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Roof collapse of shallow tunnels with limit analysis method

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Abstract: A new failure mechanism is proposed to analyze the roof collapse based on nonlinear failure criterion. Limit analysis approach and variational principle are used to obtain analytical findings concerning the stability of potential roof. Then, parametric study is carried out to derive the change rule of corresponding parameters on the influence of collapsing shape, which is of paramount engineering significance to instruct the tunnel excavations. In comparison with existing results, the findings show agreement and validity of the proposed method. The actual collapse in certain shallow tunnels is well in accordance with the proposed failure mechanism.

Key words: tunnel roofs; nonlinear failure criterion; failure mechanism; kinematical approach

1 Introduction

In the analysis of tunnel stability, linear failure criterion was widely used due to its simplicity, and it can satisfy the engineering demand to some extent [1−2]. However, many experiments have validated nonlinear mechanical characteristics of geotechnical materials rather than simply linear identities in the stress space, particularly in the range of small normal stress [3]. Meanwhile, there was a tough problem that was involved in calculating the rate of energy dissipation and the work rate done by external forces along with velocity discontinuities to adopt nonlinear failure criterion. To resolve this intractable issue, a generalized tangential methodology was introduced. It utilized a tangential line point on the nonlinear curve to reflect the characteristic of nonlinear failure criterion. Thus, the tangential parameters were formulated to compute the power of energy dissipation and external loads with ease. Since limit analysis approach was first used in 1975, the upper and lower bound theorems have been effectively compulsory methods to estimate the stability of geotechnical materials. Therefore, numerous actual engineering cases have been tackled with a reasonable range of results. A kinematical multi-block collapse mechanism was constructed to derive the upper bound solution of stability coefficient. The results had agreement for shallow tunnels in comparison with the experiments. SLOAN and ASSADI [4] derived the analytical bounds of supporting pressure on the basis of limit analysis approach and finite element (FE) technique. Thereafter, based on 3D failure mechanism, the centrifugal model tests were employed to reflect the practical failure mode of tunnels. CHAMBON and CORTÉ [5] verified the findings of supporting pressure. LAM and FREDLUND [6] utilized the 3D pattern to estimate the cylinder stress function of the slope model. Meanwhile, DONALD and CHEN [7] obtained the slope minimal safety coefficient by constructing 3D failure mechanism, based on the plane analytical method.

In the previous work, most scholars devoted to face stability analysis of tunnel excavations with analytical method and numerical simulation. ANAGNOSTOU and KOVÁRI [8] studied the stability of excavation face under drainage condition, constructing 3D failure mechanism in the excavation face of earth-pressure-balanced shield tunnel. The typical 3D conical-shaped rotational collapse mechanism was employed. In this case, the precise upper bound solution of supporting pressure was derived in the excavation face of shallow tunnels. An elliptical scope, inscribed to the circular tunnel face, was involved in failure block, and the remaining area of the tunnel face was at test. Faced with such problem, spatial discretization technology by point-to-point methodology was adopted by MOLLON et al [9] to generate a 3D failure mechanism, so as to obtain more exact solutions.

The stability of tunnel roof remained significant in the tunnel excavations. Recently, some scholars shifted the focus to this respect, and have gained fruitful achievements. FRALDI and GUARRACINO [10−11]

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firstly gained the analytical expression of velocity detaching line, which described the collapsing scope of tunnel roof in deep-buried tunnels. Then, the failure mechanism was extended from the roof to the ground with arch effect taken into account to calculate critical depth. This depth was used to distinguish the deep and shallow tunnel according to the failure mechanism. Actually, a large number of tunnels were built under water table where seepage forces exert. The pore water pressure cannot be neglected when investigating the roof stability. HUANG et al [12−13] adopted the similar collapse mechanism in deep tunnels to discuss the influence of pore pressure on collapsing shape for circular tunnels. The rate of pore pressure was incorporated into virtual equation as an external force. Under the 3D collapse mode of shallow tunnel, QIN et al [14] introduced the rate of seepage forces to estimate how seepage forces influence the roof stability, and the results were in accordance with actual engineering situation.

Many tunnels were built in shallow strata, especially in underground construction. The analysis of very shallow tunnels was of the same significance to instruct the design and in-site construction. Importantly, a new collapse mechanism should be constructed to approximate the real potential failure. OSMAN et al [15] investigated stability number and surface settlement profile in undrained clay with plastic deformation mechanism. Thereafter, ASHRAF and OSMAN [16] studied the stability problem of twin tunnels excavated in soft layer through constructing a compatible displacement field. In the limit analysis of slope stability, the failure mechanism was constructed to work out the analytical upper bound solution of stability [17−18]. In this work, a new curved collapse mechanism referring to practical failure mode was presented regardless of arch effect, since its effect in very shallow tunnels was relatively small. Meanwhile, circular and rectangle cross-sections were selected to calculate the exact velocity discontinuities with upper bound theorem and nonlinear failure criterion. Analytical solutions were presented for practical use in engineering.

2 Principle theory

2.1 Nonlinear failure criterion

Nonlinear mechanical characteristic of geotechnical material was extensively adopted in engineering. And it has been the dominant formula in nonlinear analysis of geotechnical engineering. Generally, it was written with two different forms, expressed by major and minor principal stresses and normal and shear stresses, respectively [19]. Considering the simplicities of calculating the rate of internal energy dissipation, it is very convenient to employ the latter form, expressed as

$$\tau = A\sigma_c[(\sigma_n + \sigma_t)\sigma_c^{-1}]^B \tag{1}$$

where τ and σ_n are the shear and normal stresses, respectively. A and B represent physical parameters describing the characteristics of material. σ_c and σ_t are the uniaxial compressive strength and the tensile strength, respectively. The failure criterion [19] is widely used in soil and rock mechanics.

2.2 Upper bound theorem

Based on the merits of the upper bound theorem, it has been broadly used in engineering. And it is indicated that the actual collapsing load is no more than the load gained by equating the rate of energy dissipation to the external work rate in any kinematically admissible velocity field, when the velocity boundary condition is satisfied. The upper bound theorem in the framework of limit analysis is expressed as

$$\int_\Omega \sigma_{ij}\dot{\varepsilon}_{ij}\mathrm{d}\Omega \ge \int_s T_i v_i \mathrm{d}s + \int_\Omega X_i v_i \mathrm{d}\Omega \tag{2}$$

where σ_{ij} is the stress, and $\dot{\varepsilon}_{ij}$ means strain rate in any kinematically admissible velocity field. T_i represents a surcharge load on boundary s, and X_i indicates the body force. Ω shows the volume of the potential collapsing block, and v_i is the velocity. To simplify the calculation process, certain hypothesis is introduced when estimating the stability of tunnels with the upper bound theorem. First, the geotechnical material is perfectly plastic and follows an associated flow rule. Second, the collapsing block is regarded as a rigid block without considering volumetric strain.

3 Failure mode of collapsing block

As introduced before, a new curved collapse mechanism is constructed without taking arch effect into consideration to probe into the potential roof failure with limit analysis approach. According to existing research and actual failure mode of tunnel roofs, the proposed mechanism resembles pour eight-shaped curve from the circumference of tunnel stretching to the ground surface, as shown in Fig. 1. From the mechanism, it is manifest that the collapsing area tends to increase with the increasing buried depth. The failure mode is solely suitable for those very shallow tunnels.

4 Upper bound solutions with variational method

Under the limit state, the tunnel roof is about to slip along with the detaching line. The energy dissipation at a

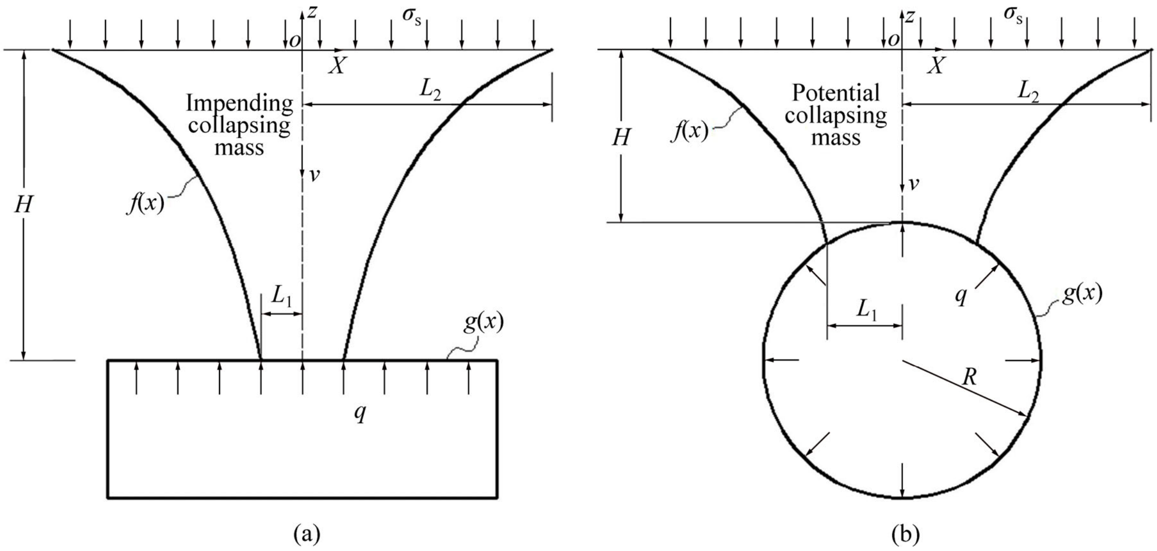

Fig. 1 Potential roof collapse of tunnels: (a) Square cross-section; (b) Circular cross-section

random point along the velocity discontinuity caused by normal and shear stresses is expressed as Eq. (9) in Ref. [10]. The total energy dissipation rate along the velocity discontinuity line can be worked out by integrating D_i over the interval (L_1, L_2):

$$D=\int_s D_i t\mathrm{d}s=\int_{L_1}^{L_2}[\sigma_\mathrm{t}-\sigma_\mathrm{c}(AB)^{1/(1-B)}(1-B^{-1})\cdot f'(x)^{1/(1-B)}]v\mathrm{d}x \tag{3}$$

where $f'(x)$ is the first derivative of $f(x)$; t means the thickness of the detaching surface; L_1 and L_2 are the widths of collapsing block, as illustrated in Fig. 1. In terms of unbiased tunnel under homogeneous and isotropous material, the failure mechanism of unbiased tunnel presents symmetry with respect to Z-axis. The work rate of collapsing block caused by weight can be computed as

$$P_\gamma=\gamma\int_{L_1}^{L_2}f(x)v\mathrm{d}x+\gamma\int_0^{L_1}g(x)v\mathrm{d}x \tag{4}$$

where γ is the unit weight of rock/soil mass, and $g(x)$ describes the tunnel cross-section, including circular and rectangle profile.

Tunnels are often built in very shallow strata. To ensure the safety and stability of tunnel structure, supporting structure is imposed inside the inner surface of tunnel. Therefore, the work rate of supporting pressure in shallow circular and rectangle tunnel remains the same pattern:

$$P_q=qL_1v \tag{5}$$

where q is equal to the supporting pressure imposed on the circumference of tunnel lining.

Surcharge load also acts on the collapsing block due to its specific mechanism. The work rate of surcharge force results in

$$P_{\sigma_\mathrm{s}}=-\sigma_\mathrm{s}L_2v \tag{6}$$

where σ_s is the surcharge force applied on the ground surface.

The virtual equation of an objective function, constituted by the rate of external work and energy dissipation, can be formulated as

$$\zeta=D-P_\gamma-P_q-P_{\sigma_\mathrm{s}} \tag{7}$$

Substituting Eqs. (3)−(6) into Eq. (7) yields

$$\begin{aligned}\zeta&=\int_{L_1}^{L_2}[\sigma_\mathrm{t}-\sigma_\mathrm{c}(AB)^{1/(1-B)}(1-B^{-1})f'(x)^{1/(1-B)}-\\&\quad\gamma f(x)]v\mathrm{d}x-\gamma\int_0^{L_1}g(x)v\mathrm{d}x-qL_1v+\sigma_\mathrm{s}L_2v\\&=\int_{L_1}^{L_2}\psi[f(x),f'(x),x]v\mathrm{d}x-\gamma\int_0^{L_1}g(x)v\mathrm{d}x-qL_1v+\sigma_\mathrm{s}L_2v\end{aligned} \tag{8}$$

where the form of $\psi[f(x), f'(x), x]$ is

$$\psi[f(x),f'(x),x]=\sigma_\mathrm{t}-\sigma_\mathrm{c}(AB)^{1/(1-B)}(1-B^{-1})\cdot f'(x)^{1/(1-B)}-\gamma f(x) \tag{9}$$

Based on upper bound theorem, there exist numerous solutions which belong to a part of upper bound solutions. As to a specific problem, an optimal finding can be derived which makes the virtual equation reach an extremum by optimizing the objective function to achieve the minimum. The integral function ψ is shifted to a differential equation by variational calculation. According to variational principle and the condition of customary regularity, the expression of ψ is translated into Euler's equation to make the integral over the interval [L_1, L_2] acquire a stationary value:

$$\delta\psi = 0 \;\Rightarrow\; \frac{\partial \psi}{\partial f(x)} - \frac{\partial}{\partial x}\left[\frac{\partial \psi}{\partial f'(x)}\right] = 0 \tag{10}$$

According to Eq. (9), the explicit form of Euler's equation is

$$\sigma_{\mathrm{c}}(AB)^{1/(1-B)}(1-B)^{-1} f'(x)^{(2B-1)/(1-B)} f''(x) + \gamma = 0 \tag{11}$$

It is a second-order homogeneous differential equation, and the final expression of the velocity discontinuity surface, $f(x)$, can be determined through twice integral calculation:

$$f(x) = -A^{-1/B}\left(\frac{\gamma}{\sigma_{\mathrm{c}}}\right)^{(1-B)/B}\left(\frac{\tau_0}{\gamma} - x\right)^{1/B} + h \tag{12}$$

where τ_0 and h represent the integration constants determined by geometric and mechanical boundary conditions, respectively. Based on mechanical condition on the ground surface, the distribution of shear stress is zero. Consequently, an implicit mechanical equilibrium equation is satisfied:

$$\tau_{xz}\left(x = L_2, z = 0\right) = 0 \tag{13}$$

Meanwhile, there still exist two geometrical boundary conditions needed to determine the explicit expression of $f(x)$. The following conditions are introduced here to calculate the constants, and build the geometrical relationship of collapsing block:

$$f(x = L_2, z = 0) = 0 \tag{14}$$

$$f(x = L_1) = g(x = L_1) \tag{15}$$

According to Eqs. (13)−(15), the analytical form of $f(x)$ is written as

$$f(x) = -A^{-1/B}\left(\gamma/\sigma_{\mathrm{c}}\right)^{(1-B)/B}\left(L_2 - x\right)^{1/B} \tag{16}$$

Then, by combining Eq. (9) and Eq. (16), the expression of ψ results in

$$\psi[f(x), f'(x), x] = \sigma_{\mathrm{t}} + A^{-1/B}B^{-1}\sigma_{\mathrm{c}}^{(B-1)/B}\gamma^{1/B}\left(L_2 - x\right)^{1/B} \tag{17}$$

As mentioned before, the circular and square tunnel profiles are considered to estimate the potential roof collapse. The collapsing scale with different cross-section presents distinctive scope. In the following analysis, the discussion is divided into two parts owing to two kinds of cross profiles, though the expression of the work rate of supporting pressure is identical.

4.1 Collapsing block in circular tunnel

According to the coordinate system illustrated in Fig. 1, the expression of $g(x)$ for describing the shape of circular profile gives

$$g(x) = -(H + R) + \sqrt{R^2 - x^2} \tag{18}$$

Substituting Eq. (18) into Eq. (15), a geometric relationship of collapsing block is built:

$$A^{-1/B}\left(\gamma/\sigma_{\mathrm{c}}\right)^{(1-B)/B}\left(L_2 - L_1\right)^{1/B} = H + R - \sqrt{R^2 - L_1^2} \tag{19}$$

In order to get the upper bound solution of potential collapse, some integral calculations should be performed, for instance,

$$\int_0^{L_1} g(x)\mathrm{d}x = -(H + R)L_1 - \frac{R^2}{2}\left[\arccos\frac{L_1}{R} - \frac{\pi}{2} - \frac{L_1\sqrt{R^2 - L_1^2}}{R^2}\right] \tag{20}$$

By combining Eqs. (17), (20) and Eq. (8), the expression of ζ turns into

$$\zeta = \sigma_{\mathrm{t}}(L_2 - L_1)v + A^{-1/B}(1+B)^{-1}\sigma_{\mathrm{c}}^{(B-1)/B}\gamma^{1/B} \cdot \left(L_2 - L_1\right)^{(1+B)/B} v + \gamma(H+R)L_1 v + \frac{\gamma R^2}{2}\left[\arccos\frac{L_1}{R} - \frac{\pi}{2} - \frac{L_1\sqrt{R^2 - L_1^2}}{R^2}\right]v - qL_1 v + \sigma_{\mathrm{s}}L_2 v \tag{21}$$

Equating virtual equation to zero means making the energy dissipation rate equivalent to the rate of external work:

$$\sigma_{\mathrm{t}}(L_2 - L_1) + A^{-1/B}(1+B)^{-1}\sigma_{\mathrm{c}}^{(B-1)/B}\gamma^{1/B} \cdot \left(L_2 - L_1\right)^{(1+B)/B} + \sigma_{\mathrm{s}}L_2 + \gamma(H+R)L_1 + \frac{\gamma R^2}{2}\left[\arccos\frac{L_1}{R} - \frac{\pi}{2} - \frac{L_1\sqrt{R^2 - L_1^2}}{R^2}\right] - qL_1 = 0 \tag{22}$$

By solving Eqs. (19) and (22) simultaneously, the solutions of L_1 and L_2 are obtained and the final results of potential roof failure in circular tunnel are determined.

4.2 Collapsing block in rectangle tunnel

The square profile is widely selected in the design of subway station. The analysis of roof stability in rectangle tunnels is inevitably a very stimulating research aspect as well. Likewise, the exact form of $g(x)$ remains a concise expression, $g(x) = -H$. Putting this into Eq. (15), the relationship of the collapsing block gives

$$A^{-1/B}\left(\gamma/\sigma_{\mathrm{c}}\right)^{(1-B)/B}\left(L_2 - L_1\right)^{1/B} = H \tag{23}$$

Again, substituting $g(x) = -H$ into Eq. (8), then

$$\zeta = \sigma_{\mathrm{t}}(L_2 - L_1)v + A^{-1/B}(1+B)^{-1}\sigma_{\mathrm{c}}^{(B-1)/B}\gamma^{1/B} \cdot$$

$$\left(L_2-L_1\right)^{(1+B)/B}v+\gamma HL_1v-qL_1v+\sigma_s L_2 v \tag{24}$$

Applying upper bound theorem and making the virtual equation approach zero, it is

$$\sigma_t(L_2-L_1)+A^{-1/B}(1+B)^{-1}\sigma_c^{(B-1)/B}\gamma^{1/B}\cdot \\ \left(L_2-L_1\right)^{(1+B)/B}+\gamma HL_1-qL_1+\sigma_s L_2=0 \tag{25}$$

After calculating Eqs. (23) and (25), the analytical expressions of collapsing block, L_1 and L_2, are given as follows:

$$L_1=A(\sigma_c/\gamma)^{(1-B)}H^B\frac{\sigma_t+(1+B)^{-1}\gamma H+\sigma_s}{q-\gamma H-\sigma_s} \tag{26}$$

$$L_2=A(\sigma_c/\gamma)^{(1-B)}H^B\frac{\sigma_t-B\left(1+B\right)^{-1}\gamma H+q}{q-\gamma H-\sigma_s} \tag{27}$$

5 Consistency of proposed failure mechanism

To evaluate the validity of suggested collapse mechanism, it should be of overriding necessity to compare the theoretical results with precious research findings and practical failure mode in site. The proposed mechanism is similar to that of OSMAN et al [15]. The results gained in this work can verify its correctness with referring to that. Since the research object is constructed in undrained clay, the failure criterion in purely clay is linear and even a constant $\tau=c$, where c is the internal cohesion. The velocity discontinuity line is an inclined straight line with $f'(x)=\cot\varphi$, where φ is an internal frictional angle. In this case, the collapse mechanism is composed of two vertical lines. Considering the symmetric characteristic, the corresponding calculation process is as follows.

Under the condition of $\tau=c$, the work rate of soil weight is computed as

$$P_\gamma=\gamma\int_0^L g(x)v\mathrm{d}x=-\gamma(H+R)Lv- \\ \frac{\gamma R^2}{2}\left[\arccos\frac{L}{R}-\frac{\pi}{2}-\frac{L\sqrt{R^2-L^2}}{R^2}\right]v \tag{28}$$

where L is the collapse width of failure block which shares the same magnitude in ground surface and at the circumference of the tunnel roof, and H shows the height of collapsing block. Meanwhile, the internal energy dissipation is expressed as

$$D=chv \tag{29}$$

With regard to the power generated by supporting pressure, it gives

$$P_q=qLv \tag{30}$$

Moreover, the work rate of surcharge load imposed on the ground surface results in

$$P_{\sigma_s}=-\sigma_s Lv \tag{31}$$

Based on the upper bound theorem of limit analysis, the virtual equation is formulated by equating the internal energy dissipation to external work rate:

$$\gamma(H+R)L+\frac{\gamma R^2}{2}\left[\arccos\frac{L}{R}-\frac{\pi}{2}-\frac{L\sqrt{R^2-L^2}}{R^2}\right]+\sigma_s L= \\ ch+qL \tag{32}$$

In this case, the supporting pressure can reach the extreme value when the collapse width L attains R. In order to compare the results gained in this work with that in Ref. [15], the parameters should be the same: γ=20 kN/m^3, σ_s=50 kPa, R=5 m, H=10 m, s_m=0 m, c=26 kPa, and H=h−R. Substituting these parameters into Eq. (32), the value of supporting pressure is q=114.92 kPa, and then the comparative results are presented. It is found that the present solution is 0.16, which approaches to the OSMAN et al's solution.

The solution obtained in this work is in accordance with that obtained in the research of OSMAN et al [15]. The relative error between the two attains 6.7%, which indicates that the analytical results of failure mechanism proposed in this work are valid, compared with the existing study.

6 Numerical results and parametric study

The nonlinear failure criterion is characterized by certain parameters, which play varying influence on the dimensions of collapsing block. As a consequence, the analysis of different parameters on the failure block is of typical significance from the perspective of engineering. Numerical calculation with single variable is considered to look insight into the sensitivity and change rule of various parameters. Based on the form of profile, the effects of parameters on the impending collapse in circular and square tunnels are illustrated in Figs. 2−5.

From the results, it is found that the change law is completely in accordance with the previous research. With the decrease of A, σ_c and σ_s, the width of collapsing block tends to decrease to varying degree. However, it follows a negative correlation with q, R and B changing.

The regular pattern of arguments in rectangle tunnels follows the similar form, though the actual value of collapse scale varies. On the whole, the range of failure block in square tunnels is a bit greater than that in circular tunnels without taking the effect of arch into account. Theoretically, the lower the values of B, q and γ, the larger the volume and the width of impending collapse. Inversely, the impact of A, σ_c and σ_s is positively proportional to the scope of potential failure volume.

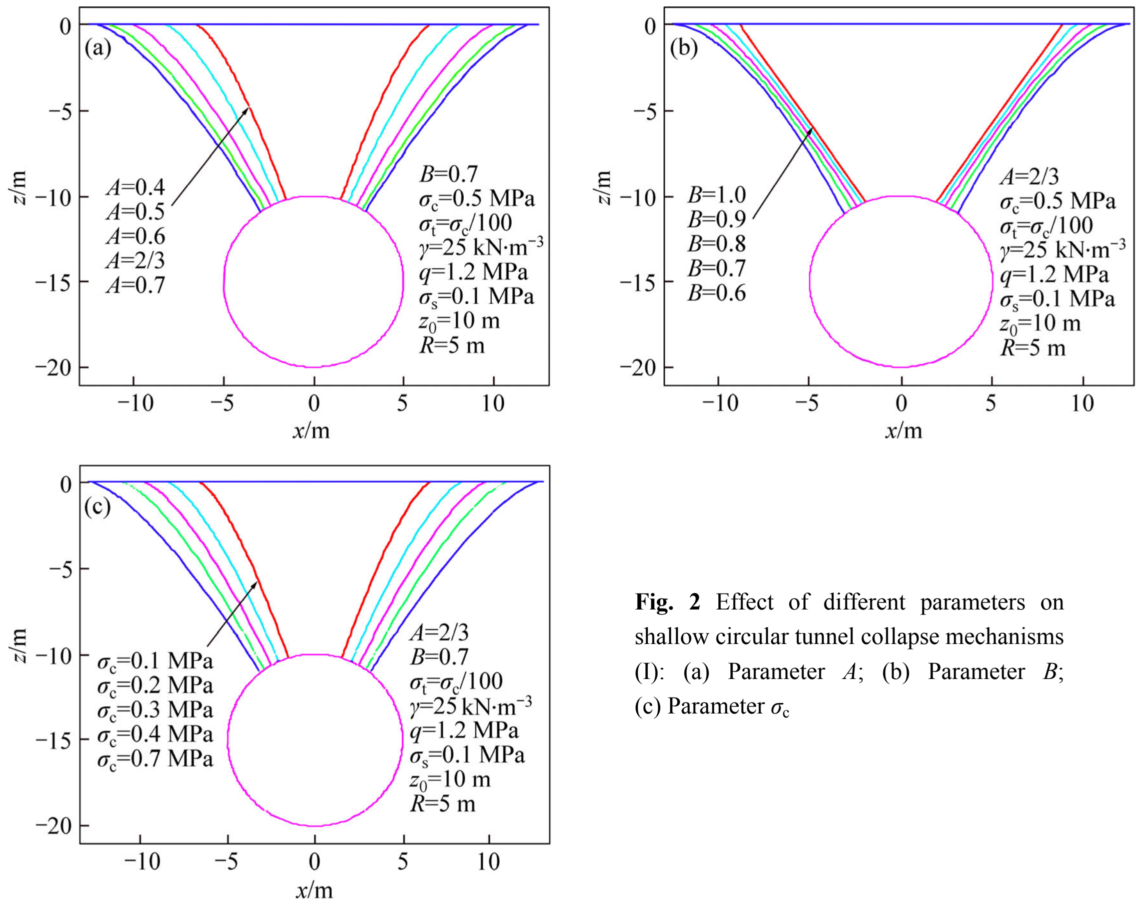

Fig. 2 Effect of different parameters on shallow circular tunnel collapse mechanisms (I): (a) Parameter A; (b) Parameter B; (c) Parameter σ_c

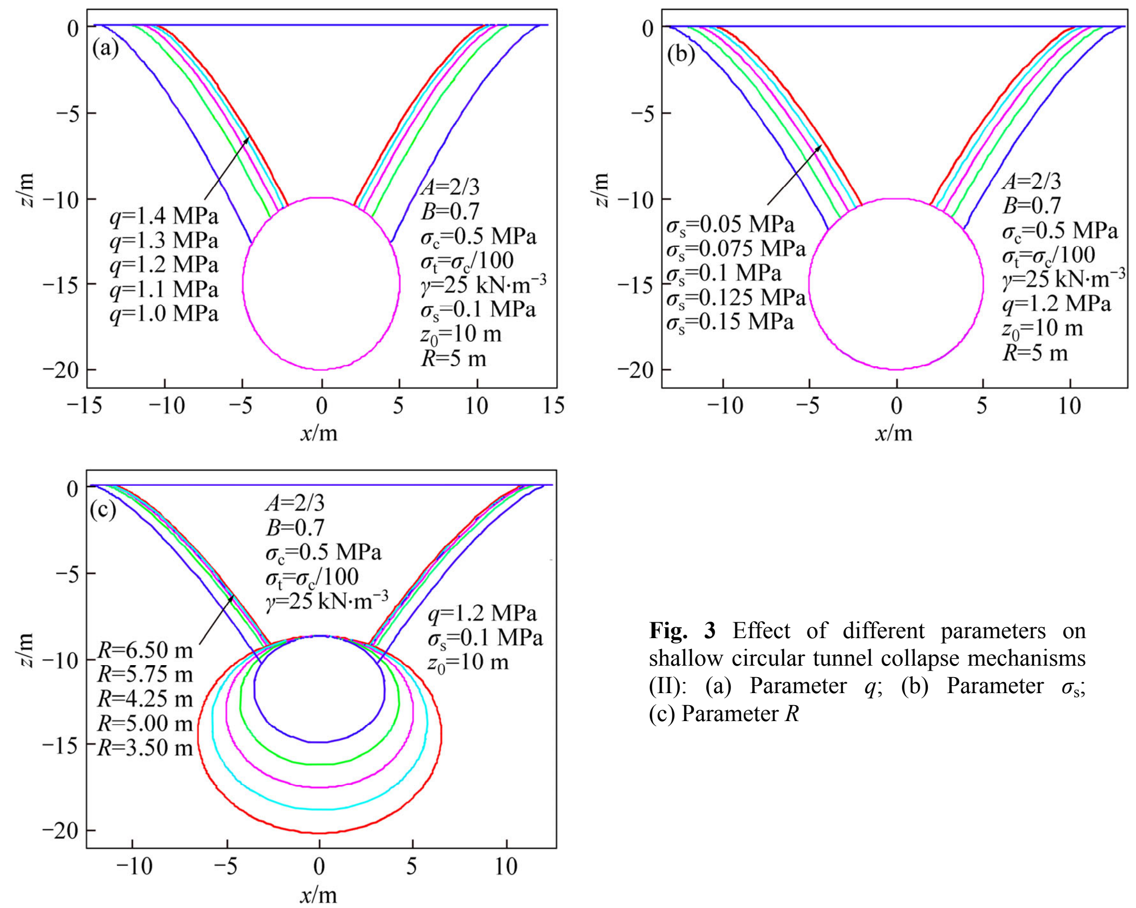

Fig. 3 Effect of different parameters on shallow circular tunnel collapse mechanisms (II): (a) Parameter q; (b) Parameter σ_s; (c) Parameter R

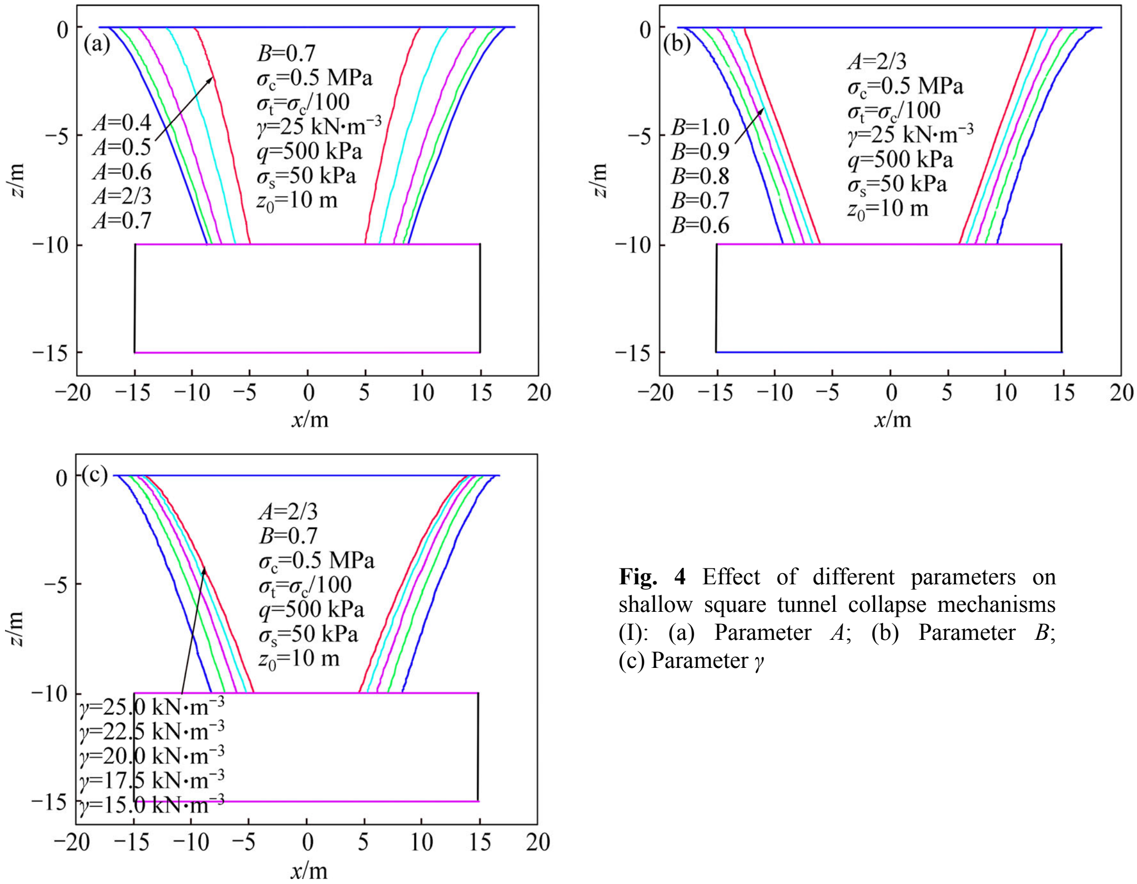

Fig. 4 Effect of different parameters on shallow square tunnel collapse mechanisms (I): (a) Parameter A; (b) Parameter B; (c) Parameter γ

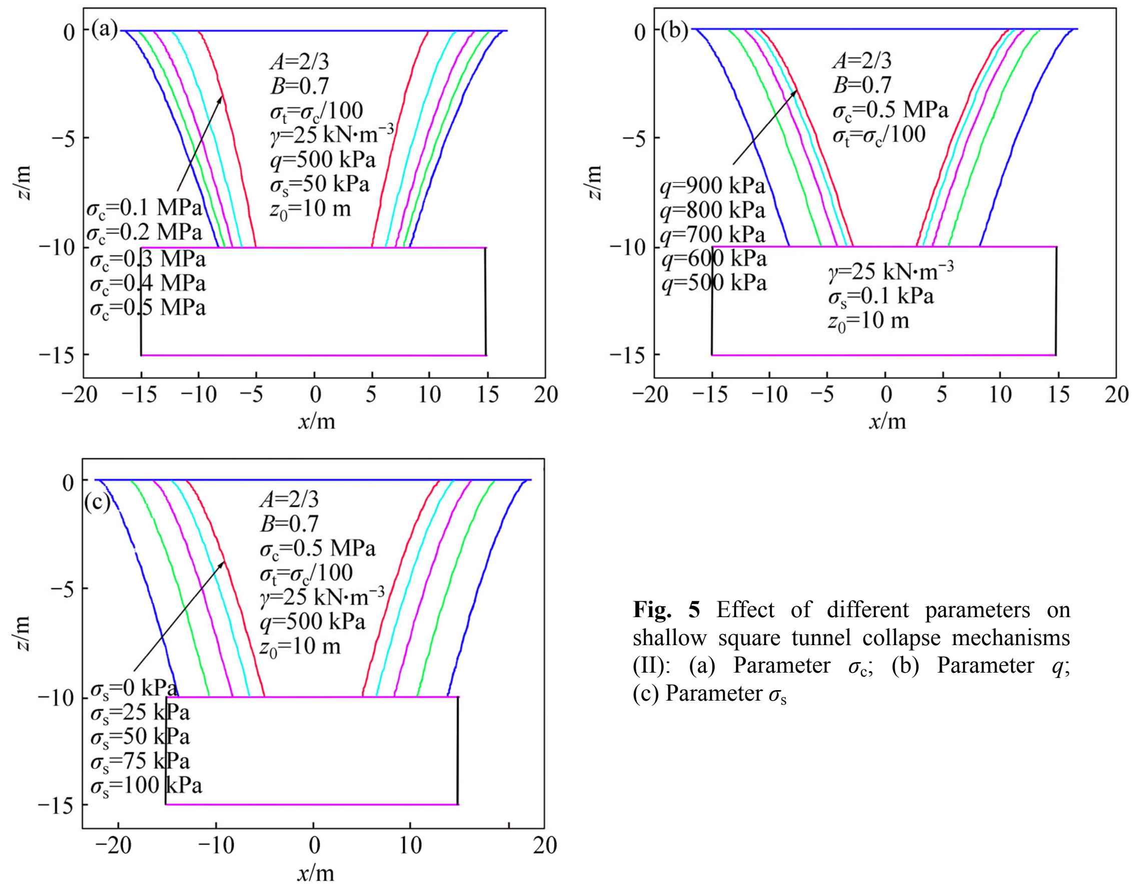

Fig. 5 Effect of different parameters on shallow square tunnel collapse mechanisms (II): (a) Parameter σ_c; (b) Parameter q; (c) Parameter σ_s

7 Conclusions

1) A new curved failure mechanism is proposed to estimate the stability of impending collapsing block under the limit state. Based on the nonlinear failure criterion, the analytical solutions are derived for circular and square tunnels with limit analysis approach and variational principle. The proposed mechanism shows great agreement and validity in comparison with existing solutions. The current engineering cases are also in accordance with the failure mode.

2) From the perspective of engineering, the analysis of influence of various parameters on the roof collapse has important significance to instruct the design and construction of underground cavities and tunnels. The results indicate that the higher values of B, q and γ and the lower values of A, σ_c and σ_s are beneficial to diminishing the scope of collapsing volume and attaining the roof stability.

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