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Robust user equilibrium model based on cumulative prospect theory under distribution-free travel time

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Abstract: The assumption widely used in the user equilibrium model for stochastic network was that the probability distributions of the travel time were known explicitly by travelers. However, this distribution may be unavailable in reality. By relaxing the restrictive assumption, a robust user equilibrium model based on cumulative prospect theory under distribution-free travel time was presented. In the absence of the cumulative distribution function of the travel time, the exact cumulative prospect value (CPV) for each route cannot be obtained. However, the upper and lower bounds on the CPV can be calculated by probability inequalities. Travelers were assumed to choose the routes with the best worst-case CPVs. The proposed model was formulated as a variational inequality problem and solved via a heuristic solution algorithm. A numerical example was also provided to illustrate the application of the proposed model and the efficiency of the solution algorithm.

Key words: user equilibrium; cumulative prospect theory; distribution-free travel time; variational inequality

1 Introduction

As widely known, the uncertainties of network travel time exist in both supply side (link capacity variation) and demand side (travel demand fluctuation). The link capacity degradations can be caused by events, such as bad weather, vehicle breakdown, traffic management and control. Such capacity variations typically lead to non-recurrent congestion. On the other hand, the travel demand between a specified origin-destination (O-D) pair varies between time of the day, days of the week, and seasons of the year. Such variations in demand always cause recurrent congestion. Due to the existence of uncertainties in transportation systems, both link and route travel time are uncertain [1].

Recently, travel time uncertainty has emerged as an important topic due to its significant impacts on travelers' route choice decision. Several attempts have been made by researchers to include travel time variability into user equilibrium (UE) models. With different behavioral assumptions on travelers' risk-taking behaviors, the proposed models included, among others, the travel time budget (TTB) or percentile travel time (PTT) model [2−3], the mean excess travel time (METT) model [4−6], the combined mean travel time (CMTT) model [7], the expected utility theory (EUT)-based model [8−9] and the cumulative prospect theory (CPT)-based model [10−12].

A common assumption that pervaded the above literature was that the distributions of the travel time on alternative routes could be known explicitly by travelers. However, travel time distributions might be unavailable (inaccurate) in reality as travelers might have no (insufficient) data to calibrate them. Therefore, such distributions were rarely known exactly to travelers within their decision making process. In most instances, only the mean and variance of the travel time are obtained by travelers. How do travelers select the optimal routes when they do not know travel time distributions?

To begin with, it was possibly believed by some people that the appropriate method to describe travelers' decision making is to use the simple heuristic rule, e.g., mean-variance or mean-standard deviation tradeoff. The standard mean-variance model was the firstly used by NOLAND et al [13] in transportation studies. However, it is well known that the mean-variance and mean-standard deviation models suffer from severe biases since in particular they generate irrational behaviors if

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the random variable (here the travel time) is not normally distributed [14]. Furthermore, it was argued by de PALMA and PICARD [9] that, for the binary distribution, the mean-variance and mean-standard deviation models could all lead to counter-intuitive results.

Next, the EUT has been applied to model travelers' route choice without the travel time distribution information. To capture risk-averse behavior, travelers' utility was assumed by MIRCHANDANI and SOROUSH [15] and YIN et al [16] to be a quadratic function of travel time. However, the quadratic utility implied increasing absolute risk aversion, which was inconsistent with the known economic and traffic phenomena [9].

Finally, the robust decision criteria have been adopted by some researchers. In the field of finance, some attempts have been made to study decision making (i.e., portfolio selection) without distribution information about variables (i.e., asset returns). A method for deriving robust solutions to portfolio selection problems was provided by POPESCU [17], based on mean-covariance information about the distributions underlying the uncertain vector of returns. Tight bounds for some risk measures were applied by CHEN et al [18] to study robust portfolio selection models with the lack of distribution information. Inspired by the worst-case value-at-risk and conditional value-at-risk measures by CHEN et al [18], the robust PTT/METT without the probability distributions of the travel time was defined by SUN and GAO [19] and the robust percentile/mean-excess stochastic traffic equilibrium model was then proposed. However, the proposed models implied that travelers are perfectly rational in minimizing their own travel times. Although this behavior assumption, is theoretically useful it on the other hand was extreme and unrealistic. Up to now, travelers' route choice behavior in the absence of the travel time distributions has rarely been studied.

The assumption that the probability distributions of the travel time are known explicitly by travelers is also relaxed in this work. Specifically, network travel times are assumed to be distribution-free. Travelers' route choice behavior under distribution-free travel time are modeled based on CPT. Advanced by psychologists and behavioral economists, CPT provides a well-supported descriptive paradigm for decision making based on limited rationality. A variety of phenomena and experimental data in the transportation context, which cannot be explained by EUT previously, has been well explained by CPT. In the CPT framework, without the specific distribution information of the travel times, the accurate CPV of each route cannot be calculated by travelers. Instead, the upper and lower bounds on the CPV can be understood by them. Specifically, the lower bound on the CPV can be interpreted as the worst-case CPV of each route. In the frame work of robust optimization, it is assumed that decision-makers aim to maximize return in the worst-case scenario [20−22]. When this concept is applied to route choice, it implies finding the best route to minimize the worst-case travel time (see Ref. [20]) or maximize the worst-case travel utility (see Section 3). Under the assumption that travelers always prepare for the worst, a robust optimization formulation to maximize the minimum expected travel time over all possible link failure scenarios was proposed by BELL and CASSIR [21]. Based on the same assumption, it was believed by ORDONEZ and STIER-MOSES [22] that travelers would select the "robust shortest route", which minimized the worst-case travel time.

Then, a robust user equilibrium (UE) model based on CPT under distribution-free travel time is presented. In the proposed model, it is assumed that travelers always choose routes with the best worst-case CPV. A robust UE is reached when no user can increase his or her worst-case CPV by unilaterally switching routes. The demand uncertainty in the traffic network which causes travel time variability is introduced in this work. In view of day-to-day travel demand fluctuation, the travel demand between each O-D pair is assumed to be a random variable. In practice it is much easier to determine the first γ moments (where γ is a user-specified positive integer) than the probability distribution of the travel demand between each O-D pair. Hence, in this work, it is not necessary to pay attention to the specific probability distributions on the random O-D demand and travel time.

2 Distribution-free travel time

2.1 Assumptions

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made.

A1: The travel demand between each O-D pair is a random variable with the given first to fourth origin moments: $\mathrm{E}(Q_r)$, $\mathrm{E}[(Q_r)^2]$, $\mathrm{E}[(Q_r)^3]$ and $\mathrm{E}[(Q_r)^4]$, where $\mathrm{E}[\cdot]$ is the expectation operator. The macroscopic characteristics of probability distribution for the random variable can be described by these origin moments, which can be easily obtained by traffic surveys and mathematical statistics.

A2: The actual O-D travel demand on any day is independently distributed across inter-zonal movements, i.e., $\mathrm{E}\left(Q_{r_1}Q_{r_2}\right)=\mathrm{E}\left(Q_{r_1}\right)\mathrm{E}\left(Q_{r_2}\right),\ \ \forall r_1,\ r_2\in\mathbf{R},\ r_1\neq r_2$.

A3: The path flow is the product of path choice proportion and the O-D travel demand, i.e., $F_r^k=p_r^kQ_r$. The path choice proportion p_r^k is a deterministic variable which can be obtained from the model's output, i.e., $p_r^k = f_r^k / q_r$, $\sum_{k \in K_r} p_r^k = 1$. This is similar to the assumption made in other previous studies (i.e. Ref. [23]).

A4: The link travel times are independent from each other. To consider the covariance, the link travel time would open a new research topic that is worthy exploring in future.

A5: The stochastic travel time on each route is still assumed to follow a continuous probability distribution in this work. However, the exact probability distribution is very difficult or not required to be known.

2.2 Derivation of traffic flow

As O-D travel demand Q_r is assumed to be a random variable, the route flow, F_r^k, and the link flow, V_a, are also random variables, which consequently induce the random link and route travel time.

The flow conservation relationships can be expressed by

$$Q_r = \sum_{k \in K_r} F_r^k, \quad \forall r \in R \tag{1}$$

$$F_r^k \geq 0, \ \forall r \in R, k \in K_r \tag{2}$$

$$V_a = \sum_{r \in R} \sum_{k \in K_r} \delta_r^{ka} F_r^k, \ \forall a \in A \tag{3}$$

By taking the expectation, it follows from Eqs. (1)−(3) that

$$q_r = \sum_{k \in K_r} f_r^k, \ \forall r \in R \tag{4}$$

$$f_r^k \geq 0, \ \forall r \in R, k \in K_r \tag{5}$$

$$v_a = \sum_{r \in R} \sum_{k \in K_r} \delta_r^{ka} f_r^k, \ \forall a \in A \tag{6}$$

According to Eq. (3) and Assumption **A3**, the γth moment (where γ is a user-specified positive integer) of the link flow is

$$\mathrm{E}\left[(V_a)^\gamma\right] = \mathrm{E}\left[\left(\sum_{r \in R} \sum_{k \in K_r} \delta_r^{ka} F_r^k\right)^\gamma\right] = \mathrm{E}\left[\left(\sum_{r \in R} \sum_{k \in K_r} \delta_r^{ka} p_r^k Q_r\right)^\gamma\right], \ \forall a \in A \tag{7}$$

Using the Taylor-series expansion, Eq. (7) can be calculated. For γ=2 or 4, it follows that [19]

$$\mathrm{E}\left[(V_a)^2\right] = \sum_{k \in K_r} \delta_r^{ka} \left(p_r^k\right)^2 \mathrm{E}\left[(Q_r)^2\right] + 2 \sum_{k \in K_r} \sum_{\substack{k' \in K_{r'}, r,r' \in R \\ k > k'}} \left[\delta_r^{ka} \delta_{r'}^{k'a} p_r^k p_{r'}^{k'} \mathrm{E}\left(Q_r Q_{r'}\right)\right], \ \forall a \in A \tag{8}$$

$$\begin{aligned}\mathrm{E}\left[(V_a)^4\right] = & \sum_{k \in K_r} \delta_r^{ka} \left(p_r^k\right)^4 \mathrm{E}\left[(Q_r)^4\right] + 4 \sum_{k \in K_r} \sum_{\substack{k' \in K_{r'} \\ k \neq k'}} \left\{\delta_r^{ka} \delta_{r'}^{k'a} \left(p_r^k\right)^3 p_{r'}^{k'} \mathrm{E}\left[(Q_r)^3 Q_{r'}\right]\right\} + \\ & 6 \sum_{k \in K_r} \sum_{\substack{k' \in K_{r'} \\ k > k'}} \left\{\delta_r^{ka} \delta_{r'}^{k'a} \left(p_r^k\right)^2 \left(p_{r'}^{k'}\right)^2 E\left[(Q_r)^2 \cdot (Q_{r'})^2\right]\right\} + 12 \sum_{k \in K_r} \sum_{\substack{k' \in K_{r'} \\ k' \neq k}} \sum_{\substack{k'' \in K_{r''} \\ k'' \neq k \\ k'' > k'}} \left\{\delta_r^{ka} \delta_{r'}^{k'a} \delta_{r''}^{k''a} \cdot \right. \\ & \left. \left(p_r^k\right)^2 p_{r'}^{k'} p_{r''}^{k''} E\left[(Q_r)^2 Q_{r'} Q_{r''}\right]\right\} + 24 \sum_{k \in K_r} \sum_{\substack{k' \in K_{r'} \\ k' > k}} \cdot \sum_{\substack{k'' \in K_{r''} \\ k'' > k'}} \sum_{\substack{k''' \in K_{r'''} \\ k''' > k''}} \left[\delta_r^{ka} \delta_{r'}^{k'a} \delta_{r''}^{k''a} \delta_{r'''}^{k'''a} p_r^k p_{r'}^{k'} p_{r''}^{k''} p_{r'''}^{k'''} \cdot \right. \\ & \left. \mathrm{E}\left(Q_r Q_{r'} Q_{r''} Q_{r'''}\right)\right], \ \forall a \in A\end{aligned} \tag{9}$$

2.3 Derivation of link and route travel time

The commonly used BPR function for the link travel time is adopted as

$$T_a = t_a^0 \left[1 + m\left(\frac{V_a}{c_a}\right)^n\right], \ \forall a \in A \tag{10}$$

where m and n are parameters in the BPR function.

The first and second moments of the link travel time can be expressed as

$$\mathrm{E}\left(T_a\right) = t_a^0 \left[1 + m \frac{E\left(V_a\right)^n}{\left(c_a\right)^n}\right], \ \forall a \in A \tag{11}$$

$$\begin{aligned}\mathrm{E}\left[(T_a)^2\right] &= \mathrm{E}\left\{\left(t_a^0\right)^2 \left[1 + m\left(\frac{V_a}{c_a}\right)^n\right]^2\right\} \\ &= \left(t_a^0\right)^2 \left\{1 + 2m \frac{\mathrm{E}\left[(V_a)^n\right]}{\left(c_a\right)^n} + m^2 \frac{\mathrm{E}\left[(V_a)^{2n}\right]}{\left(c_a\right)^{2n}}\right\}, \\ & \forall a \in A\end{aligned} \tag{12}$$

Then, the variance of the link travel time can be represented as

$$\mathrm{Var}\left(T_a\right) = \mathrm{E}\left[(T_a)^2\right] - \left[\mathrm{E}\left(T_a\right)\right]^2, \ \forall a \in A \tag{13}$$

The route travel time variable can be expressed by simply summing the corresponding link travel time variables. Thus, the mean and variance of the route travel time can be written as

$$\mathrm{E}\left(T_r^k\right) = \sum_{a \in A} \mathrm{E}\left(T_a\right) \delta_r^{ka}, \ \forall r \in R, k \in K_r \tag{14}$$

$$\mathrm{Var}\left(T_r^k\right)=\sum_{a\in A} Var\left(T_a\right)\delta_r^{ka},\ \forall r\in R, k\in K_r \tag{15}$$

The second moment of the route travel time is

$$\mathrm{E}\left[\left(T_r^k\right)^2\right]=\left[\mathrm{E}\left(T_r^k\right)\right]^2+Var\left(T_r^k\right),\ \forall r\in R, k\in K_r \tag{16}$$

The central limit theorem was applied in some papers to deducing that the path travel time followed normal distribution regardless of the link travel time distributions. However, it should be noted that the central limit theorem is not applicable herein due to the limited number of links constituting a route. Further analysis can be carried out to estimate the route travel time distribution using the fitting distribution method [24−25]. However, it is very difficult and time-consuming to obtain the higher moments (e.g., skewness and kurtosis) of the route travel time. Therefore, it is reasonable to declare that the route travel time distribution cannot be known by travelers.

3 Cumulative prospect theory-based robust user equilibrium model

In short, CPT maintains the framework of the EUT but incorporates the following features that have been observed in numerous behavioral experiments [26]:

1) People distinguish gains from losses before making choices. The payoffs are framed as gains or losses compared with some reference points (RPs).

2) The loss looms larger than the gain, i.e., people generally care more about potential losses than potential gains. At the same time, they are risk averse over gains and risk seeking over losses.

3) People tend to overweight extreme, but unlikely events. At the same time, they underweight “average” events.

Therefore, CPT can be viewed as an extension to the EUT with the following three modifications:

1) Replacing the final wealth with payoffs relative to the RP.

2) Replacing the utility function with a value function to capture individual’s risk attitude.

3) Replacing cumulative probabilities with weighted cumulative probabilities.

In this section, the basic value function used in the CPT model and the worst-case CPV of each route are provided. Then, a robust traffic equilibrium model based on CPT is presented. It should be noted that the probability weighting function is not applicable herein due to the nonexistence of the travel time distribution. This is similar to the considerations made in other previous economic studies [18] and transport studies [27−28].

3.1 Worst-case cumulative prospect value

In the CPT, the payoff level is considered to be a gain or loss from the RP. Compared with a RP, the outcome of a trip may be considered by travelers as a gain, if the travel time is less than the RP; as a loss if otherwise. The S-shaped value function $g_r(u)$ is

$$g_r(u)=\begin{cases}\left(\lambda_r-u\right)^{\alpha}, & u\le\lambda_r,\ \forall r\in R\\ -\eta\left(u-\lambda_r\right)^{\beta}, & u>\lambda_r,\ \forall r\in R\end{cases} \tag{17}$$

where u represents the route travel time, parameters α and β measure the degrees of diminishing sensitivity of value function. Typically, $0<\alpha$, $\beta<1$ and thus the value function exhibits risk aversion over gains and risk seeking over losses. The parameter $\eta\geq1$ is called “loss-aversion” coefficient, indicating that the individuals are more sensitive to losses than gains.

Consequently, the relative payoff for choosing a route k can be defined as $\lambda_r-T_r^k$ and the CPV of route k can be calculated through Eq. (18).

$$\begin{aligned}U_r^k&=\mathrm{E}\left[g_r\left(T_r^k\right)\right]\\&=\int_{\underline{t}_r^k}^{\lambda_r} g_r(u)\,\mathrm{d}\psi_r^k(u)+\int_{\lambda_r}^{\bar{t}_r^k} g_r(u)\,\mathrm{d}\psi_r^k(u),\\&\quad\forall r\in R, k\in K_r\end{aligned} \tag{18}$$

where $\underline{t}_r^k$ and $\bar{t}_r^k$ are the lower and upper bounds on the travel time of route k between O-D pair r, respectively. In this work, the former is assumed to be the free-flow route travel time and the latter is the route travel time when all mean O-D demand is assigned to it.

To determine the CPV of each route, the CDF of the route travel time is required. However, in real applications, the distribution information is generally unknown. Thus, the exact CPV for each route cannot be obtained by travelers under distribution-free travel time. Instead, the upper and lower bounds on the CPV for each route are known to travelers.

Definition 1: The worst-case CPV of route k between O-D pair r is defined as the lower bound on the CPV.

According to the well known inequality of Tchebycheff [18], the upper and lower bounds on the CDF of the route travel time are given as

$$0\le\psi_r^k(u)\le\frac{\mathrm{Var}(T_r^k)}{\mathrm{Var}(T_r^k)+\left[\mathrm{E}(T_r^k)-u\right]^2},\ 0\le u\le\mathrm{E}(T_r^k) \tag{19}$$

and

$$1-\frac{\mathrm{Var}(T_r^k)}{\mathrm{Var}(T_r^k)+\left[\mathrm{E}(T_r^k)-u\right]^2}\le\psi_r^k(u)\le1,\ u\ge\mathrm{E}(T_r^k) \tag{20}$$

The worst-case CPV of route k between OD pair r

can be defined as follows (see Appendix A).

If $\underline{t}_r^k < \lambda_r \le \mathrm{E}(T_r^k)$,

$$\underline{U}_r^k = -\left(\lambda_r - \underline{t}_r^k\right)^{\alpha} \underline{\Lambda} - \eta\left(\bar{t}_r^k - \lambda_r\right)^{\beta} + \eta\beta\int_{\mathrm{E}\left(T_r^k\right)}^{\bar{t}_r^k} (1-\Lambda)\cdot (u-\lambda_r)^{\beta-1}\,\mathrm{d}u,\ \forall r\in R, k\in K_r \tag{21}$$

If $\bar{t}_r^k > \lambda_r > \mathrm{E}\left(T_r^k\right)$,

$$\underline{U}_r^k = -\left(\lambda_r - \underline{t}_r^k\right)^{\alpha} \underline{\Lambda} - \eta\left(\bar{t}_r^k - \lambda_r\right)^{\beta} + \eta\beta\int_{\lambda_r}^{\bar{t}_r^k} (u-\lambda_r)^{\beta-1}(1-\Lambda)\mathrm{d}u + \int_{E\left(T_r^k\right)}^{\lambda_r} \alpha(\lambda_r-u)^{\alpha-1}(1-\Lambda)\,\mathrm{d}u,\ \forall r\in R, k\in K_r \tag{22}$$

3.2 Robust user equilibrium model under cumulative prospect theory

Definition 2: A user traveling between O-D pair r always selects the path with the best worst-case CPV. The robust user equilibrium state is reached by allocating the O-D demands to the network such that no traveler can improve his/her worst-case CPV by unilaterally changing routes. The worst-case CPVs of all utilized routes are equal, and are the same or greater than the ones of any unutilized routes, namely, the following conditions hold

$$\begin{cases} f_r^k > 0 \quad \underline{U}_r^k = \Pi_r,\ \forall k\in K_r, r\in R \\ f_r^k = 0 \quad \underline{U}_r^k \le \Pi_r,\ \forall k\in K_r, r\in R \end{cases} \tag{23}$$

where $\Pi_r = \max\limits_{k\in\boldsymbol{K}_r}\left\{\underline{U}_r^k\right\} = \max\limits_{k\in\boldsymbol{K}_r}\left\{\min\limits_{k\in\boldsymbol{K}_r}\left\{U_r^k\right\}\right\}$.

The problem of finding robust UE route flows Eq. (23) may be formulated as a variational inequality problem VI($\boldsymbol{f}$, $\boldsymbol{\Omega}$) as follows: find a vector $\boldsymbol{f}^* \in \boldsymbol{\Omega}$, such that

$$\underline{\boldsymbol{U}}(\boldsymbol{f}^*)^{\mathrm{T}}(\boldsymbol{f}-\boldsymbol{f}^*) \le 0,\ \forall \boldsymbol{f}\in\boldsymbol{\Omega} \tag{24}$$

where $\boldsymbol{f} = \left(f_r^k\right)_{r\in R,k\in K_r}$; $\underline{U}(\boldsymbol{f}) = \left(\underline{U}_r^k\left(f_r^k\right)\right)_{r\in R,k\in K_r}$; $\boldsymbol{\Omega}$ represents the feasible route set defined by Eqs. (4)−(6).

The following propositions give the equivalence of the VI formulation and the proposed UE model as well as the existence of the equilibrium solutions.

Proposition 1: The solution of the VI problem Eq. (24) is equivalent to the equilibrium solution of the robust UE model based on CPT.

Proof: Note that $\boldsymbol{f}^*$ is a solution of the VI problem if and only if it is a solution of the following linear program

$$\max_{\boldsymbol{f}\in\boldsymbol{\Omega}} \underline{U}(\boldsymbol{f}^*)^{\mathrm{T}}\boldsymbol{f} \tag{25}$$

By considering the primal-dual optimality conditions of Eq. (25), it follows that

$$f_r^{k*}\left(\underline{U}_r^k\left(f_r^{k*}\right) - \Pi_r\right) = 0,\ \forall k\in K_r, r\in R \tag{26}$$

$$\underline{U}_r^k\left(f_r^{k*}\right) - \Pi_r \le 0,\ \forall k\in K_r, r\in R \tag{27}$$

It is easy to see that the robust UE conditions Eq. (23) are satisfied. This completes the proof.

Proposition 2: Assume the worst-case CPV function $\underline{\boldsymbol{U}}(\boldsymbol{f})$ is continuous, then the robust UE problem has at least one solution.

Proof: According to **Proposition 1**, the equivalent VI formulation is only needed to consider. Note that the feasible set $\boldsymbol{\Omega}$ is nonempty and convex. Furthermore, consider the link travel time function, value function and worst-case CPV function, it is reasonable to give the continuous assumption of the mapping $\underline{\boldsymbol{U}}(\boldsymbol{f})$. Thus, the VI problem (Eq. (24)) has at least one solution. This completes the proof. However, note that the solution to the VI problem is not unique in general. The reason is that the mapping in the VI formulation may be not strictly monotone due to the complicated function for the worst-case CPV.

4 Solution algorithm based on method of successive average

The method of successive average (MSA) can be adopted to solve the equilibrium assignment defined in Eq. (24) [11−12]. From the VI condition in Eq. (24), an equivalent optimization problem can be defined as

$$\min_{\boldsymbol{f}\in\boldsymbol{\Omega}}\{G(\boldsymbol{f})\} \tag{28}$$

where $G(\boldsymbol{f}) = \max\limits_{g\in\boldsymbol{\Omega}}\left(\underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}g - \underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}\boldsymbol{f}\right)$ is the gap function of the VI. For a given $\boldsymbol{f}$, the descending direction to minimize the gap function is the solution of the inner problem $\max\limits_{g\in\boldsymbol{\Omega}}\left(\underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}g - \underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}\boldsymbol{f}\right)$. The inner problem reduces to $\max\limits_{g\in\boldsymbol{\Omega}}\underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}g$ since $\underline{\boldsymbol{U}}(\boldsymbol{f})^{\mathrm{T}}\boldsymbol{f}$ is a constant term. The solution of this inner problem is to assign all mean O-D demands to a route connecting that O-D pair with the highest worst-case CPV evaluated at $\boldsymbol{f}$. The MSA algorithm is briefly described as follows.

Step 1: Initialization. Set l=1 and specify an initial route flow pattern $\boldsymbol{f}^{(l)} = \left(f_r^{k(l)}\right)_{r\in R,k\in K_r}$.

Step 2: CPV calculation. Calculate the worst-case CPV for each route, $\underline{\boldsymbol{U}}(\boldsymbol{f}^{(l)}) = \left(\underline{U}_r^k\left(f_r^{k(l)}\right)\right)_{r\in R,k\in K_r}$.

Step 3: Search direction finding. For each O-D pair, find the route with the maximum worst-case CPV. Define an route flow pattern $\boldsymbol{g}^{(l)}=\left(g_r^{k(l)}\right)_{r\in R,k\in K_r}$ with $g_r^k=\mathrm{E}\left(Q_r\right)$ if $\underline{U}_r^{k(l)}=\Pi_r^{(l)}$ and 0 otherwise.

Step 4: Check convergence. Evaluate $\boldsymbol{G}(\boldsymbol{f}^{(l)})=\underline{\boldsymbol{U}}(\boldsymbol{f}^{(l)})^{\mathrm{T}}\cdot(\boldsymbol{g}^{(l)}-\boldsymbol{f}^{(l)})$, terminate the algorithm if $M(\boldsymbol{f}^{(l)})=G(\boldsymbol{f}^{(l)})/\|\boldsymbol{f}^{(l)}\|\le\varepsilon$ or $l>l_{\max}$ where ε is the convergence criterion and $l_{\max}$ is the pre-set maximum number of iteration.

Step 5: Route flow updating. Update the route flow pattern as $\boldsymbol{f}^{(l+1)}=\boldsymbol{f}^{(l)}+s^{(l)}(\boldsymbol{g}^{(l)}-\boldsymbol{f}^{(l)})$, where step size $s^{(l)}=1/l$. Set $l=l+1$, go to **step 2**.

5 Numerical example

To illustrate the proposed robust UE model under CPT and the solution algorithm, the well-known Nguyen-Dupuis network is adopted in the numerical experiment. The network is shown in Fig. 1, which contains 13 nodes, 19 directed links, 4 O-D pairs, and 25 routes. The first to fourth moments of the O-D travel demand are given in Table 1. The fixed link free-flow travel time and capacity can be found in Ref. [12]. The coefficients of the BPR function in Eq. (10) are m=0.15 and n=2. The parameters for value function in Eq. (17) are set as α=β=0.88, and η=2.25 (see Ref. [11]). The base RP in the CPT-based model is set at 60. Note that it is assumed that the RP is the same for all O-D pairs. In reality different O-D pairs may have different travel time and hence should have different RPs. The proposed model formulation and the solution algorithm can accommodate this in general cases. The convergence

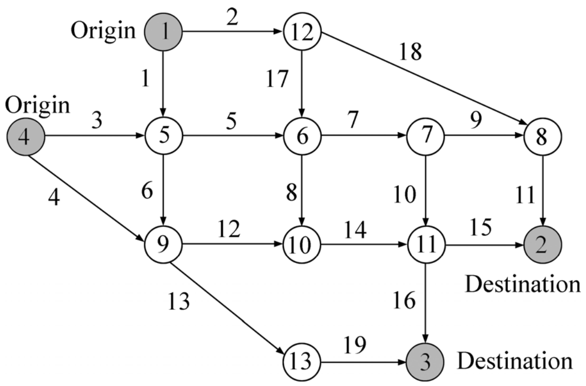

Fig. 1 Nguyen-Dupuis network

Table 1 First to fourth moments of O-D travel demand

O-D pair	First moment	Second moment	Third moment	Fourth moment
(1, 2)	8.00×10^2	8.32×10^2	7.17×10^5	6.37×10^8
(4, 2)	5.00×10^2	5.45×10^2	3.18×10^5	1.96×10^8
(1, 3)	6.00×10^2	6.24×10^2	4.03×10^5	2.69×10^8
(4, 3)	6.00×10^2	6.54×10^2	4.57×10^5	3.38×10^8

criterion and the maximum iteration number are set as ε=0.25 and $l_{\max}$=500.

The assignment results for the robust UE model under CPT are given in Table 2. From these results, the convergence of the algorithm is considered satisfactory since the worst-case CPVs of all used routes between each O-D pair are almost equivalent. The worst-case CPVs of unused routes are lower than the ones of used routes. The mean and standard deviation (SD) of each route travel time are also displayed in Table 2. The trade-off between the mean and SD of the travel time can also be observed from the results.

The convergence of the solution algorithm is shown in Fig. 2. To further demonstrate the convergence of the proposed solution procedure, without loss of generality, route 1 connecting O-D pair (1, 2) is examined. The evolution of the route flow and the route worst-case CPV during the iteration process are depicted in Figs. 3 and 4. From these figures, it can be seen that the algorithm quickly converges to the required solution precision. The robust UE solution is achieved as M is almost equal to zero after 368 iterations. At the same time, the worst-case CPVs of used routes for a given O-D pair are getting closer to each other during the iteration process. Furthermore, it can be observed that the fluctuation of M is more frequent and larger at the early iterations, and getting smaller along with the iteration process. It is demonstrated that the proposed algorithm has the ability to reach a stable equilibrium solution.

The upper and lower bounds on the CDF of travel time on route 1 between O-D pair (1,2) are depicted in Fig. 5. The ones for other routes can also be obtained. No matter the exact probability distribution of the travel time, the curve of the CDF lies in the area defined by the upper and lower bounds in the figure.

At equilibrium, the sensitivity of the exact CPVs (or worst-case CPVs) of all O-D pairs to the value of RP with (or without) the probability distribution of the travel time is given in Fig. 6. For comparison, it is assumed that the route travel time follows normal distribution, giving the mean and variance of the route travel time. Then, the exact CPV can be calculated by travelers. It can be seen from Fig. 6 that as the value of RP grows from 60 to 100, both the exact and the worst-case equilibrium CPVs all increase. The reason is that travelers are very likely to experience “gain” trips with a long reference time. The larger the value of RP, the higher the exact CPV (or worst-case CPV) on the used route. Moreover, with the increase in the value of RP, the difference between the exact and worst-case CPV will narrow sharply. Obviously, with a larger value of RP, travelers become more conservative.

Table 2 CPT-based robust user equilibrium route flow patterns

OD pair	Route	Link sequence	Worst-case CPV	Mean route flow	Travel time	
					Mean	Standard deviation
(1, 2)	1	2−18−11	−37.85	403.27	76.62	5.34
	2	1−5−7−9−11	−37.14	45.78	79.04	3.96
	3	1−5−7−10−15	−37.87	32.69	79.41	3.59
	4	1−5−8−14−15	−40.88	0	79.39	4.61
	5	1−6−12−14−15	−37.75	318.26	79.84	3.26
	6	2−17−7−9−11	−39.29	0	77.84	4.98
	7	2−17−7−10−15	−39.35	0	78.81	4.48
	8	2−17−8−14−15	−39.15	0	78.19	4.90
(4, 2)	9	4−12−14−15	−49.23	85.83	81.65	8.10
	10	3−5−7−9−11	−49.96	332.43	84.39	6.25
	11	3−5−7−10−15	−49.97	17.71	84.75	6.03
	12	3−5−8−14−15	−49.97	19.07	84.74	6.04
	13	3−6−12−14−15	−49.14	44.96	85.19	5.84
(1, 3)	14	1−6−13−19	−50.63	0	88.82	7.57
	15	1−5−7−10−16	−47.73	106.27	86.16	4.04
	16	1−5−8−14−16	−49.74	0	86.14	5.06
	17	1−6−12−14−16	−47.70	269.75	86.59	3.75
	18	2−17−7−10−16	−47.82	160.22	84.96	4.85
	19	2−17−8−14−16	−47.83	63.76	84.95	4.96
(4, 3)	20	4−13−19	−59.08	362.94	84.62	10.60
	21	4−12−14−16	−59.22	19.62	88.40	8.31
	22	3−6−13−19	−61.27	0	88.16	9.90
	23	3−5−7−10−16	−59.16	142.23	91.51	6.31
	24	3−5−8−14−16	−59.16	35.97	91.49	7.32
	25	3−6−12−14−16	−59.34	39.24	91.94	6.12

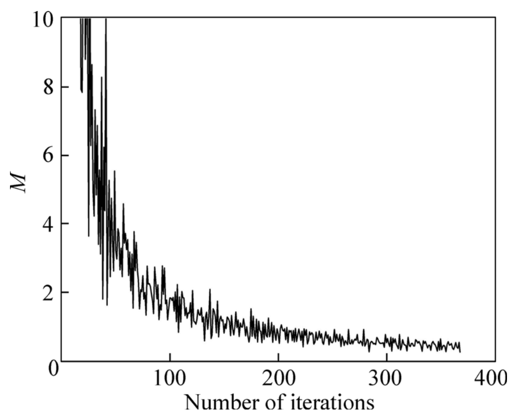

Fig. 2 Convergence of proposed solution algorithm

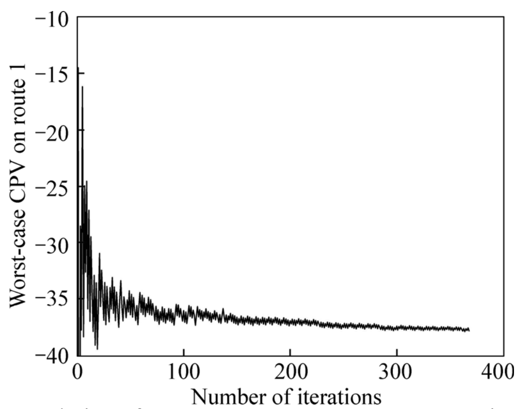

Fig. 4 Evolution of worst-case CPV on route 1 connecting O-D pair (1, 2)

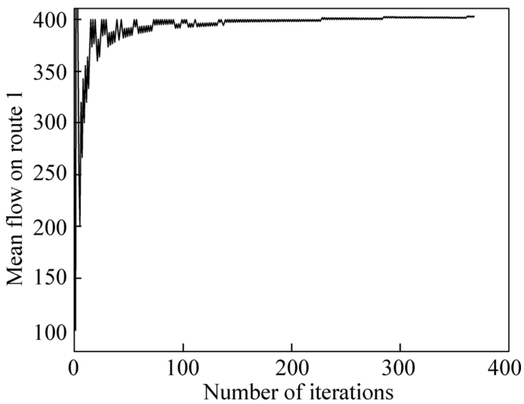

Fig. 3 Evolution of mean flow on route 1 connecting O-D pair (1, 2)

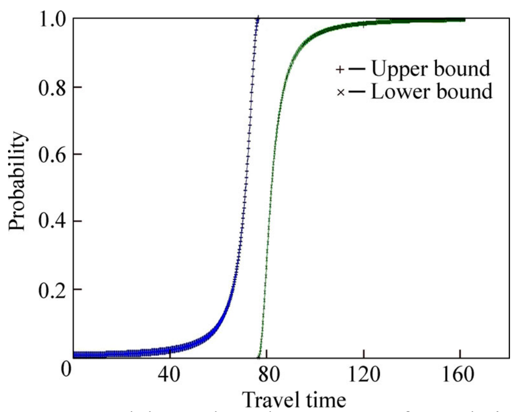

Fig. 5 Upper and lower bounds on CDF of travel time on route 1

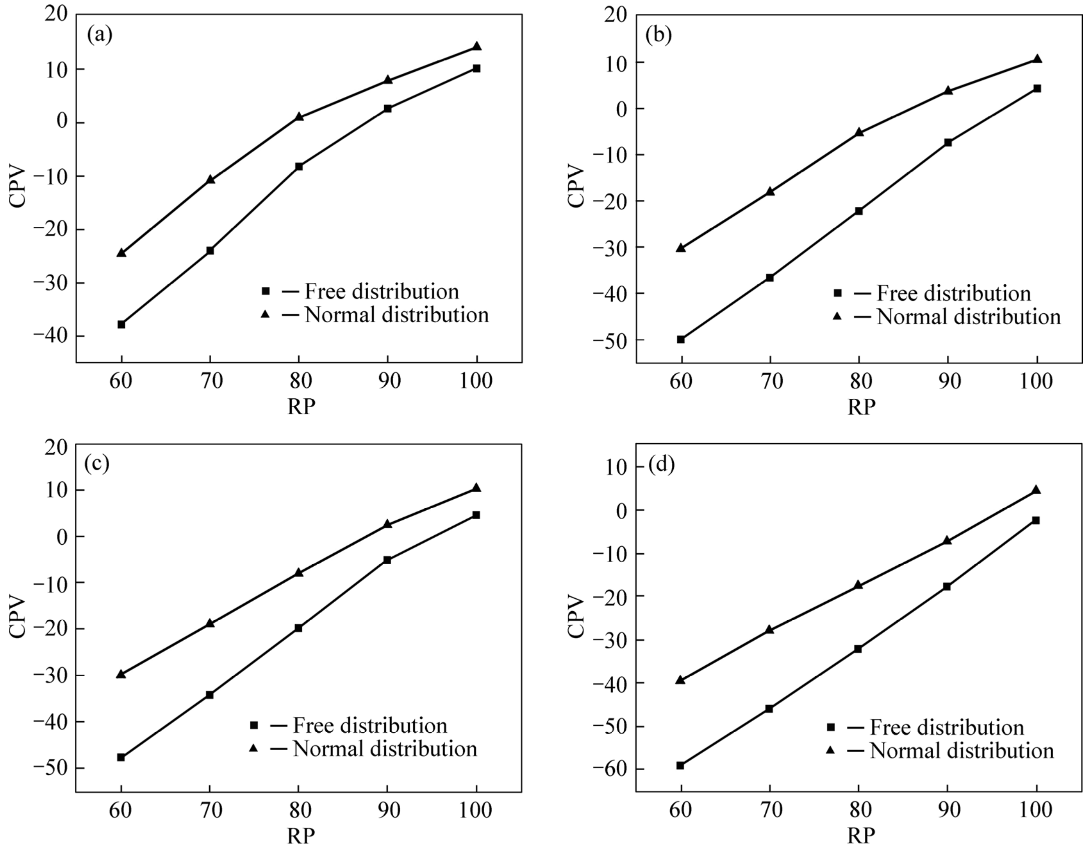

Fig. 6 Sensitivity of exact and worst-case CPV to value of RP: (a) O-D pair (1, 2); (b) O-D pair (4, 2); (c) O-D pair (1, 3); (d) O-D pair (4, 3)

6 Conclusions

1) The rather restrictive assumption that travel time distributions are known exactly by travelers is relaxed. Only the mean and variance of the travel time are obtained by travelers. The upper and lower bounds on the CDF of the route travel time are given.

2) The exact CPV for each route is not obtained by travelers with the lack of the travel time distributions. However, the upper and lower bounds on the CPV can be calculated, respectively. The worst-case CPV is adopted as a route choice criterion. A robust UE model based on CPT under distribution-free travel time is presented. The proposed model is formulated as a VI problem. Qualitative properties, such as equivalence and existence of the solution, are also rigorously proved. A route-based traffic assignment algorithm is adopted to solve the proposed model.

3) A numerical example is also provided to illustrate the essential ideas of the proposed model and the applicability of the solution algorithm. It is found that the proposed model is capable of describing travelers' route choice behaviors under distribution-free travel time, which is more efficient than the mean-variance, mean-standard deviation, and EUT-based models.

4) For future research, the endogenous RP should be considered in the CPT-based UE model under distribution-free travel time. In addition, it is interesting but challenging to present the application of the proposed model in congestion pricing and traffic network design.

Appendix A: Upper and lower bounds on cumulative prospect value

In the first case, it is assumed that $\underline{t}_r^k < \lambda_r \le \mathrm{E}\left(T_r^k\right)$.

Using the method of integration by parts, the first term in Eq. (18) can be obtained as

$$\int_{\underline{t}_r^k}^{\lambda_r} g_r\left(u\right) d\psi_r^k\left(u\right) = \left[g_r\left(u\right)\psi_r^k\left(u\right)\right]_{\underline{t}_r^k}^{\lambda_r} - \int_{\underline{t}_r^k}^{\lambda_r} \psi_r^k\left(u\right) \mathrm{d}g_r\left(u\right)$$
$$= -\left(\lambda_r - \underline{t}_r^k\right)^{\alpha} \psi_r^k\left(\underline{t}_r^k\right) + \int_{\underline{t}_r^k}^{\lambda_r} \alpha\left(\lambda_r - u\right)^{\alpha-1} \psi_r^k\left(u\right) \mathrm{d}u \qquad (29)$$

For Eq. (29), the inequality $\underline{t}_r^k \le u \le \lambda_r \le \mathrm{E}\left(T_r^k\right)$ is true. According to Eq. (19), one obtains $0 \le \psi_r^k\left(\underline{t}_r^k\right) \le \underline{\Lambda}$ and $0 \le \psi_r^k\left(u\right) \le \Lambda$, in which $\underline{\Lambda} = \dfrac{\mathrm{Var}(T_r^k)}{\mathrm{Var}(T_r^k) + \left[E(T_r^k) - \underline{t}_r^k\right]^2}$, $\Lambda = \dfrac{\mathrm{Var}(T_r^k)}{\mathrm{Var}(T_r^k) + \left[E(T_r^k) - u\right]^2}$.

With $\left(\lambda_r-\underline{t}_r^k\right)^\alpha \ge 0$ and $\alpha\left(\lambda_r-u\right)^{\alpha-1}\ge 0$, it follows that

$$-\left(\lambda_r-\underline{t}_r^k\right)^\alpha \underline{\Lambda} \le -\left(\lambda_r-\underline{t}_r^k\right)^\alpha \left(\psi_r^k\left(\underline{t}_r^k\right)\right)\le 0 \tag{30}$$

$$0\le \int_{\underline{t}_r^k}^{\lambda_r}\alpha\left(\lambda_r-u\right)^{\alpha-1}\psi_r^k\left(u\right)\mathrm{d}u \le \int_{\underline{t}_r^k}^{\lambda_r}\alpha\left(\lambda_r-u\right)^{\alpha-1}\Lambda\mathrm{d}u \tag{31}$$

Then,

$$-\left(\lambda_r-\underline{t}_r^k\right)^\alpha \underline{\Lambda} \le \int_{\underline{t}_r^k}^{\lambda_r} g_r\left(u\right)\mathrm{d}\psi_r^k\left(u\right)\le \int_{\underline{t}_r^k}^{\lambda_r}\alpha\left(\lambda_r-u\right)^{\alpha-1}\Lambda\mathrm{d}u \tag{32}$$

The second term in Eq. (18) can be formulated as follows

$$\int_{\lambda_r}^{\bar{t}_r^k} g_r\left(u\right)\mathrm{d}\psi_r^k\left(u\right)=-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta \psi_r^k\left(\bar{t}_r^k\right)+ \eta\beta\int_{\lambda_r}^{\mathrm{E}\left(T_r^k\right)}\left(u-\lambda_r\right)^{\beta-1}\psi_r^k\left(u\right)\mathrm{d}u+ \eta\beta\int_{\mathrm{E}\left(T_r^k\right)}^{\bar{t}_r^k}\left(u-\lambda_r\right)^{\beta-1}\psi_r^k\left(u\right)\mathrm{d}u \tag{33}$$

For the first term in Eq. (33), the inequality $\bar{t}_r^k > \mathrm{E}\left(T_r^k\right)\ge\lambda_r$ is true. According to Eq. (20), one obtains $1-\bar{\Lambda}\le\psi_r^k\left(\bar{t}_r^k\right)\le 1$, where $\bar{\Lambda}=[\mathrm{Var}(T_r^k)]/\{\mathrm{Var}(T_r^k)+[\mathrm{E}(T_r^k)-\bar{t}_r^k]^2\}$. In view of $-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta<0$, the following inequalities hold

$$-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta \le -\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta\psi_r^k\left(\bar{t}_r^k\right)\le -\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta\left(1-\bar{\Lambda}\right) \tag{34}$$

For the second term in Eq. (33), if $\lambda_r<u<\mathrm{E}\left(T_r^k\right)$, then $0\le\psi_r^k\left(u\right)\le\Lambda$. Given $\eta\beta\left(u-\lambda_r\right)^{\beta-1}\ge 0$, it follows that

$$0\le\eta\beta\int_{\lambda_r}^{\mathrm{E}\left(T_r^k\right)}\left(u-\lambda_r\right)^{\beta-1}\psi_r^k\left(u\right)\mathrm{d}u\le \eta\beta\int_{\lambda_r}^{\mathrm{E}\left(T_r^k\right)}\left(u-\lambda_r\right)^{\beta-1}\Lambda\mathrm{d}u \tag{35}$$

For the last term in Eq. (33), if $\lambda_r\le\mathrm{E}\left(T_r^k\right)<u<\bar{t}_r^k$, then $1-\Lambda\le\psi_r^k\left(u\right)\le 1$. Since $\eta\beta\left(u-\lambda_r\right)^{\beta-1}\ge 0$, one obtains the expression as

$$\eta\beta\int_{\mathrm{E}\left(T_r^k\right)}^{\bar{t}_r^k}\left(1-\Lambda\right)\left(u-\lambda_r\right)^{\beta-1}\mathrm{d}u\le \eta\beta\int_{\mathrm{E}\left(T_r^k\right)}^{\bar{t}_r^k}\left(u-\lambda_r\right)^{\beta-1}\psi_r^k\left(u\right)\mathrm{d}u\le \eta\beta\left[\left(\bar{t}_r^k-\lambda_r\right)^\beta-\left(\mathrm{E}\left(T_r^k\right)-\lambda_r\right)^\beta\right] \tag{36}$$

It follows from Eqs. (34)−(36) that

$$\int_{\lambda_r}^{\bar{t}_r^k} g_r\left(u\right)\mathrm{d}\psi_r^k\left(u\right)\ge -\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta+ \eta\beta\int_{\mathrm{E}\left(T_r^k\right)}^{\bar{t}_r^k}\left(1-\Lambda\right)\left(u-\lambda_r\right)^{\beta-1}\mathrm{d}u \tag{37}$$

and

$$\int_{\lambda_r}^{\bar{t}_r^k} g_r\left(u\right)\mathrm{d}\psi_r^k\left(u\right)\le -\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta\left(1-\bar{\Lambda}\right)+ \eta\beta\int_{\lambda_r}^{\mathrm{E}\left(T_r^k\right)}\left(u-\lambda_r\right)^{\beta-1}\Lambda\mathrm{d}u+\eta\beta\left[\left(\bar{t}_r^k-\lambda_r\right)^\beta- \left(\mathrm{E}\left(T_r^k\right)-\lambda_r\right)^\beta\right] \tag{38}$$

Based on Eqs. (32) (37) and (38), the upper and lower bounds on the CPV can be obtained as follows:

$$\bar{U}_r^k=\int_{\underline{t}_r^k}^{\lambda_r}\alpha\left(\lambda_r-u\right)^{\alpha-1}\Lambda\mathrm{d}u-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta\left(1-\bar{\Lambda}\right)+ \eta\beta\int_{\lambda_r}^{\mathrm{E}\left(T_r^k\right)}\left(u-\lambda_r\right)^{\beta-1}\Lambda\mathrm{d}u+\eta\beta\left[\left(\bar{t}_r^k-\lambda_r\right)^\beta- \left(\mathrm{E}\left(T_r^k\right)-\lambda_r\right)^\beta\right] \tag{39}$$

$$\underline{U}_r^k=-\left(\lambda_r-\underline{t}_r^k\right)^\alpha\underline{\Lambda}-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta+ \eta\beta\int_{E\left(T_r^k\right)}^{\bar{t}_r^k}\left(1-\Lambda\right)\left(u-\lambda_r\right)^{\beta-1}\mathrm{d}u \tag{40}$$

In the other case, it is assumed that $\bar{t}_r^k>\lambda_r>E\left(T_r^k\right)$. Using the same method above, the upper and lower bounds on the CPV can be calculated as follows:

$$\bar{U}_r^k=\int_{\underline{t}_r^k}^{\mathrm{E}\left(T_r^k\right)}\alpha\left(\lambda_r-u\right)^{\alpha-1}\Lambda\mathrm{d}u+\left[\lambda_r-\mathrm{E}\left(T_r^k\right)\right]^\alpha- \eta\left(\bar{t}_r^k-\lambda_r\right)^\beta\left(1-\bar{\Lambda}\right)+\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta \tag{41}$$

$$\underline{U}_r^k=-\left(\lambda_r-\underline{t}_r^k\right)^\alpha\underline{\Lambda}-\eta\left(\bar{t}_r^k-\lambda_r\right)^\beta+ \eta\beta\int_{\lambda_r}^{\bar{t}_r^k}\left(u-\lambda_r\right)^{\beta-1}\left(1-\Lambda\right)\mathrm{d}u+ \int_{\mathrm{E}\left(T_r^k\right)}^{\lambda_r}\alpha\left(\lambda_r-u\right)^{\alpha-1}\left(1-\Lambda\right)\mathrm{d}u \tag{42}$$

Abbreviations

TTB	Travel time budget
PTT	Percentile travel time
METT	Mean excess travel time
CMTT	Combined mean travel time
EUT	Expected utility theory
CPT	Cumulative prospect theory
CPV	Cumulative prospect value
MSA	Method of successive average
CDF	Cumulative distribution function
RP	Reference point

UE User equilibrium

VI Variational inequality

O-D Origin-destination

SD Standard deviation

Notations

Considering a strongly connected transportation network $G=(N, A)$, where N and A denote the sets of nodes and links, respectively. The notations used throughout the paper are listed as follows unless otherwise specified.

R Set of O-D pairs

K_r Set of routes between O-D pair $r \in R$

Q_r Travel demand for O-D pair $r \in R$

q_r Mean travel demand for O-D pair $r \in R$

F_r^k Traffic flow on route $k \in K_r$

f_r^k Mean traffic flow on route $k \in K_r$

V_a Traffic flow on link $a \in A$

v_a Mean traffic flow on link $a \in A$

T_a Travel time on link $a \in A$

T_r^k Travel time on route $k \in K_r$

$\hat{t}_r$ Upper bound of the travel time on route $k \in K_r$

$\underline{t}_r^k$ Lower bound of the travel time on route $k \in K_r$

δ_r^{ka} Indicator variable that is equal to 1 if path $k \in K_r$ contains link $a \in A$, and 0 otherwise

p_r^k Path choice proportion on route $k \in K_r$

t_a^0 Free-flow travel time on link $a \in A$

c_a Link a's capacity

U_r^k CPV of route $k \in K_r$

$\underline{U}_r^k$ Lower bound on the CPV of route $k \in K_r$

$\psi_r^k(\cdot)$ Cumulative distribution function (CDF) of the travel time on route $k \in K_r$

λ_r Reference point (RP) for a trip between O-D pair r

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