},\quad t\in[-\tau,\quad 0] \end{cases} \tag{8}$$

where
$$\begin{bmatrix} \Delta\bar{A}_{\sigma(t)} & \Delta\bar{A}_{\mathrm{d}\sigma(t)} \end{bmatrix}=\bar{H}_{\sigma(t)}\bar{F}_{\sigma(t)}\begin{bmatrix} \bar{E}_{1\sigma(t)} & \bar{E}_{2\sigma(t)} \end{bmatrix},$$
$$\bar{F}^{\mathrm{T}}_{\sigma(t)}\bar{F}_{\sigma(t)}\le I.$$

According to the structure of system (8), the desired observer is described by

$$\begin{cases} \dot{\hat{x}}(t)=\bar{A}_{\sigma(t)}\hat{x}(t)+\bar{A}_{\mathrm{d}\sigma(t)}\hat{x}(t-\tau(t))+ \\ \qquad K_{\sigma(t)}(y(t)-\hat{y}(t)) \\ \hat{f}(t)=\bar{C}_w\hat{x}(t) \\ \hat{y}(t)=\bar{C}_{\sigma(t)}\hat{x}(t)+\bar{C}_{\mathrm{d}\sigma(t)}\hat{x}(t-\tau(t)) \end{cases} \tag{9}$$

where $K_{\sigma(t)}$ is the gain matrix to be determined; $\hat{x}(t)$ and $\hat{y}(t)$ are the state and output of the observer, respectively; $\hat{f}(t)$ denotes the fault estimator.

Let $\varepsilon(t)=\bar{x}(t)-\hat{x}(t)$, $e(t)=f(t)-\hat{f}(t)$. From Eqs. (8) and (9), error system (10) is obtained:

$$\begin{cases} \dot{\varepsilon}(t)=(\bar{A}_{\sigma(t)}-K_{\sigma(t)}\bar{C}_{\sigma(t)})\varepsilon(t)+ \\ \qquad (\bar{A}_{\mathrm{d}\sigma(t)}-K_{\sigma(t)}\bar{C}_{\mathrm{d}\sigma(t)})\varepsilon(t-\tau(t))+ \\ \qquad (\bar{B}_{\sigma(t)}-K_{\sigma(t)}\bar{D}_{\sigma(t)})w(t)+ \\ \qquad \Delta\bar{A}_{\sigma(t)}\bar{x}(t)+\Delta\bar{A}_{\mathrm{d}\sigma(t)}\bar{x}(t-\tau(t)) \\ e(t)=\bar{C}_w\varepsilon(t)+\bar{D}_w w(t) \end{cases} \tag{10}$$

Define the following matrices given by

$$\tilde{x}^{\mathrm{T}}(t)=\begin{bmatrix} \bar{x}^{\mathrm{T}}(t) & \varepsilon^{\mathrm{T}}(t) \end{bmatrix},$$
$$v^{\mathrm{T}}(t)=\begin{bmatrix} w^{\mathrm{T}}(t) & 0 \end{bmatrix},$$
$$\tilde{A}_{\sigma(t)}=\begin{bmatrix} \bar{A}_{\sigma(t)} & 0 \\ 0 & \bar{A}_{\sigma(t)}-K_{\sigma(t)}\bar{C}_{\sigma(t)} \end{bmatrix},$$
$$\tilde{A}_{\mathrm{d}\sigma(t)}=\begin{bmatrix} \bar{A}_{\mathrm{d}\sigma(t)} & 0 \\ 0 & \bar{A}_{\mathrm{d}\sigma(t)}-K_{\sigma(t)}\bar{C}_{\mathrm{d}\sigma(t)} \end{bmatrix},$$
$$\tilde{B}_{\sigma(t)}=\begin{bmatrix} \bar{B}_{\sigma(t)} & 0 \\ \bar{B}_{\sigma(t)}-K_{\sigma(t)}\bar{D}_{\sigma(t)} & 0 \end{bmatrix},$$
$$\Delta\tilde{A}_{\sigma(t)}=\begin{bmatrix} \Delta\bar{A}_{\sigma(t)} & 0 \\ \Delta\bar{A}_{\sigma(t)} & 0 \end{bmatrix},$$
$$\Delta\tilde{A}_{\mathrm{d}\sigma(t)}=\begin{bmatrix} \Delta\bar{A}_{\mathrm{d}\sigma(t)} & 0 \\ \Delta\bar{A}_{\mathrm{d}\sigma(t)} & 0 \end{bmatrix},$$
$$\tilde{C}_w=\begin{bmatrix} 0 & \bar{C}_w \end{bmatrix},$$
$$\tilde{D}_w=\begin{bmatrix} \bar{D}_w & 0 \end{bmatrix},$$
$$\tilde{H}_{\sigma(t)}=\begin{bmatrix} \bar{H}_{\sigma(t)} \\ \bar{H}_{\sigma(t)} \end{bmatrix},$$
$$\tilde{F}_{\sigma(t)}=\bar{F}_{\sigma(t)},$$
$$\tilde{E}_{1\sigma(t)}=\begin{bmatrix} \bar{E}_{1\sigma(t)} & 0 \end{bmatrix},$$
$$\tilde{E}_{2\sigma(t)}=\begin{bmatrix} \bar{E}_{2\sigma(t)} & 0 \end{bmatrix} \tag{11}$$

Then, the following augment error system is constructed via Eqs. (8) and (10):

$$\begin{cases} \dot{\tilde{x}}(t)=(\tilde{A}_{\sigma(t)}+\Delta\tilde{A}_{\sigma(t)})\tilde{x}(t)+ \\ \qquad (\tilde{A}_{\mathrm{d}\sigma(t)}+\Delta\tilde{A}_{\mathrm{d}\sigma(t)})\tilde{x}(t-\tau(t))+\tilde{B}_{\sigma(t)}v(t) \\ e(t)=\tilde{C}_w\tilde{x}(t)+\tilde{D}_w v(t) \end{cases} \tag{12}$$

where
$$\begin{bmatrix} \Delta\tilde{A}_{\sigma(t)} & \Delta\tilde{A}_{\mathrm{d}\sigma(t)} \end{bmatrix}=\tilde{H}_{\sigma(t)}\tilde{F}_{\sigma(t)}\begin{bmatrix} \tilde{E}_{1\sigma(t)} & \tilde{E}_{2\sigma(t)} \end{bmatrix},$$
$$\tilde{F}^{\mathrm{T}}_{\sigma(t)}\tilde{F}_{\sigma(t)}\le I.$$

The main task of this work is that constructing observer (9) guarantees: 1) when $v(t)$=0, system (12) is stable; 2) under initial condition $\phi(t)$=0, the H_∞ performance index should satisfy

$$\sup_{v(t)\ne 0}\frac{\|e(t)\|_2}{\|v(t)\|_2}\le\gamma \quad \text{for} \quad \|v(t)\|_2\in[0,+\infty) \tag{13}$$

If observer (9) could be built successfully, then the observer-based fault estimator $\hat{\boldsymbol{f}}(t)$ could be obtained from observer (9).

3 Robust fault estimator design

In this section, the sufficient conditions on the existence of fault estimator would be proposed, and the desired fault estimator would be built.

Lemma 2: For given scalars $\gamma>0$, $\lambda>0$ and $\rho\geq1$, if there exist positive symmetric matrices $\boldsymbol{T}_p$, $\boldsymbol{P}_p$ and $\boldsymbol{R}_p$, and positive scalar ε such that

$$\begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}} \\ * & \boldsymbol{\Psi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}} \\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0 \\ * & * & * & -\boldsymbol{I} & 0 & 0 \\ * & * & * & * & -\varepsilon^{-1}\boldsymbol{I} & 0 \\ * & * & * & * & * & -\varepsilon\boldsymbol{I} \end{bmatrix}<0 \tag{14}$$

$$\boldsymbol{T}_i\leq\rho\boldsymbol{T}_j,\ \boldsymbol{P}_i\leq\rho\boldsymbol{P}_j,\ \boldsymbol{R}_i\leq\rho\boldsymbol{R}_j\ i,j\in\underline{m} \tag{15}$$

and the ADT satisfies

$$\tau_{\mathrm{a}}>\frac{\ln\rho}{\lambda} \tag{16}$$

then there exists observer (9) such that system (12) meets the requirements (1) and (2).

$$\boldsymbol{\Psi}_{11}=\tilde{\boldsymbol{A}}_p{}^{\mathrm{T}}\boldsymbol{T}_p+\boldsymbol{T}_p\tilde{\boldsymbol{A}}_p+\lambda\boldsymbol{T}_p+\boldsymbol{P}_p+(\tau-\frac{\exp(-\lambda\tau)}{\tau})\boldsymbol{R}_p$$

$$\boldsymbol{\Psi}_{12}=\boldsymbol{T}_p\tilde{\boldsymbol{A}}_{\mathrm{d}p}+\frac{\exp(-\lambda\tau)}{\tau_1}\boldsymbol{R}_p$$

$$\boldsymbol{\Psi}_{22}=(d-1)\exp(-\lambda\tau)\boldsymbol{P}_p-\frac{\exp(-\lambda\tau)}{\tau}\boldsymbol{R}_p$$

Proof: Construct multiple Lyapunov-Krasovskii function for system (12) as follows:

$$\boldsymbol{V}_{\sigma(t)}(t)=\boldsymbol{V}_{\sigma(t),1}(t)+\boldsymbol{V}_{\sigma(t),2}(t)+\boldsymbol{V}_{\sigma(t),3}(t) \tag{17}$$

where

$$\boldsymbol{V}_{\sigma(t),1}(t)=\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{T}_{\sigma(t)}\tilde{\boldsymbol{x}}(t)$$

$$\boldsymbol{V}_{\sigma(t),2}(t)=\int_{t-\tau(t)}^{t}\exp\{\lambda(s-t)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{P}_{\sigma(t)}\tilde{\boldsymbol{x}}(s)\mathrm{d}s$$

$$\boldsymbol{V}_{\sigma(t),3}(t)=\int_{-\tau}^{0}\int_{t+\theta}^{t}\exp\{\lambda(s-t)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_{\sigma(t)}\tilde{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta$$

Let t_k represent the instant of the kth switching and t_k^- denote the instant just before t_k. Assume the pth subsystem is activated over $t\in[t_k,t_{k+1})$. Take the derivative of $\boldsymbol{V}_p(t)$ with respect to t along the trajectory of system (12) over $t\in[t_k,t_{k+1})$, and then

$$\begin{aligned}\dot{\boldsymbol{V}}_{p,1}(t)=&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)[(\tilde{\boldsymbol{A}}_p+\Delta\tilde{\boldsymbol{A}}_p)^{\mathrm{T}}\boldsymbol{T}_p+\boldsymbol{T}_p(\tilde{\boldsymbol{A}}_p+\Delta\tilde{\boldsymbol{A}}_p)+\\&\lambda\boldsymbol{T}_p]\tilde{\boldsymbol{x}}(t)+\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{T}_p\tilde{\boldsymbol{B}}_p\boldsymbol{v}(t)+\\&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{T}_p(\tilde{\boldsymbol{A}}_{\mathrm{d}p}+\Delta\tilde{\boldsymbol{A}}_{\mathrm{d}p})\tilde{\boldsymbol{x}}(t-\tau(t))+\\&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))(\tilde{\boldsymbol{A}}_{\mathrm{d}p}+\Delta\tilde{\boldsymbol{A}}_{\mathrm{d}p})^{\mathrm{T}}\boldsymbol{T}_p\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)+\\&\boldsymbol{v}^{\mathrm{T}}(t)\tilde{\boldsymbol{B}}_p^{\mathrm{T}}\boldsymbol{T}_p\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)-\lambda\boldsymbol{V}_{p,1}(t)\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{\boldsymbol{V}}_{p,2}(t)=&(\dot{\tau}(t)-1)\exp\{-\lambda\tau(t)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{P}_p\tilde{\boldsymbol{x}}(t-\tau(t))+\\&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_p\tilde{\boldsymbol{x}}(t)-\lambda\boldsymbol{V}_{p,2}(t)\\\leq&(d-1)\exp\{-\lambda\tau(t)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{P}_p\tilde{\boldsymbol{x}}(t-\tau(t))+\\&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_p\tilde{\boldsymbol{x}}(t)-\lambda\boldsymbol{V}_{p,2}(t)\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{\boldsymbol{V}}_{p,3}(t)\leq&-\lambda\boldsymbol{V}_{p,3}(t)+\tau\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}_p\tilde{\boldsymbol{x}}(t)-\frac{\exp\{\lambda(s-t)\}}{\tau}\cdot\\&[\tilde{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{R}_p[\tilde{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}(t-\tau(t))]\end{aligned} \tag{20}$$

By the Jensen inequality, inequality (21) is obtained from inequality (20):

$$\begin{aligned}\dot{\boldsymbol{V}}_{p,3}(t)\leq&-\lambda\boldsymbol{V}_{p,3}(t)+\tau\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}_p\tilde{\boldsymbol{x}}(t)-\frac{\exp\{\lambda(s-t)\}}{\tau}\cdot\\&[\tilde{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{R}_p[\tilde{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}(t-\tau(t))]\end{aligned} \tag{21}$$

From Eq. (18), and inequalities (19) and (21), it follows that

$$\begin{aligned}\dot{\boldsymbol{V}}_p(t)\leq-\lambda\boldsymbol{V}_p(t)+&\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\\tilde{\boldsymbol{x}}(t-\tau(t))\\\boldsymbol{v}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{S}_{11}&\boldsymbol{S}_{12}&\boldsymbol{T}_p\tilde{\boldsymbol{B}}_p\\*&\boldsymbol{S}_{22}&0\\*&*&0\end{bmatrix}\cdot\\&\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\\tilde{\boldsymbol{x}}(t-\tau(t))\\\boldsymbol{v}(t)\end{bmatrix}\end{aligned} \tag{22}$$

where

$$\begin{aligned}\boldsymbol{S}_{11}=&(\tilde{\boldsymbol{A}}_p+\Delta\tilde{\boldsymbol{A}}_p)^{\mathrm{T}}\boldsymbol{T}_p+\boldsymbol{T}_p(\tilde{\boldsymbol{A}}_p+\Delta\tilde{\boldsymbol{A}}_p)+\lambda\boldsymbol{T}_p+\boldsymbol{P}_p+\\&\tau\boldsymbol{R}_p-\frac{\exp\{\lambda(s-t)\}}{\tau}\boldsymbol{R}_p\end{aligned}$$

$$\boldsymbol{S}_{12}=\boldsymbol{T}_p(\tilde{\boldsymbol{A}}_{\mathrm{d}p}+\Delta\tilde{\boldsymbol{A}}_{\mathrm{d}p})+\frac{\exp\{\lambda(s-t)\}}{\tau}\boldsymbol{R}_p$$

$$\boldsymbol{S}_{22}=(d-1)\exp\{\lambda(s-t)\}\boldsymbol{P}_p-\frac{\exp\{\lambda(s-t)\}}{\tau}\boldsymbol{R}_p$$

First, when $\boldsymbol{v}(t)=0$, the stability of system (12) is considered.

Noting that $\boldsymbol{v}(t)=0$, inequality (22) could be written as

$$\dot{\boldsymbol{V}}_p(t)\leq-\lambda\boldsymbol{V}_p(t)+\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\\tilde{\boldsymbol{x}}(t-\tau(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{S}_{11}&\boldsymbol{S}_{12}\\*&\boldsymbol{S}_{22}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\\tilde{\boldsymbol{x}}(t-\tau(t))\end{bmatrix} \tag{23}$$

According to Schur complement, inequality (24) is obtained from inequality (14):

$$\begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} \\ * & \boldsymbol{\Psi}_{22} & 0 & 0 \\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} \\ * & * & * & -\boldsymbol{I} \end{bmatrix} + \varepsilon^{-1}\begin{bmatrix} \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}} \\ \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tilde{\boldsymbol{E}}_{1p} & \tilde{\boldsymbol{E}}_{2p} & 0 & 0 \end{bmatrix} + \varepsilon\begin{bmatrix} \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tilde{\boldsymbol{H}}_p^{\mathrm{T}}\boldsymbol{T}_p^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix} < 0 \tag{24}$$

where

$$\boldsymbol{\Psi}_{12} = \boldsymbol{T}_p\tilde{\boldsymbol{A}}_{\mathrm{d}p} + \frac{\exp\{\lambda(s-t)\}}{\tau_1}\boldsymbol{R}_p$$

$$\boldsymbol{\Psi}_{22} = (d-1)\exp\{\lambda(s-t)\}\boldsymbol{P}_p - \frac{\exp\{\lambda(s-t)\}}{\tau}\boldsymbol{R}_p$$

According to Lemma 1 and inequality (24), then

$$\begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} \\ * & \boldsymbol{\Psi}_{22} & 0 & 0 \\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} \\ * & * & * & -\boldsymbol{I} \end{bmatrix} + \begin{bmatrix} \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p \\ 0 \\ 0 \\ 0 \end{bmatrix}\tilde{\boldsymbol{F}}\begin{bmatrix} \tilde{\boldsymbol{E}}_{1p} & \tilde{\boldsymbol{E}}_{2p} & 0 & 0 \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}} \\ \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\tilde{\boldsymbol{F}}^{\mathrm{T}}\begin{bmatrix} \tilde{\boldsymbol{H}}_p^{\mathrm{T}}\boldsymbol{T}_p^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix} < 0 \tag{25}$$

Obviously, inequality (25) is equivalent to

$$\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} \\ * & \boldsymbol{S}_{22} & 0 & 0 \\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} \\ * & * & * & -\boldsymbol{I} \end{bmatrix} < 0 \tag{26}$$

By Schur complement and inequality (26), then

$$\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ * & \boldsymbol{S}_{22} \end{bmatrix} < 0 \tag{27}$$

Consequently,

$$\dot{\boldsymbol{V}}_p(t) + \lambda\boldsymbol{V}_p(t) < 0$$

$$\begin{aligned} \boldsymbol{V}_p(t) &< \exp\{-\lambda(t-t_k)\}\boldsymbol{V}_p(t_k) \\ &< \rho\exp\{-\lambda(t-t_k)\}\boldsymbol{V}_{\sigma(t_{k-1})}(t_k^-) \\ &= \rho\exp\{-\lambda(t-t_k)\}\boldsymbol{V}_{\sigma(t_{k-1})}(t_{k-1}) \end{aligned} \tag{28}$$

By iterative calculation, it follows that

$$\boldsymbol{V}_p(t) \le \rho^k\exp\{-\lambda(t-t_0)\}\boldsymbol{V}_{\sigma(t_0)}(t_0) \tag{29}$$

Let $\kappa_1 = \max_{p\in\underline{m}}\lambda(\boldsymbol{T}_p)$, $\kappa_2 = \max_{p\in\underline{m}}\lambda(\boldsymbol{P}_p)$, $\kappa_3 = \max_{p\in\underline{m}}\lambda(\boldsymbol{R}_p)$ and $\kappa_4 = \min_{p\in\underline{m}}\lambda(\boldsymbol{T}_p)$, where $\lambda(\boldsymbol{T}_p)$ denotes all eigenvalues of matrix $\boldsymbol{T}_p$.

$$\begin{aligned} \boldsymbol{V}_{\sigma(t_0)}(t_0) &= \tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\boldsymbol{T}_{\sigma(t_0)}\tilde{\boldsymbol{x}}(t_0) + \\ &\int_{t_0-\tau(t_0)}^{t_0}\exp\{\lambda(-t_0+s)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\boldsymbol{P}_{\sigma(t_0)}\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s + \\ &\int_{-\tau}^{0}\int_{t_0+\theta}^{t_0}\exp\{\lambda(-t_0+s)\}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\boldsymbol{R}_{\sigma(t_0)}\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s\mathrm{d}\theta \\ &\le \tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_1\tilde{\boldsymbol{x}}(t_0) + \int_{t_0-\tau}^{t_0}\exp\{\lambda(-t_0+s)\}\cdot \\ &\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_2\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s + \int_{-\tau}^{0}\int_{t_0-\tau}^{t_0}\exp\{\lambda(-t_0+s)\}\cdot \\ &\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_3\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{30}$$

Since $s \in [t_0-\tau, t_0]$ and $\exp\{\lambda(-t_0+s)\} \in [\exp(-\lambda\tau), 1]$,

$$\begin{aligned} \boldsymbol{V}_{\sigma(t_0)}(t_0) &\le \tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_1\tilde{\boldsymbol{x}}(t_0) + \int_{t_0-\tau}^{t_0}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_2\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s + \\ &\int_{-\tau}^{0}\int_{t_0-\tau}^{t_0}\tilde{\boldsymbol{x}}^{\mathrm{T}}(t_0)\kappa_3\tilde{\boldsymbol{x}}(t_0)\mathrm{d}s\mathrm{d}\theta \\ &= \kappa_1\|\tilde{\boldsymbol{x}}(t_0)\| + \tau\kappa_2\|\tilde{\boldsymbol{x}}(t_0)\| + \tau^2\kappa_3\|\tilde{\boldsymbol{x}}(t_0)\| \end{aligned} \tag{31}$$

Inequality (32) is derived from inequalities (29), (31) and the definition of $\boldsymbol{V}_{\sigma(t)}(t)$:

$$\begin{aligned} \|\tilde{\boldsymbol{x}}(t)\| &\le \frac{\kappa_1+\tau\kappa_2+\tau^2\kappa_3}{\kappa_4}\rho^k\exp\{-\lambda(t-t_0)\}\|\tilde{\boldsymbol{x}}(t_0)\| \\ &\le \frac{\kappa_1+\tau\kappa_2+\tau^2\kappa_3}{\kappa_4}\exp\{(\frac{\ln\rho}{\tau_{\mathrm{a}}}-\lambda)(t-t_0)\}\|\tilde{\boldsymbol{x}}(t_0)\| \end{aligned} \tag{32}$$

Define $\alpha = \dfrac{(\kappa_1+\tau\kappa_2+\tau^2\kappa_3)}{\kappa_4}$, and $\beta = \lambda - \dfrac{\ln\rho}{\tau_{\mathrm{a}}}$. Obviously, $\alpha>0$. Since $\tau_{\mathrm{a}} > \ln\rho/\lambda$, $\beta>0$. According to Definition 1, system (12) is GUES with an ADT (16).

Next, under initial condition $\boldsymbol{\phi}(t_0)=0$, consider the following performance index:

$$J = \int_{t_0}^{+\infty}[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) - \gamma^2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{v}(t)]\mathrm{d}t \tag{33}$$

For all non-zero $\boldsymbol{v}(t) \in l_2[0, +\infty)$,

$$\begin{aligned} J &= \int_{t_0}^{+\infty}[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) - \gamma^2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{v}(t)]\mathrm{d}t \\ &= \int_{t_0}^{+\infty}\left\{\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) - \gamma^2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{v}(t) + \dot{\boldsymbol{V}}_p(t)\right\}\mathrm{d}t - \\ &\lim_{t\to+\infty}\boldsymbol{V}_p(t) + \boldsymbol{V}_p(t_0) \end{aligned}$$

$$=\int_{t_0}^{+\infty}\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\\ \boldsymbol{v}(t)\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Pi}\begin{bmatrix}\tilde{\boldsymbol{x}}(t)\\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-t(t))\\ \boldsymbol{v(t)}\end{bmatrix}\mathrm{d}t-\lim_{t\to+\infty}\boldsymbol{V}_p(t)+\boldsymbol{V}_p(t_0) \tag{34}$$

where

$$\boldsymbol{\Pi}=\begin{bmatrix}\boldsymbol{S}_{11} & \boldsymbol{S}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p\\ * & \boldsymbol{S}_{22} & 0\\ * & * & -\gamma^2\boldsymbol{I}\end{bmatrix}+\begin{bmatrix}\tilde{\boldsymbol{C}}_w^{\mathrm{T}}\\ 0\\ \tilde{\boldsymbol{D}}_w^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{C}}_w & 0 & \tilde{\boldsymbol{D}}_w\end{bmatrix}$$

Furthermore,

$$\boldsymbol{\Pi}=\begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p\\ * & \boldsymbol{\Psi}_{22} & 0\\ * & * & -\gamma^2\boldsymbol{I}\end{bmatrix}+\begin{bmatrix}\boldsymbol{T}_p\tilde{\boldsymbol{H}}_p\\ 0\\ 0\end{bmatrix}\tilde{\boldsymbol{F}}_p\begin{bmatrix}\tilde{\boldsymbol{E}}_{1p} & \tilde{\boldsymbol{E}}_{2p} & 0\end{bmatrix}+\left\{\begin{bmatrix}\boldsymbol{T}_p\tilde{\boldsymbol{H}}_p\\ 0\\ 0\end{bmatrix}\tilde{\boldsymbol{F}}_p\begin{bmatrix}\tilde{\boldsymbol{E}}_{1p} & \tilde{\boldsymbol{E}}_{2p} & 0\end{bmatrix}\right\}^{\mathrm{T}}+\begin{bmatrix}\tilde{\boldsymbol{C}}_w^{\mathrm{T}}\\ 0\\ \tilde{\boldsymbol{D}}_w^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{C}}_w & 0 & \tilde{\boldsymbol{D}}_w\end{bmatrix} \tag{35}$$

According to Schur complement and Lemma 1, inequality (36) could be obtained:

$$\boldsymbol{\Pi}=\begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{T}_p\tilde{\boldsymbol{B}}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}\\ * & \boldsymbol{\Psi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}}\\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0\\ * & * & * & -\boldsymbol{I} & 0 & 0\\ * & * & * & * & -\varepsilon^{-1}\boldsymbol{I} & 0\\ * & * & * & * & * & -\varepsilon\boldsymbol{I}\end{bmatrix}<0 \tag{36}$$

Since $\boldsymbol{V}_p(t_0)=0$ and $\lim_{t\to+\infty}\boldsymbol{V}_p(t)>0$, then $J<0$, which implies $\dfrac{\|\boldsymbol{e}(t)\|_2}{\|\boldsymbol{v}(t)\|_2}<\gamma$. This completes the proof of Lemma 2.

Remark 1: Matrices $\tilde{\boldsymbol{B}}_p$, $\tilde{\boldsymbol{A}}_p$ and $\tilde{\boldsymbol{A}}_{\mathrm{d}p}$ contain unknown gain matrix $\boldsymbol{K}_p$, therefore $\tilde{\boldsymbol{A}}_p$, $\tilde{\boldsymbol{A}}_{\mathrm{d}p}$ and $\tilde{\boldsymbol{B}}_p$ are unknown matrices. Then, two unknown matrix variables $\boldsymbol{K}_p$ and $\boldsymbol{T}_p$ exist simultaneously in product terms of $\boldsymbol{T}_p\tilde{\boldsymbol{A}}_p$, $\tilde{\boldsymbol{A}}_p^{\mathrm{T}}\boldsymbol{T}_p$, $\boldsymbol{T}_p\tilde{\boldsymbol{A}}_{\mathrm{d}p}$ and $\boldsymbol{T}_p\tilde{\boldsymbol{B}}_p$. In addition, ε^{-1} also exists in inequality (14). Thus, it would generate product term $\boldsymbol{T}_p\tilde{\boldsymbol{A}}_p$. Consequently, inequality (14) is not a LMIS. Next, Lemma 3 is proposed to solve this problem.

Lemma 3: For given scalars $\gamma>0$ and $\lambda>0$, if there exist positive symmetric matrices $\boldsymbol{T}_p$, $\boldsymbol{P}_p$ and $\boldsymbol{R}_p$, and matrices $\boldsymbol{\Omega}_p$, $\boldsymbol{W}_p$ and $\boldsymbol{\Xi}_p$, and positive scalars a and b such that

$$\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{11} & \boldsymbol{\Xi}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}b\\ * & \boldsymbol{\Phi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}}b\\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0\\ * & * & * & -\boldsymbol{I} & 0 & 0\\ * & * & * & * & -a\boldsymbol{I} & 0\\ * & * & * & * & * & \boldsymbol{\Phi}_{66}\end{bmatrix}<0 \tag{37}$$

then inequality (14) holds.

$$\boldsymbol{\Phi}_{11}=\boldsymbol{\Omega}_p^{\mathrm{T}}+\boldsymbol{\Omega}_p+\lambda\boldsymbol{T}_p+\boldsymbol{P}_p+(\tau-\frac{\exp(-\lambda\tau)}{\tau})\boldsymbol{R}_p$$

$$\boldsymbol{\Phi}_{22}=-(1-d)\exp(-\lambda\tau)\boldsymbol{P}_p-\frac{\exp(-\lambda\tau)}{\tau}\boldsymbol{R}_p$$

$$\boldsymbol{\Phi}_{66}=(a-2b)\boldsymbol{I}$$

Proof: Since $a>0$,

$$\begin{cases}(a-b)^2a^{-1}\geq 0\\ a-2b\geq -ba^{-1}b\end{cases} \tag{38}$$

It follows from inequalities (37) and (38) that

$$\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{11} & \boldsymbol{\Xi}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}b\\ * & \boldsymbol{\Phi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}}b\\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0\\ * & * & * & -\boldsymbol{I} & 0 & 0\\ * & * & * & * & -a\boldsymbol{I} & 0\\ * & * & * & * & * & -ba^{-1}b\boldsymbol{I}\end{bmatrix}<0 \tag{39}$$

Pre-multiply diag $\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},b^{-1}\boldsymbol{I}\}$ and post-multiply diag $\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},b^{-1}\boldsymbol{I}\}$ to inequality (39):

$$\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{11} & \boldsymbol{\Xi}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}b\\ * & \boldsymbol{\Phi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{2p}^{\mathrm{T}}b\\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0\\ * & * & * & -\boldsymbol{I} & 0 & 0\\ * & * & * & * & -a\boldsymbol{I} & 0\\ * & * & * & * & * & -a^{-1}\boldsymbol{I}\end{bmatrix}<0 \tag{40}$$

Let $a=\varepsilon^{-1}$, $\boldsymbol{\Omega}_p=\boldsymbol{T}_p\tilde{\boldsymbol{A}}_p$, $\boldsymbol{W}_p=\boldsymbol{T}_p\tilde{\boldsymbol{A}}_{\mathrm{d}p}$ and $\boldsymbol{\Xi}_p=\boldsymbol{T}_p\tilde{\boldsymbol{B}}_p$, then inequality (14) is established. This completes the proof.

Remark 2: In Lemma 3, $\boldsymbol{T}_p\tilde{\boldsymbol{A}}_p$, $\boldsymbol{T}_p\tilde{\boldsymbol{A}}_{\mathrm{d}p}$ and $\boldsymbol{T}_p\tilde{\boldsymbol{B}}_p$ are replaced by $\boldsymbol{\Omega}_p$, $\boldsymbol{W}_p$ and $\boldsymbol{\Xi}_p$, respectively. Positive variables a and b are introduced to eliminate the product terms generated by ε^{-1}. By this way, inequality (14) is transformed into LMIS (37).

Theorem 1: For given scalars $\gamma>0$, $\lambda>0$ and $\rho\geq 1$, if there exist positive symmetric matrices $\boldsymbol{T}_p$, $\boldsymbol{P}_p$ and $\boldsymbol{R}_p$, and matrices $\boldsymbol{\Omega}_p$, $\boldsymbol{W}_p$ and $\boldsymbol{\Xi}_p$, and positive scalars a and b such that

$$\begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{11} & \boldsymbol{\Xi}_p & \tilde{\boldsymbol{C}}_w^{\mathrm{T}} & \boldsymbol{T}_p\tilde{\boldsymbol{H}}_p & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}b \\ * & \boldsymbol{\Phi}_{22} & 0 & 0 & 0 & \tilde{\boldsymbol{E}}_{1p}^{\mathrm{T}}b \\ * & * & -\gamma^2\boldsymbol{I} & \tilde{\boldsymbol{D}}_w^{\mathrm{T}} & 0 & 0 \\ * & * & * & -\boldsymbol{I} & 0 & 0 \\ * & * & * & * & -a\boldsymbol{I} & 0 \\ * & * & * & * & * & \boldsymbol{\Phi}_{66} \end{bmatrix} < 0 \tag{41}$$

$$\boldsymbol{T}_i \le \rho\boldsymbol{T}_j,\quad \boldsymbol{P}_i \le \rho\boldsymbol{P}_j,\ \boldsymbol{R}_i \le \rho\boldsymbol{R}_j,\quad i,j\in \underline{m} \tag{42}$$

and the ADT satisfies

$$\tau_{\mathrm{a}} > \frac{\ln\rho}{\lambda} \tag{43}$$

then there is observer (9) such that system (12) is GUES, and

$$\sup_{\boldsymbol{v}(t)\ne 0}\frac{\|\boldsymbol{e}(t)\|_2}{\|\boldsymbol{v}(t)\|_2} \le \gamma \ \text{ holds for } \ \|\boldsymbol{v}(t)\|_2 \in [0,+\infty) \tag{44}$$

Proof: According to Lemma 3 and inequality (41), inequality (14) holds. Furthermore, by Lemma 2, inequalities (42) and (43), there exists observer (9) such that system (12) is GUES and $\sup_{\boldsymbol{v}(t)\ne 0}\frac{\|\boldsymbol{e}(t)\|_2}{\|\boldsymbol{v}(t)\|_2} \le \gamma$ for $\|\boldsymbol{v}(t)\|_2 \in [0,+\infty)$. This completes the proof.

Remark 3: First, $\boldsymbol{\Omega}_p$ could be obtained via solving LMIs (41). Then, $\tilde{\boldsymbol{A}}_p = \boldsymbol{T}_p^{-1}\boldsymbol{\Omega}_p$. Finally, from Eqs. (7) and (11), $\boldsymbol{K}_p$ is obtained. By this way, the fault observer is built and the fault estimator $\hat{\boldsymbol{f}}(t)$ is also obtained.

4 Numerical example

In this section, a numerical example is presented to illustrate the validity of the obtained result. Consider the switched linear system (1) with following subsystems and fault system.

1) Subsystem 1:

$$\begin{cases} \boldsymbol{A}_1 = \begin{bmatrix} -0.5 & 1.0 \\ 0.1 & -0.3 \end{bmatrix} \\ \boldsymbol{A}_{d1} = \begin{bmatrix} 0.2 & 0.3 \\ -0.5 & -0.1 \end{bmatrix} \\ \boldsymbol{B}_1 = \boldsymbol{G}_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \\ \boldsymbol{C}_1 = [0.2 \quad 0.3] \\ \boldsymbol{C}_{d1} = [0.1 \quad 0.2] \\ \boldsymbol{D}_1 = 0.1 \\ \boldsymbol{J}_1 = 0.5 \end{cases} \tag{45}$$

2) Subsystem 2:

$$\begin{cases} \boldsymbol{A}_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & 0.4 \end{bmatrix} \\ \boldsymbol{A}_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ -0.3 & -0.4 \end{bmatrix} \\ \boldsymbol{B}_2 = \boldsymbol{G}_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \\ \boldsymbol{C}_2 = [0.2 \quad 0.1] \\ \boldsymbol{C}_{d2} = [0.1 \quad 0.1] \\ \boldsymbol{D}_2 = 0.2 \\ \boldsymbol{J}_2 = 0.4 \end{cases} \tag{46}$$

3) Fault system:

$$\begin{cases} \boldsymbol{A}_w = \begin{bmatrix} 0 & 1 \\ -0.2 & -0.8 \end{bmatrix} \\ \boldsymbol{B}_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \boldsymbol{C}_w = [0.2 \quad 0] \\ \boldsymbol{D}_w = 0.2 \end{cases} \tag{47}$$

Let γ=0.2, λ=0.2 and ρ=1.1. From Theorem 1 and Remark 3, it is easy to get

$$\begin{cases} \tau_{\mathrm{a}} > 0.4766 \\ \boldsymbol{K}_1 = \begin{bmatrix} 2.0062 \\ 3.2242 \\ -9.0552 \\ -1.9527 \end{bmatrix} \\ \boldsymbol{K}_2 = \begin{bmatrix} -7.2691 \\ 6.5349 \\ -4.2853 \\ 2.5209 \end{bmatrix} \end{cases} \tag{48}$$

Over time interval [0, 10], the results of the simulation are shown in Figs. 1−3.

In Fig 1, one could find τ_{a}>0.4766, which implies

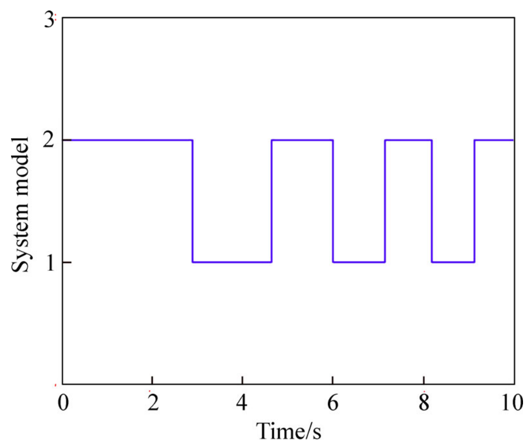

Fig. 1 Simulation of switching law

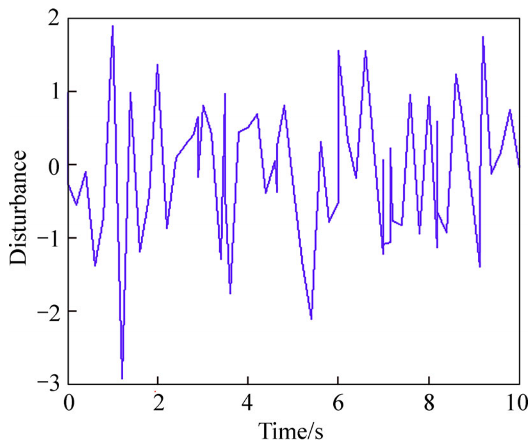

Fig. 2 Simulation of disturbance

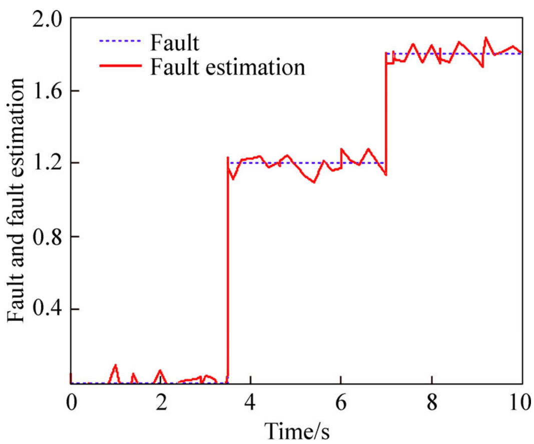

Fig. 3 Simulation of fault and fault estimation

ADT satisfies inequality (43). The random disturbance is shown in Fig. 2. From Fig. 3, one can find that the fault estimation approximates the fault. The error between them is very small. Besides, the fault estimator could tract the trajectory of fault timely. Thus, the designed fault estimator could estimate the fault exactly and meet the requirements.

In most of other works, the model uncertainty has not been taken into account. However, the designed fault estimator in this work is robust to uncertainty. Compared with other works, such as Refs. [26] and [27], the fault estimation in this work approximates the fault more exactly.

5 Conclusions

1) By constructing an observer-based estimator, this problem is formulated as a H_∞ problem. Based on the multiple Lyapunov-Krasovskii functions and average dwell-time method, the sufficient conditions on the existence of robust fault estimator are obtained. Furthermore, the sufficient conditions are presented in form of LMIs via a novel method.

2) It should be pointed out that the multiply co-positive Lyapunov-Krasovskii functions used in this work are traditional. In the future work, we will try to utilize the new kind of function to deal with the fault estimation problem for switched positive system, and make a comparative analysis with the traditional functions.

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