

Damage constitutive model of different age concretes under impact load

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Abstract: In order to investigate the mechanical properties and stress–strain curves of concrete at different ages under impact load, the impact compression tests of concrete at age of 1, 3, 7, 14 and 28 d were conducted with a large diameter split Hopkinson pressure bar, respectively. Based on statistical damage theory and Weibull distribution, combining the analysis of the change laws of stress–strain curves and viscosity coefficient of concrete with age, a damage constitutive model that can reflect the variation in dynamic mechanical properties with age was proposed. The stress–strain curves calculated from the proposed model are in good agreement with those from experimental data directly.

Key words: concrete; constitutive model; age; split Hopkinson pressure bar

1 Introduction

Large span concrete structures have been ever-increasingly adopted in mountain tunnel construction. Drilling and blasting method is still the usual way to excavate tunnel with large section. In order to speed up the construction process, blasting operation is often conducted following the spraying concrete operation in a very short time interval, while the repeated blasting impact loading will probably cause damage to the concrete that is newly formed or has not reached stable age near the excavation face, affecting the strength of early age concrete. DOUGILL [1] firstly introduced damage to study the behavior of concrete materials in 1976, since then many researchers have focused their attention on concrete damage theory. LEMAITRE [2] proposed the famous strain equivalence hypothesis, introduced damage variable to stress–strain relationship and improved traditional elastic–plastic and viscoelastic constitutive models to describe the growth process of cracks in concrete [3], which advanced the further development of phenomenological damage theory based on statistical damage mechanics and statistical theory [4–5]. However, as the strength of early age concrete is very low compared with mature concrete, and which would lead more difficulties to meet the requires such as one-dimensional stress wave propagation, stress equilibrium state of specimen. In addition, in order to

make the specimen-bar contact as perfect as possible, specimens should be prepared carefully with grinding and smoothing such that the error in transmitted stress wave due to the non-perfect contact is negligible. As a result, the concrete damage model involved in previous studied concerned mainly on concrete at stable age [6–8], Little literature has been reported about damage model of concrete at early age subjected to impacting.

This work was focused on damage constitutive model of different age concretes under impact load, and impact compression experiments for concrete at early ages were scheduled and conducted according to the practical situation of large section tunnelling construction. Based on the statistical damage constitutive model, a new damage constitutive model consisting of concrete age variable was proposed, which could describe the variation in dynamic mechanical properties with age. Finally, the new-built constitutive model was verified by comparing stress–strain curves obtained from the new model and the experimental stress–strain curves.

2 Impact compressive experiments for different ages concretes

The experimental program was scheduled according to the practical situation of large section tunnelling construction. Impact compression tests were conducted on C20 concrete specimens at ages of 1, 3, 7, 14 and 28 d, respectively. Cylindrical specimens with the size of

Foundation item: Project(2010CB732004) supported by National Basic Research Program of China; Project(50934006) supported by the National Natural Science Foundation of China

Received date: 2013–10–28; **Accepted date:** 2014–02–25

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$\Phi 75 \text{ mm} \times 37.5 \text{ mm}$ were used in the tests [9]. The cement used in the experiments is ordinary Portland cement (ASTM Type I), and river sand was used as the fine aggregate while crushed limestone gravel passing the 16 mm sieve was used as the coarse aggregate [9–10]. Mixture proportion of concrete is given in Table 1, which was the same with that of sprayed concrete in tunnelling construction. Specimens were prepared in accordance with standard regulations [11], and the whole preparation process included mixing, tamping, trowelling, curing, mould removing, trowelling again and curing again.

Table 1 Concrete mixture proportions

$w(\text{Water})/$ $w(\text{Cement})$	Unit weight/($\text{kg} \cdot \text{m}^{-3}$)				Accelerator ratio/%
	Cement	Aggregate	Sand	Water	
0.46	397	853	786	182	4

Figure 1 shows the SHPB testing system which is characterized by its well adaptation to study the behavior of heterogeneous brittle materials under medium to high strain rate loading, and a spindle-shaped striker is adopted to obtain half-sine stress wave with constant loading strain rate [12]. The impacting experiments process mainly included the following steps: 1) Select the loading strain rate. According to the previous research, the loading strain rate in this test is $20\text{--}80 \text{ s}^{-1}$ (controlled by adjusting impact pressure). 2) Place the specimens between two elastic bars, and the end-friction effect was minimized through smearing butter between the interfaces. 3) Make sure that the smoothness and contact conditions of two elastic bars and specimen are good, if not, then go back to step 2). 4) Prepare the impact launching valves and set strain rate by adjusting impact air pressure. 5) Make sure that the data collection systems are ready, waiting for triggering. 6) Launch the striker, collect data and signals and save. Figure 2 shows the typical waveform of compression test. The dynamic mechanical parameters were calculated indirectly by three-wave method, with the results listed in Table 2.

3 Experimental results and discussions

Figure 3 shows the stress–strain curves of concrete

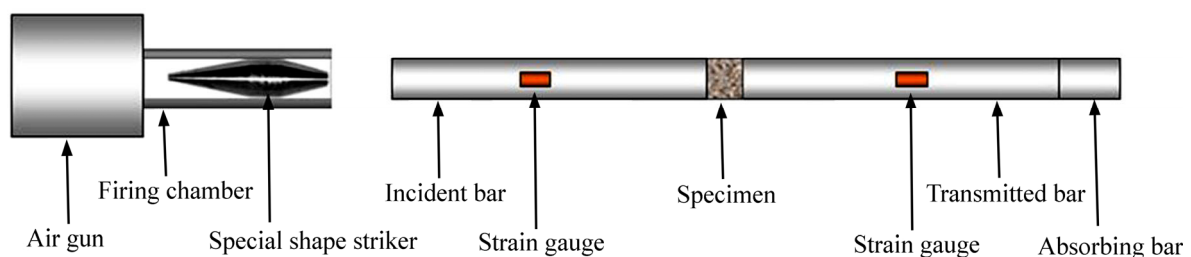


Fig. 1 Sketch of SHPB experimental system

at different ages subjected to uniaxial compressive impact loading. Conclusions can be drawn from Fig. 3 and Table 2 as follows.

1) Dynamic uniaxial compressive strength of concrete at different ages increases with the increase of loading strain rate.

2) Elastic modulus of concrete at the same age has no obvious change with the increase of loading strain rate. While under a similar loading condition (strain rate), the elastic modulus of concrete increase with the growth of age.

3) The critical strain and failure strain are relatively great for early age concrete and decrease rapidly with the growth of age, while have no obvious change for concrete after 7 d age. This indicates that concrete behaves stronger viscoelasticity before 7 d age, while gradually behaves quasi-brittle properties with the growth of age.

From the above analysis, the mechanical property of concrete will change distinctly from initial formation stage to stable age stage after curing for quite a long time. Hence, the change of mechanical property with age needs to be fully considered in the research of damage constitutive model and a statistical damage model based on Weibull distribution is introduced in the work.

4 Establishment and verification of constitutive model of different ages concretes

4.1 Dynamic statistical damage constitutive model

Since KRAJGINOVIC [3] proposed the statistical damage model, statistical damage theory has been developed rapidly in the research of rock-like material damage behavior. According to the randomness of micro flaws distribution in rock materials, continuum damage theory was combined with statistical strength theory [3, 13–14] and the strength of rock micro element obeys Gaussian distribution or Weibull distribution was considered, widening the application range of statistical damage model based on Weibull distribution in quasi-brittle materials [15–16].

The statistical damage model defines that the

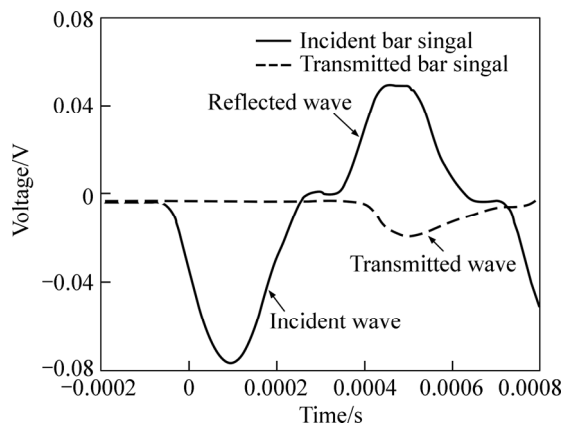


Fig. 2 Typical waveform of compression test

Table 2 Experiment results of different age concretes under impact load

Serial No.	Strain rate/s ⁻¹	Strength/ MPa	Elastic modulus/ GPa	Failure shape
1-1	33	6.98	2.09	Local crack
1-2	46	8.45	2.67	Crush
1-3	47	10.4	2.07	Crush
1-4	20	4.38	2.11	Intact
1-5	22	3.77	2.21	Intact
3-1	41	13.15	4.01	Crush
3-2	21	6.89	3.36	Intact
3-3	33	10.19	3.66	Fragmentation with two large degrees
3-4	31	9.81	3.42	Local crack
7-1	35	17.85	5.92	Crush
7-2	44	18.71	5.2	Crush
7-3	30	8.89	3.37	Intact
7-4	33	8.44	3.32	Intact
7-5	34	12.2	4.56	Intact
14-1	27	7.83	2.86	Intact
14-2	29	12.93	5.1	Intact
14-3	48	21.43	8.29	Fragmentation with large degrees
14-4	37	15.3	4.7	Intact
14-5	46	20.38	7.93	Crush
28-1	33	29.97	8.88	Fragmentation with three large degrees
28-2	25	21.87	8.51	Intact
28-3	30	27.08	9.06	Cracks throughout
28-4	31	23.7	8.85	Fragmentation with small degrees
28-5	41	31.09	9.23	Crush

Note: The first number of serial No. indicates the specimen age, and the second one indicates the specific number.

progressive failure features of rock-like materials due to microcracks and microvoids can be expressed by damage variable (D). It is provided that micro-element strength of material obeys Weibull distribution, then the damage variable also obeys Weibull distribution. The probability density function $p(F)$ can be expressed as

$$p(F) = \frac{m}{F_0} \cdot \left(\frac{F}{F_0}\right)^{m-1} \cdot \exp\left[-\left(\frac{F}{F_0}\right)^m\right] \quad (1)$$

where F is the strength distribution variable of rock micro-element, m and F_0 are both parameters of Weibull distribution. So, damage variable can be expressed as

$$D = \int_0^F P(y)dy = 1 - \exp\left[-\left(\frac{F}{F_0}\right)^m\right] \quad (2)$$

$$F = \alpha_0 I_1 + \sqrt{J_2} \quad (3)$$

where

$$\alpha_0 = \sin \varphi / \sqrt{9 + 3 \sin^2 \varphi} \quad (4)$$

is the parameter of micro-element strength which can be obtained from Drucker–Prager yield criterion [17]; I_1 and J_2 are the first and second invariants of stress, respectively, with the expressions as

$$I_1 = \sigma'_x + \sigma'_y + \sigma'_z = \sigma'_1 + \sigma'_2 + \sigma'_3 \quad (5)$$

$$J_2 = \frac{1}{6} \left[(\sigma'_x - \sigma'_y)^2 + (\sigma'_x - \sigma'_z)^2 + (\sigma'_z - \sigma'_y)^2 \right] + (\tau'_{xy})^2 + (\tau'_{xz})^2 + (\tau'_{yz})^2 = \frac{1}{6} \left[(\sigma'_1 - \sigma'_2)^2 + (\sigma'_1 - \sigma'_3)^2 + (\sigma'_2 - \sigma'_3)^2 \right] \quad (6)$$

where φ is the internal friction angle; σ'_1 , σ'_2 , σ'_3 are the effective stress corresponding to nominal stresses of σ_1 , σ_2 and σ_3 . The following equation can be got from Hooke’s law.

$$\varepsilon_1 = \frac{\sigma'_1 - 2\mu\sigma'_3}{E} \quad (7)$$

$$\sigma'_1 = \frac{\sigma_1}{1-D} \quad (8)$$

$$\sigma'_2 = \sigma'_3 = \frac{\sigma_3}{1-D} \quad (9)$$

Combining Eqs. (4)–(9) gives

$$I_1 = \frac{(\sigma_1 + 2\sigma_3)E\varepsilon_1}{\sigma_1 - 2\mu\sigma_3} \quad (10)$$

$$\sqrt{J_2} = \frac{(\sigma_1 - \sigma_3)E\varepsilon_1}{\sqrt{3}(\sigma_1 - 2\mu\sigma_3)} \quad (11)$$

Under one-dimensional stress state, $\sigma_1 = \sigma_2 = 0$, $\varepsilon_1 = \varepsilon$. So, Eq. (3) can be written as

$$F = (\alpha_0 + 1/\sqrt{3})\sigma_1 = (\alpha_0 + 1/\sqrt{3})E\varepsilon \quad (12)$$

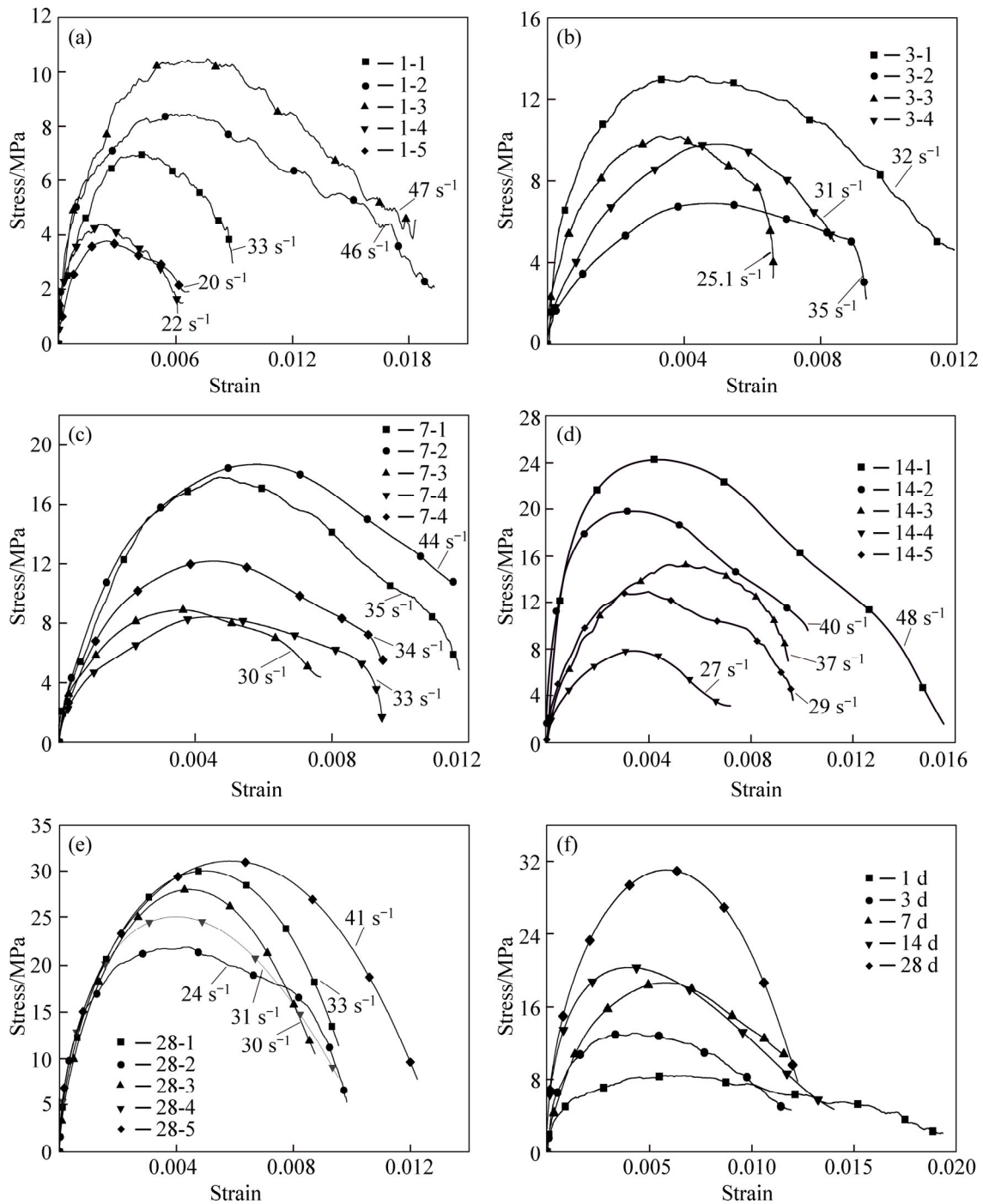


Fig. 3 Stress–strain curves of different age concretes under impact load: (a) 1 d; (b) 3 d; (c) 7 d; (d) 14 d; (e) 28 d; (f) Loading strain rate around 40 s^{-1}

Previous dynamic constitutive model can only describe the dynamic properties of rock but can't express the damage deformation properties. This work adopted modified time-dependency damage model of rock [18] on the basis of KELVIN model [19] (Fig. 4), and the constitutive equation is

$$\sigma = E \cdot \varepsilon(1 - D) + \eta \frac{d\varepsilon}{dt} \tag{13}$$

where E is the initial elastic modulus of material in

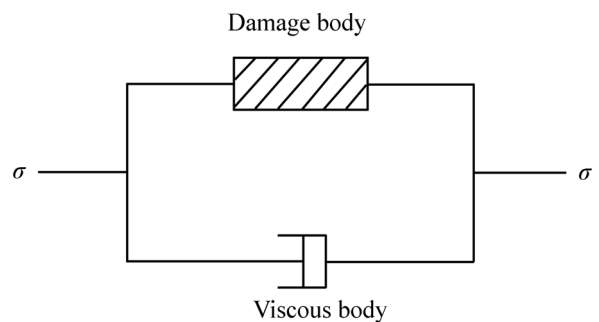


Fig. 4 Time-dependent damage model

undamaged state, and η is the coefficient of viscosity.

Equation (13) combining with Eq. (2) gives

$$\sigma = E \cdot \varepsilon \exp\left[-\left(\frac{F}{F_0}\right)^m\right] + \eta \frac{d\varepsilon}{dt} \quad (14)$$

From Eq. (14), with $d\varepsilon/dt=0$ at the peak point (σ_m, ε_m) of the stress–strain curve, Eq.(14) can be written as

$$F_0 = (\alpha_0 + 1/\sqrt{3})E\varepsilon_m \cdot m^{\frac{1}{m}} \quad (15)$$

where σ_m is the peak strength, ε_m is the peak strain.

So, Eq. (15) combining with Eq. (14) gives

$$m = 1/[\ln(E\varepsilon_m) - \ln(\sigma_m - \eta \cdot \frac{d\varepsilon}{dt})] \quad (16)$$

$$\sigma = E \cdot \varepsilon \exp\left[-\frac{1}{m}\left(\frac{\varepsilon}{\varepsilon_m}\right)^m\right] + \eta \frac{d\varepsilon}{dt} \quad (17)$$

Equation (16) is the dynamic statistical damage constitutive relation with m that can be easily determined by experiments.

4.2 Dynamic statistical damage constitutive model for different age concretes

It can be seen from the stress–strain curve of different age concretes (Detailed in Fig. 3) that, the mechanical properties of concrete change from viscoelasticity to quasi-brittle with growth of age, and it is expected that the viscosity coefficient has also been changing during the process. So, it is well-founded to introduce the age variable into constitutive model given above by coefficient of viscosity, which provides a feasible way to describe the constitutive relation for concrete at different ages. To introduce the age variable d into the dynamic statistical damage constitutive equation, the relationship between the age variable d and the viscosity coefficient η needs to be defined.

In the practice of measuring velocity by ultrasonic method, when the circular frequency is so large that the wave length of ultrasonic is less than half a minimum lateral dimension of specimen, the velocity of longitudinal wave in the specimen is deemed to be the same

as that in infinite, homogeneous materials. The velocity of longitudinal wave spreading in infinite, homogeneous materials can be written as [20]

$$V_{pu} = \sqrt{\frac{2E[1+(1.2R_u)^2]}{0.9\rho[1+\sqrt{1+(1.2R_u)^2}]} \quad (18)$$

where E is elastic modulus, ρ is density of material, and $R_u = \eta\omega_u/E$.

When the circular frequency is so small that the wave length of ultrasonic is far more than lateral dimension of specimen, the velocity of longitudinal wave in the specimen is deemed to be the same as that in one-dimensional elastic rod. And the velocity of longitudinal wave spreading in one-dimensional elastic wave can be written as

$$V_{pr} = \sqrt{E/\rho} \quad (19)$$

Therefore, the viscosity coefficient of concrete can be obtained only if two kinds of velocities are detected under above-mentioned corresponding frequency conditions. The velocity of longitudinal wave in concrete at different ages was detected by employing geotechnical engineering quality detector CE-9201, which is applicable of measuring longitudinal wave velocity of various materials with a wide adjustable frequency range of 1 kHz to 100 kHz. Now that the velocity of concrete before 14 d age is less than 3750 m/s, the maximum wave length is 37.5 mm (half of specimen’s diameter) when the emitting frequency is set as 100 kHz, and 3750 mm (far more than specimen’s diameter) when the emitting frequency is 1 kHz. So, V_{pu} and V_{pr} of concrete at different ages were measured by adjusting the frequency of emitting supersonic as listed in Table 3. Combining Eqs. (18) and (19), the expression of viscosity coefficient can be written as Eq. (20), and the results in Table 3 combining with Eq. (20) gives the viscosity coefficient of concrete at different ages, listed in Table 4.

$$\eta = \frac{E \sqrt{\left[0.9(dV) + \sqrt{0.81(dV)^2 + 7.2(dV)}\right]^2 - 16}}{4.8\omega_u} \quad (20)$$

Table 3 Variation in wave velocities of different frequencies with age

Serial No.	Density/ (kg·m ⁻³)	V/(m·s ⁻¹), (Frequency 1 kHz)				V/(m·s ⁻¹), (Frequency 100 kHz)			
		1 d	3 d	7 d	14 d	1 d	3 d	7 d	14 d
28-1	2188	2840	2982	3263	3450	3089	3216	3490	3679
28-2	2201	2938	3136	3391	3596	3101	3302	3570	3706
28-3	2185	2806	2944	3120	3355	3000	3147	3295	3699
28-4	2189	2912	3086	3308	3565	3163	3327	3547	3746
28-5	2257	2918	3096	3297	3526	3159	3348	3519	3745

Table 4 Variation in viscosity coefficient with age

Serial No.	Viscosity coefficient/(MPa·s)			
	1 d	3 d	7 d	14 d
28-1	0.0426	0.0405	0.0396	0.0358
28-2	0.0458	0.0451	0.0430	0.0391
28-3	0.0415	0.0394	0.0362	0.0338
28-4	0.0448	0.0434	0.0407	0.0382
28-5	0.0463	0.0450	0.0417	0.0385

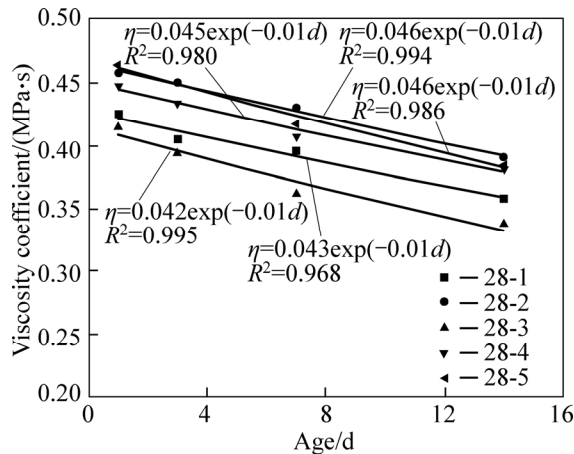


Fig. 5 Variation in viscosity coefficient with age

where $dV = \left(\frac{V_{pu}}{V_{pr}} \right)^2$.

The change laws of viscosity coefficient are well fitted by exponential functions with the related coefficient of more than 0.95 (Detailed in Fig. 5). It's worth noting that the viscosity coefficient of concrete at 28 d age calculated by the fitting expression accords with practical situation, too [21]. The relationship between viscosity coefficient and age can be expressed as $\eta = a \exp(bd)$, so the statistical damage constitutive model considering the factor of concrete age can be written as

$$\sigma = E \cdot \varepsilon \exp \left[-\frac{1}{m} \left(\frac{\varepsilon}{\varepsilon_m} \right)^m \right] + a \exp(bd) \cdot \frac{d\varepsilon}{dt} \quad (21)$$

where a and b are coefficients, a is obtained as the average value of 0.045 and b is equal to -0.01 from Fig. 5. Then, Eq. (21) is written as

$$\sigma = E \cdot \varepsilon \exp \left[-\frac{1}{m} \left(\frac{\varepsilon}{\varepsilon_m} \right)^m \right] + 0.045 \exp(-0.01d) \cdot \frac{d\varepsilon}{dt} \quad (22)$$

4.3 Verification of newly-built constitutive model

To verify whether the newly-built constitutive model can describe the dynamic stress–strain relation of concrete at different ages or not, the theoretical stress–strain curves of newly constitutive model were

compared with experimental stress–train curves as shown in Fig. 6, and an error analysis which can be obtained by Eq. (23) was conducted, listed in Table 5.

$$\delta = \left| \frac{\sigma_{\text{model}} - \sigma_{\text{test}}}{\sigma_{\text{model}} + \sigma_{\text{test}}} \right| \times 100\% \quad (23)$$

From Fig. 6, for concrete specimens at different ages, the stress–strain curves from newly-built constitutive model are in good agreement with the stress–strain curves obtained from experiments, indicating the model not only can fully illustrate the relation between concrete strength and strain and strain rate, but well satisfy the mechanical properties variation of early age concrete with the growth of age.

Meanwhile, there are some drawbacks in the newly-built model. Combining Fig. 6 and Table 5, it is observed that if the concrete specimens maintain unbroken or just fragment into big blocks due to impact, the two groups of stress–strain curves can match well, no matter pre- or post-peak strength. The average relative error of specimens that still maintain intact after impact is less than 0.15 basically, and the average relative error of specimens that critical damage or partial fragmentation after impact is substantially less than 0.25. However, once the specimens are fractured into pieces by impacting, the two groups of curves perform quite differently at the post-peak strength, and the stress observed from the model curve is greater than that from experiments for the post-peak part. Besides the average relative error is large, individual even reached 0.342. The characterization of post-peak curve has always been a problem in the research of constitutive model of quasi-brittle materials [22], because the failure process and mechanism is extremely complicated due to the microcracks and microvoids in quasi-brittle materials. In the work, once the specimen was locally fractured due to the strong locally stressed effect at post-peak stage, the stress condition of specimen is extremely complicated (causing larger average relative error of crushing failure specimen), the resistance against external force lowered accordingly, which means the specimen couldn't be taken as a whole any longer. While the newly-built constitutive was established on the condition that the specimen was basically unbroken. So, this is probably the reason why the stress observed from the model curve is greater than that from experimental curve, especially when the specimen was comminuted fragmented. In the future research of constitutive model for quasi-brittle materials, particularly in characterization of post-peak curve, if the factor of failure shape can be taken into consideration in constitutive model, better fitting performance will achieve.

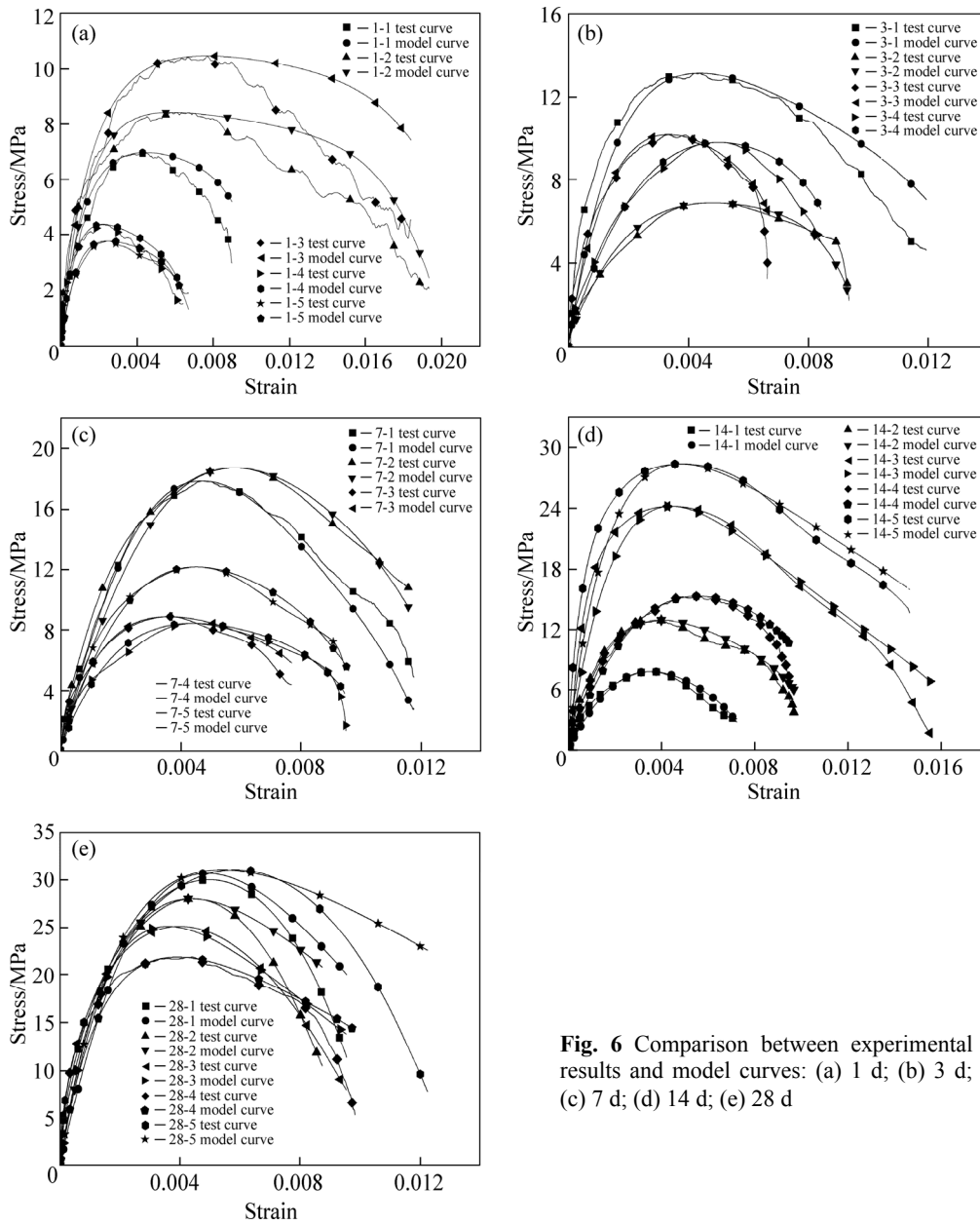


Fig. 6 Comparison between experimental results and model curves: (a) 1 d; (b) 3 d; (c) 7 d; (d) 14 d; (e) 28 d

Table 5 Error analysis between experimental and model curves

Serial No.	Average relative error	Failure shape	Serial No.	Average relative error	Failure shape
1-1	0.236	Local crack	7-4	0.113	Intact
1-2	0.287	Crush	7-5	0.079	Intact
1-3	0.342	Crush	14-1	0.114	Intact
1-4	0.168	Intact	14-2	0.231	Intact
1-5	0.067	Intact	14-3	0.162	Fragmentation, with large degrees
3-1	0.190	Crush	14-4	0.167	Intact
3-2	0.076	Intact	14-5	0.126	Crush
3-3	0.226	Fragmentation with two large degrees	28-1	0.272	Fragmentation with three large degrees
3-4	0.117	Local crack	28-2	0.188	Intact
7-1	0.234	Crush	28-3	0.285	Cracks throughout
7-2	0.114	Crush	28-4	0.215	Fragmentation, with small degrees
7-3	0.133	Intact	28-5	0.304	Crush

5 Conclusions

Impact compression tests of concrete at five different early ages are conducted with SHPB, and the mechanical properties of concrete subjected to impact at the age of 1, 3, 7, 14, 28 d are comparatively analyzed. Under the impact load, the dynamic uniaxial compressive strength of concrete at different ages increases with the increase of loading strain rate. The Young's modulus of concrete at the same age has no obvious change with the increase of loading strain rate, while rises with the growth of age due to the similar loading strain rate. Critical strain and failure strain of early age concrete are large, but it decreases rapidly with the growth of age and after the age of 7 d, critical strain and failure strain did not change significantly with the growth of age, which indicate the mechanical properties of concrete change from viscoelasticity to quasi-brittle with the growth of age.

Based on the statistical damage constitutive model, a new damage constitutive model consisted of concrete age variable is proposed, which could describe the constitutive relation for different ages concrete. The new-built constitutive model is verified by comparing stress–strain curves obtained from the new model and experimental stress–strain curves, indicating that there is a good agreement with two groups of stress–strain curves. However, in describing mechanical properties of the crushed specimen, the stress observed from the model curve is greater than that from experiments for the post-peak part. That is because once the specimen is locally fractured due to the strong locally stressed effect at post-peak stage, the stress condition of specimen is extremely complicated, the resistance against external force lowered accordingly, which means the specimen couldn't be taken as a whole any longer. While the newly-built constitutive was established on the condition that the specimen is basically unbroken. So, this is probably the reason why the stress observed from the model curve is greater than that from experimental curve, especially when the specimen is comminuted fragmented.

In the future research of constitutive model for quasi-brittle materials, particularly in characterization of post-peak curve, if the factor of failure shape can be taken into consideration in constitutive model, better fitting performance will be achieved.

In addition, the newly-built constitutive model is proposed by taking C20 concrete as research object, and more work needs to be developed to verify the adaptation of the model to other concretes.

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(Edited by DENG Lü-xiang)