

# Self-excited vibration problems of maglev vehicle-bridge interaction system

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**Abstract:** The self-excited vibration problems of maglev vehicle-bridge interaction system were addressed, which greatly degrades the stability of the levitation control, decreases the ride comfort, and restricts the cost of the whole system. Firstly, the coupled model containing the quintessential parts was built, and the mechanism of self-excited vibration was explained in terms of energy transmission from levitation system to bridge. Then, the influences of the parameters of the widely used integral-type proportion and derivation (PD) controller and the delay of signals on the stability of the interaction system were analyzed. The result shows that the integral-type PD control is a nonoptimal approach to solve the self-excited vibration completely. Furthermore, the differential-type PD controller can guarantee the passivity of levitation system at full band. However, the differentiation of levitation gap should be filtered by a low-pass filter due to noise of gap differentiation. The analysis indicates that a well tuned low-pass filter can still keep the coupled system stable.

**Key words:** maglev; vehicle-bridge interaction system; self-excited vibration; energy

## 1 Introduction

With the development of national economy, the city's traffic of China is increasingly congested [1]. Developing public transit is an effective way to improve the urban traffic conditions. Compared with conventional rail-way systems, the electromagnetic maglev system (EMS) has advantages of lower noise, higher speed, and the ability to climb steeper slopes [2], which is a new kind of urban transport that has been widely concerned in recent years. Germany [3–4], Japan [5–7], the UK [8], US [9], South Korea [10], China [11] and many other countries have carried out research and made great progress.

The development of low-speed EMS is more than thirty years [12]. On March 6, 2005, at the Expo in Aichi, Japan, the operation of TuboKyuryo Linimo (TKL) line marked that the low-speed EMS has entered the commercial operation stage [13]. In China, the Beijing S1 line with completely independent intellectual property rights is under construction.

The rapid development and enormous advantages of maglev sketch a bright future for the commercial applications. However, it should be pointed out that there are still some problems at present. For example, the self-excited vibration of vehicle-bridge interaction system is a

burning issue to be solved [14]. It occurs when the vehicle is suspended upon the guideway, standing still or moving at very slow speed (<5 km/h). The inability to achieve stable levitation is attributed to flexibility of the bridge, and the widely accepted conclusion is that the bridge is too flexible to permit stable levitation [15]. When the self-excited vibration occurs, both the vehicle and the guideway begin to vibrate, mainly in vertical direction. It can be divided into two subclasses: the electromagnet–steel track self-excited vibration and the vehicle-bridge self-excited vibration.

The former generally arises between a single electromagnet and a segment of elastic steel track. The vibration amplitude of steel track is about 0.1 mm, but the frequency is up to 100 Hz, which always makes an annoying noise. Luckily, this kind of vibration doesn't affect the stability of the levitation system obviously. The vibration is mainly due to the loosening of fixing bolt between the sleeper and steel track, which decreases the support stiffness. Generally, periodic inspections and tightening of the fixing bolt are effective to avoid it.

The latter occurs between one or more bogies and an elastic bridge at frequency below 20 Hz, which always leads to the large vibration amplitude of bridge. Although this kind of vibration doesn't make an annoying noise, its power is much larger than the former,

and it degrades the safety of bridge and durability of bridge. Furthermore, the self-excited vibration greatly deteriorates the stability of the levitation system, even leading to a control failure. Japanese HSST03 [17], German' TR04 [18], Tangshan's CMS04, Shanghai's TR08 [19], American AMT [20] all have experienced the self-excited vibration and instability of the coupled system.

For the self-excited vibration problem, ZOU et al [21] carried out the derivation of bifurcation equations and numerical simulation using the center manifold method. They believed that the bifurcations, such as the homoclinic, Hopf bifurcation, secondary Hopf bifurcation and chaos are the causes of self-excited vibration.

LI and MENG [22] and ZHAO [10] pointed out that the self-excited vibration is due to the improper frequency relationship between various components of the system. WANG et al [23] believed that the self-excited vibration is more likely to occur if the difference between the modal frequency of bridge and the natural frequency of controller is sufficiently small. Increasing the stiffness of sleepers and concrete bridges helps to suppress the vibration.

The influence of signal delay on the stability of nonlinear levitation system is studied [24–29]. The analysis shows that when the time delay reaches a critical value, the system undergoes a sub-critical Hopf bifurcation, and the periodic vibration occurs.

According to the existing literatures, scholars of this field explain the mechanism of self-excited vibration from different perspectives, but the existing research methods involve the homoclinic, Hopf bifurcation, chaos and other nonlinear theory. Considering that the improper frequency relationship between various components of the system is the source of self-excited vibration, further theoretical basis is needed.

In order to solve the problem of self-excited vibration, various methods are proposed from the perspective of optimization bridge design, such as increasing the modal damping, mass and stiffness [23], decreasing the modal frequency of bridge [22, 30]. However, these methods increase the cost, and are not suitable for already completed bridges. If the reasonable selection of the control strategy and parameters can avoid the self-excited vibration, the cost will be greatly saved and the competitiveness of maglev will be improved, which are significant to promote the commercialization process.

LI et al [31] used a general Kalman filter to estimate the frequency of the vibration, and then adjusted the parameters of phase lead compensator. The simulation results verify the feasibility of this method. ZHOU et al [16] proposed a novel concept of a virtual tuned mass

damper (TMD), which uses the electromagnetic force to emulate the force of a real TMD acting on bridge. In the presence of the time delay caused by the actuator, the stability proof of the levitation system combined with the virtual TMD becomes complex, and the numerical simulations indicate that the virtual TMD can make the levitation system passive.

Due to the clear physical meaning of parameters, excellent transient response, good robustness and easy engineering implementation, PD control is still the one of the most popular controllers in the maglev engineering [14, 32]. Considering that the parameters of PD controller, the types of damping and the delay of signals all have great influences on the stability of the interaction system, if we can theoretically explain the mechanism of vibration and the influences of the above factors and propose a feasible solution to solve the self-excited vibration issue, it will be more significant. This work is based on this point to carry out the research on the self-excited vibration of vehicle-bridge interaction system.

## 2 Vehicle–bridge interaction modeling

Considering the complexity of self-excited vibration, the overall dynamic models of the coupled system with details will result in a difficult analysis to draw useful conclusions. Hence, a minimum coupled model containing the quintessential parts with the self-excited vibration occurring is presented, which is composed of a flexible bridge and a single levitation electromagnet. The nonlinearity response of the bridge is not considered because the amplitude of the vibration is sufficiently small compared with the span of a bridge. The bridge is considered to be a Bernoulli-Euler beam due to the fact that the length of the bridge is much larger than the size of other dimensions. Besides, the bridge is assumed to be simply supported and have constant cross-section. The interaction force between electromagnet and bridge is simplified as a concentrated force and the action point is the geometric center of the electromagnet. The kinetics coupling between adjacent levitation units and the dynamics of air springs are neglected.

### 2.1 Model of bridge

Based on the above assumptions, the simplified interaction model is shown in Fig. 1, where  $y_0$  and  $y_1$  are the vertical displacements of bridge and electromagnet, respectively;  $\delta$  is the levitation gap measured by the gap sensor;  $m_1$  is the equivalent mass of electromagnet.

The motion of bridge can be described by the following differential equation:

$$EI_b \frac{\partial^4 y(x, t)}{\partial x^4} + \rho_b \frac{\partial^2 y(x, t)}{\partial t^2} = f(x, t) \quad (1)$$

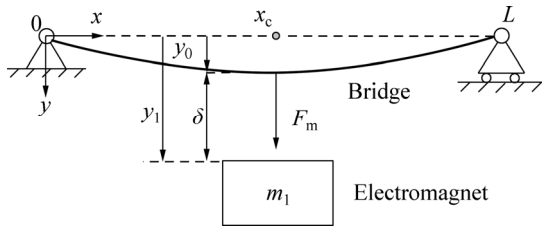


Fig. 1 Simplified interaction model

where  $EI_b$  is the bending stiffness;  $\rho_b$  is the mass per unit length;  $f(x, t)$  is the electromagnetic force acting on the bridge. For a simply supported bridge, the  $k$ -th natural frequency  $\omega_k$  and mode shape  $\phi_k(x)$  are

$$\omega_k = \lambda_k^2 \sqrt{\frac{EI_b}{\rho_b}} \tag{2}$$

$$\phi_k(x) = \sin \lambda_k x \tag{3}$$

where  $\lambda_k = k\pi/L$ . Using modal analysis method, the solutions of Eq. (1) can be expressed by the linear superposition of modal shapes as

$$y(x, t) = \sum_{k=1}^{\infty} \phi_k(x) q_k(t) \tag{4}$$

Substituting Eq. (4) into Eq. (1), multiplying both sides of the resultant equation by  $\sin(n\pi x/L)$ , and integrating both sides from 0 to  $L$ , it is given

$$\ddot{q}_k(t) + \omega_k^2 q_k(t) = \frac{2}{\rho_b L} \int_0^L f(x, t) \phi_k(x) dx \tag{5}$$

Considering that the length of the electromagnet is approximately 1/15 of the bridge, the electromagnetic force can be viewed as a concentrated force. Then,

$$\ddot{q}_k(t) + \omega_k^2 q_k(t) = \frac{2\phi_k(x_c)}{\rho_b L} F_m(t) \tag{6}$$

Taking the modal damping of bridge into account, Eq. (6) can be given by

$$\ddot{q}_k(t) + 2\xi_k \omega_k \dot{q}_k(t) + \omega_k^2 q_k(t) = \frac{2\phi_k(x_c)}{\rho_b L} F_m(t) \tag{7}$$

Denoting  $A = 2\phi_k^2(x_c) \rho_b^{-1} L^{-1}$ ,  $y_k(t)$  and  $v_k(t)$  are the  $k$ -th modal displacement and velocity of bridge, respectively. Multiplying both sides of the resultant equation by  $\phi_k(x_c)$  gives

$$\ddot{y}_k(t) + 2\xi_k \omega_k \dot{y}_k(t) + \omega_k^2 y_k(t) = AF_m(t) \tag{8}$$

Therefore,

$$H(s) = \frac{v_k(s)}{F_m(s)} = \frac{As}{s^2 + 2\xi_k \omega_k s + \omega_k^2} \tag{9}$$

Equation (9) can be considered as the response of vertical velocity of bridge inspired by the electromagnetic force.

### 2.2 Levitation system model

Suppose the number of turns of a single electromagnet is  $N$ , the pole area is  $A$ , and the magnetic permeability of vacuum is  $\mu_0$ , and then for a single electromagnet, the relationship between the controlled voltage  $U(t)$  and current  $I(t)$  is

$$\frac{U(t)}{2} = I(t)R + \frac{\mu_0 AN^2}{2\delta(t)} I(t) - \frac{\mu_0 AN^2 I(t)}{2\delta^2(t)} \dot{\delta}(t) \tag{10}$$

where  $R$  is the DC resistance. The electromagnetic force  $F_m(t)$  acting on the bridge is

$$F_m(t) = \frac{\mu_0 AN^2}{2} \left( \frac{I(t)}{\delta(t)} \right)^2 \tag{11}$$

Considering the vibration isolation of air springs, the dynamics of sprung mass is neglected. Then the movement of electromagnet is

$$m_1 \frac{d^2 y_m(t)}{dt^2} = -F_m(t) + (M_1 + m_1)g \tag{12}$$

where  $g$  is the acceleration of gravity and  $M_1$  is the sprung mass. According to Eqs. (11) and (12), the steady current of the electromagnet is

$$I_0 = \frac{z_0}{N} \sqrt{\frac{2(M_1 + m_1)g}{\mu_0 A}} \tag{13}$$

where  $z_0$  is the desired levitation gap. The integral-type PD control (damping signal  $\dot{y}(t)$  is the integral of the output of accelerometer fixed on the electromagnet) is the most widely used controller in engineering practice, which is preferred in this study. The integral-type PD controller can be expressed as

$$I_e(t) = k_p [y_1(t) - y_0(t) - z_0] + k_d \dot{y}_1(t) + I_0 \tag{14}$$

where  $I_e(t)$  is the desired current flowing through the electromagnet,  $k_p$  and  $k_d$  are the coefficients of the proportionality and damping, respectively. To accelerate the response of the actuator, a cascaded current controller is used [33–34]:

$$u(t) = k_c [I_e(t) - I(t)] \tag{15}$$

The DC resistance shares a part of control voltage, which results in the steady error between the expected current and actual current of the coils. In order to remove the steady error, a current feed forward control strategy is added. Then

$$u(t) = k_c [I_e(t) - I(t)] + 2RI(t) \tag{16}$$

The motion of the levitation system is determined by Eqs. (10)–(16). Here, the interaction model of vehicle-bridge vibration is built.

### 3 Mechanism of self-excited vibration

If the levitation controller can guarantee the stability of the levitation system under the rigid bridge, but fails to guarantee the stability under the flexible bridge, then we can draw the conclusion that the vibration of bridge occurs. As we all know, the bridge needs to absorb energy from the rest state to vibration state. The interaction system only includes the levitation system and bridge. Hence, the energy absorbed by the bridge is from the exportation of levitation system. Furthermore, the energy absorbed by the bridge should be greater than the energy consumed by the modal damping. In this case, the energy of the bridge continues to accumulate, resulting in the growth of vibration. Therefore, from the perspective of energy transmission, the mechanism of self-excited vibration can be explained clearly.

#### 3.1 Energy mechanism with integral-type PD controller

When discussing the stability of the equilibrium point, the use of linearized model can greatly simplify the analysis. The linearized system in the frequency domain can be expressed as

$$u(s) = k_p k_c [y_1(s) - y_0(s)] + k_d k_c s y_1(s) - k_c I(s) + 2RI(s) \tag{17}$$

$$U(s) = 2L_0 s I(s) + 2RI(s) - 2F_i s y_1(s) + 2F_i s y_0(s) \tag{18}$$

$$F_m(s) = 2F_i I(s) - 2F_z y_1(s) + 2F_z y_0(s) \tag{19}$$

$$F_m(s) = -m_1 s^2 y_1(s) \tag{20}$$

where  $L_0$  is the inductance coefficient of the equilibrium point and  $L_0 = 0.5\mu_0 AN^2 z_0^{-1}$ ,  $F_i$  is the partial differential coefficient of current to the electromagnetic force and  $F_i = 0.5\mu_0 AN^2 i_0 z_0^{-2}$ ,  $F_z$  is the partial differential coefficient of levitation gap to the electromagnetic force and  $F_z = 0.5\mu_0 AN^2 i_0^2 z_0^{-3}$ . Eliminating  $u(s)$ ,  $I(s)$  and  $y_1(s)$  of Eqs. (17)–(20), the transfer function between  $F_m(s)$  and  $v_0(s)$  can be deduced as

$$G_0(s) = \frac{F_m(s)}{v_0(s)} = -\frac{\eta_0 m_1 s}{m_1 L_0 s^3 + 0.5 m_1 k_c s^2 + F_i k_c k_d s + \eta_0} \tag{21}$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ . Equation (21) is viewed as the response of electromagnetic force of levitation system acting on the bridge inspired by the velocity of bridge, and Eq. (9) can be seen as the response of velocity of bridge roused by the electromagnetic force. Therefore, the levitation system and the bridge are interactive.

In order to explain the self-excited mechanism from the perspective of energy quantitatively, we assume that

the frequency of vibration is  $\omega_{vib}$ , and the amplitude of vibration is 1 mm. With the passage of time, if the amplitude of vibration becomes smaller, and converges to zero at last, then the interaction system is stable, whereas the control strategy should be improved to avoid the instability. The above assumptions give

$$y_0(t) = 0.001 \sin(\omega_{vib} t) \tag{22}$$

$$v_0(t) = 0.001 \omega_{vib} \cos(\omega_{vib} t) \tag{23}$$

According to Eq. (21), the electromagnetic force acting on the bridge is

$$F_m(t) = 0.001 \omega_{vib} |G_0(j\omega_{vib})| \cos(\omega_{vib} t + \angle G_0(j\omega_{vib})) \tag{24}$$

Therefore, the average power of the electromagnetic force acting on the bridge is

$$P_m(\omega_{vib}) = \frac{1}{T} \int_0^T v_0(t) F_m(t) d\tau = \frac{(0.001 \omega_{vib})^2}{2 \operatorname{Re}[G_0(j\omega_{vib})]} \tag{25}$$

where  $T$  is the period of self-excited vibration. When  $k_p=4700$ ,  $k_d=40$  and  $k_c=40$ , the relationship between the vibration frequency and the exportation power ( $P_m$ ) of levitation system is shown in Fig. 2. Defining  $\omega_c$  as the critical frequency of the levitation system, where  $\operatorname{Re}[G_0(j\omega_c)] = P_m(\omega_c) = 0$ .

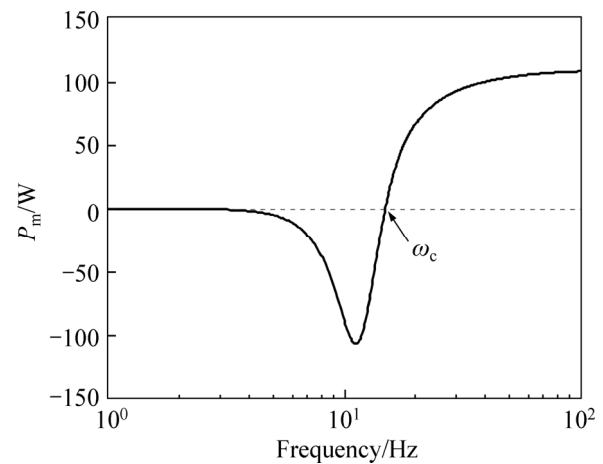


Fig. 2 Exportation power versus vibration frequency

Assuming that the self-excited vibration occurs and the frequency of vibration is less than the critical frequency of the levitation system. According to Fig. 2, the levitation system absorbs the vibration energy of the bridge. Together with the modal damping, the vibration energy will be consumed out, which makes the amplitude of the vibration decay to zero. The interaction system is stable below  $\omega_c$ . Hence, regardless of how large the modal frequency and the modal damping are, if the self-excited vibration occurs, the frequency of vibration is above  $\omega_c$ .

### 3.2 Influences of controller parameters on stability of interaction system

To investigate the influences of PD controller parameters on the stability of interaction system, based on the well tuned control parameters  $k_p=4700$ ,  $k_d=40$ , and  $k_c=40$ , we increase the parameters by 1.5 times, respectively. The relationship between the exportation power of levitation system and frequency is shown in Fig. 3.

According to Fig. 3, increasing the coefficient of proportionality does not change the critical frequency of levitation system. Conversely, the coefficients increment of damping and current loop can improve the critical frequency, which helps to extend the stable frequency range and is an effective method to improve the stability of the vehicle-bridge interaction system.

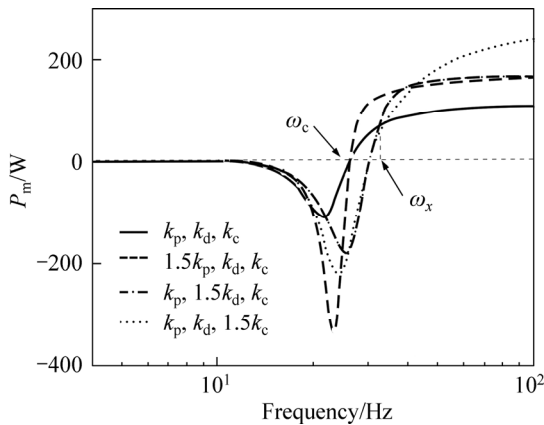


Fig. 3 Exportation power versus frequency curve with increased parameters

In addition to the critical frequency, the exportation power of levitation system above the critical frequency  $\omega_c$  is concerned. If the exportation power is greater than the power consumed by the damping in the neighborhood of the equilibrium point, less than the power consumed by the damping when the system is away from the equilibrium, a periodic oscillation with constant amplitude will occur. What is more, if the exportation power is sufficiently large, the interaction system will be unstable.

According to Fig. 3, a reasonable increment of the coefficients of damping and current loop can extend the critical frequency of the levitation system, but it will lead to greater exportation energy when the frequency is greater than the cross frequency  $\omega_x$  shown in Fig. 3. Reducing the coefficient of proportionality can reduce the exportation power of levitation system. However, it will deteriorate the guideway tractability and robustness against the external disturbance. Therefore, tuning the parameters of the integral-type PD controller is a nonoptimal approach to solve the self-excited vibration completely.

### 3.3 Influences of signal delays on stability of interaction system

The levitation gap sensors operate at a complex electromagnetic environment. The target signal of gap sensor is weak and the transmission process of signals will pull in noise. To improve noise ratio of signals, the first-order analog RC filter is widely used to weaken the high-frequency noises. However, the RC filter will lead to time delay. In addition, the AD sampling, signal transmission, calculation of control schemes and output of pulse width modulation (PWM) all pull in time delay. The influences of signal delay on the stability of nonlinear levitation system were studied [23–28]. It is believed that the system will undergo a sub-critical Hopf bifurcation when the time lag is greater than a critical value, and result in a periodic vibration. The above methods provide a good idea for exploring the delay on the stability of the levitation system from the perspective of homoclinic, Hopf bifurcation and chaos. In terms of energy, the influences of signal delay on the stability of interaction system are explained.

#### 3.3.1 Delay of gap channel

Taking CMS04 for example, the time constant of the first-order RC filter of the levitation gap signal is  $\tau_1=0.66$  ms, then the transfer function of RC filter is  $G_1(s) = (\tau_1 s + 1)^{-1}$ .

Considering the delay of AD sampling, signal transmission, and calculation of control scheme, the output of PWM is about 0.5 ms, and the transfer function of delay is

$$G_2(s) = e^{-\tau_2 s} \tag{26}$$

Considering that  $\tau_2$  is a small constant, Eq. (26) can be simplified as a first-order low pass filter,  $G_2(s) \approx (\tau_2 s + 1)^{-1}$ . Therefore, the transfer function from the initial signal to the output of PWM is

$$G_3(s) = \frac{1}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + 1} \tag{27}$$

Considering that  $\tau_1$  and  $\tau_2$  are two small constants,  $\tau_1 \tau_2$  is a second-order infinitesimal, which can be ignored here. Then, the above transfer function can be further simplified as

$$G_3(s) \approx \frac{1}{(\tau_1 + \tau_2) s + 1} \tag{28}$$

When considering the delay of levitation gap, Eq. (17) can be modified as

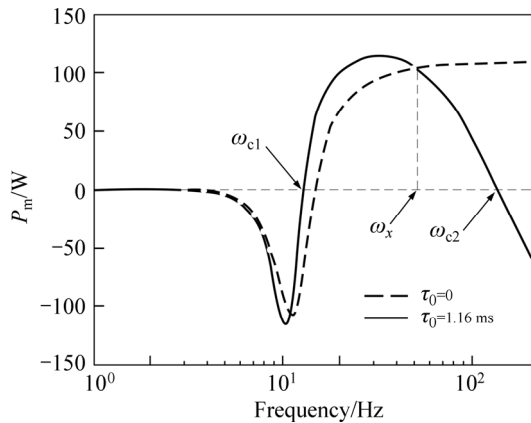
$$u(s) = k_p k_c G_3(s) [y_1(s) - y_0(s)] + k_d k_c s y_1(s) - k_c I(s) + 2RI(s) \tag{29}$$

Eliminating  $U(s)$ ,  $I(s)$  and  $y_1(s)$  of Eqs. (18)–(20), (29), the transfer function between the electromagnetic

force  $F_m(s)$  and the vertical velocity  $v_0(s)$  of vibration is

$$G_0(s) = \frac{-F_z k_c \tau_0 m_1 s^2 + \eta m_1 s}{m_1 L_0 \tau_0 s^4 + \eta_3 s^3 + \eta_2 s^2 + \eta_1 s + \eta_0} \quad (30)$$

where  $\tau_0 = \tau_1 + \tau_2$ ,  $\eta_0 = F_i k_c k_p - F_z k_c$ ,  $\eta_1 = F_i k_d k_c - F_z k_c \tau_0$ ,  $\eta_2 = 0.5 m_1 k_c + F_i k_d k_c \tau_0$ , and  $\eta_3 = m_1 L_0 + 0.5 m_1 k_c \tau_0$ . According to Eq. (30), it can be seen that the time delay of levitation gap changes the characteristic polynomial of transfer function, and adds a new zero. When the parameters of controller are  $k_p = 4700$ ,  $k_d = 40$  and  $k_c = 40$ , the exportation power of levitation system with delay of gap channel is shown in Fig. 4.



**Fig. 4** Exportation power versus frequency curve with delay of gap channel

According to Fig. 4, the delay of levitation gap channel decreases the critical frequency of the levitation system. This is to say, the stable frequency range is compressed, which is bad for the stability of the vehicle-bridge interaction system. Thereby, the delay of levitation gap channel should be reduced as much as possible.

When the frequency of vibration belongs to  $(\omega_x, \omega_{c2})$  as shown in Fig. 4, the delay of levitation gap channel results in less exportation power of the levitation system, which is in favor of the stability of the interaction system.

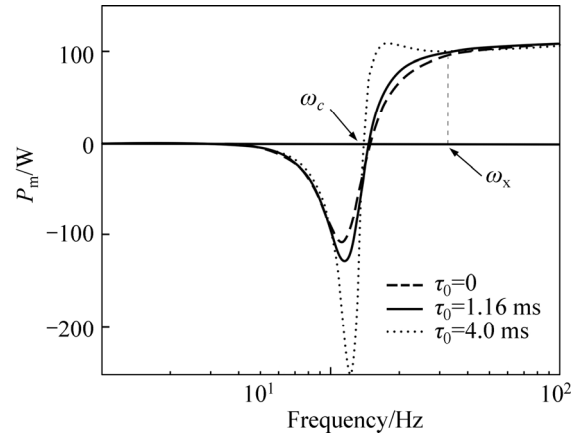
When the frequency of vibration is greater than  $\omega_{c2}$ , the levitation system absorbs the energy of bridge vibration. i.e., the interaction system keeps stable again above the frequency  $\omega_{c2}$ . Therefore, if the frequency of self-excited vibration is up to 100 Hz, such as the self-excited vibration of electromagnet-steel rail, appropriate delay of levitation gap channel is an effective way to avoid it.

### 3.3.2 Delay of damping channel

When considering the delay of the damping channel, the transfer function between the electromagnetic force and the vertical velocity of vibration is

$$G_0(s) = \frac{F_E(s)}{v_0(s)} = \frac{m_1 \eta_0 s (\tau_0 s + 1)}{m_1 L_0 \tau_0 s^4 + \eta_3 s^3 + \eta_2 s^2 + \eta_1 s + \eta_0} \quad (31)$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ ,  $\eta_1 = F_i k_p k_c \tau_0 + F_i k_d k_c - F_z k_c \tau_0$ ,  $\eta_2 = 0.5 m_1 k_c$ , and  $\eta_3 = m_1 L_0 + 0.5 m_1 k_c \tau_0$ . The exportation power of levitation system with the delay of damping channel is shown in Fig. 5.



**Fig. 5** Exportation power versus frequency curve with delay of damping channel

According to Fig. 5, the delay of the damping channel slightly reduces the critical frequency of the levitation system. From the point of the stable frequency range, it has little effect on its stability. However, it should be noted that when the self-excited vibration occurs and its frequency is greater than the critical frequency  $\omega_c$ , the levitation system will pull more energy into the bridge. Besides, in the neighborhood of  $\omega_c$ , the gradient of the exportation power is much larger, this is to say, the stability of the interaction system to the frequency is more sensitive. It is harmful for the stationary of the bridge. In all, the delay of damping channel should be avoided as much as possible.

### 3.3.3 Delay of current channel

When considering the delay of the current channel, the transfer function between the electromagnetic force  $F_m(s)$  and the vertical velocity  $v_0(s)$  of vibration is

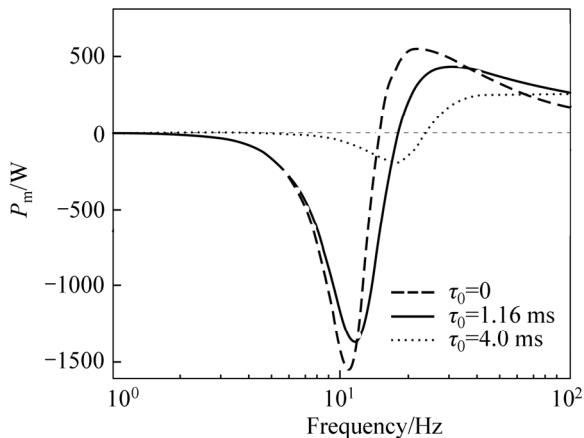
$$G_0(s) = \frac{m_1 F_i k_p k_c \tau_0 s^2 + m_1 \eta_0 s}{m_1 L_0 \tau_0 s^4 + m_1 L_0 s^3 + \eta_2 s^2 + \eta_1 s + \eta_0} \quad (32)$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ ,  $\eta_1 = F_i k_p k_c \tau_0 + F_i k_d k_c$  and  $\eta_2 = F_i k_d k_c \tau_0 + 0.5 m_1 k_c$ . The exportation power of levitation system with the delay of current channel is shown in Fig. 6.

For the bridge whose modal frequency is greater than the critical frequency of the levitation system, CHENG et al [2] pointed out that if the self-excited vibration occurs, the frequency of the self-excited vibration is

$$\omega_{vib} = \sqrt{\omega_n^2 + k_c \xi_n L^{-1}} \quad (33)$$

where  $k_c$  is the coefficient of the current loop,  $\xi_n$  and  $\omega_n$  are the modal damping ratio and modal frequency of the bridge, and  $L$  is the span of bridge. Generally,



**Fig. 6** Exportation power versus frequency curve with delay of current channel

$\omega_n^2 \gg k_c \xi_n L^{-1}$ , this is to say, the vibration frequency is mainly determined by the modal frequency of the bridge.

According to Fig. 6, different from the delay of the above two channels, the delay of current channel can extend the critical frequency of the levitation system. The filed experiments at Tangshan maglev test base show that the first modal frequency of bridge is 13–16 Hz. If the critical frequency of the levitation system can be upgraded from 15 Hz to 24 Hz by 4.0 ms delay of the current channel, the levitation system will be passive in the neighborhood of the first-order modal frequency, which is favorable for the stability of the vehicle-bridge interaction system.

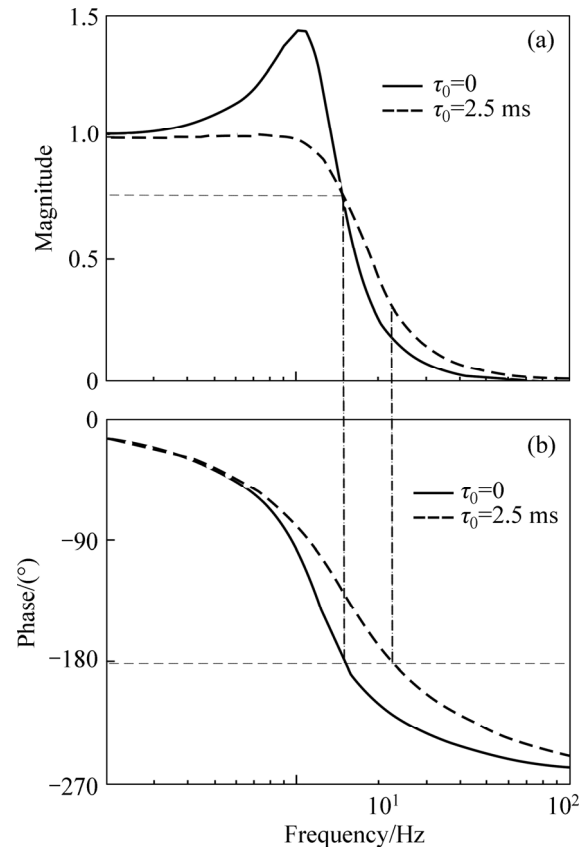
The active delay introduction of current channel is an innovative way for the suppression of self-excited vibration. For maglev engineering, whether the delay will degrade the guideway traceability needs further theoretical research.

When considering the delay of the current channel, the transfer function between the displacement of electromagnet  $y_1(s)$  and the reference trajectory  $y_0(s)$  is

$$\frac{y_1(s)}{y_0(s)} = \frac{F_i k_p k_c \tau_0 s + \eta_0}{m_1 L_0 \tau_0 s^4 + m_1 L_0 s^3 + \eta_2 s^2 + \eta_1 s + \eta_0} \quad (34)$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ ,  $\eta_1 = F_i k_p k_c \tau_0 + F_i k_d k_c$ , and  $\eta_2 = F_i k_d k_c \tau_0 + 0.5 m_1 k_c$ . The traceability of levitation system with the delay of current channel is shown in Fig. 7.

For the levitation system, the low frequency guide-way irregularities should be tracked as much as possible to guarantee the safety of the vehicle, and the high frequency guide-way irregularities should be isolated to improve the riding quality. According to Fig. 7, the above specifications can be achieved by the integral-type PD controller. However, when the frequency of guideway irregularities is in the neighborhood of 11 Hz, the levitation system without current delay leads to a resonant peak at 11 Hz. This



**Fig. 7** Traceability of levitation system with delay of current channel

means that per unit of 11 Hz irregularities will result in 1.45 times fluctuation of the electromagnet, which is harmful for the traceability of the levitation system.

The delay of current channel can eliminate the resonant peak effectively and keep the bandwidth (magnitude of 0.707) the same as the levitation system without the current delay. Moreover, the phase margin is significantly improved.

Based on the above analysis, a suitable delay of the current channel is conducive to the suppression of the self-excited vibration. Meanwhile, it improves the traceability of levitation system. This finding is significant for maglev engineering and accelerating the commercialization process.

### 3.4 Influences of differential-type PD controller on stability of interaction system

Based on the above analysis, adjusting the parameters of the integral-type PD controller and introducing reasonable delay of current channel to a certain extent can improve the stability of the coupled system, but it cannot guarantee the levitation passive at full band. After the eigenvalues analysis of the interaction model, ALBERTS and OLESZCZUK [35] believed that the differential-type PD controller can avoid the self-excited vibration. Here we would like to

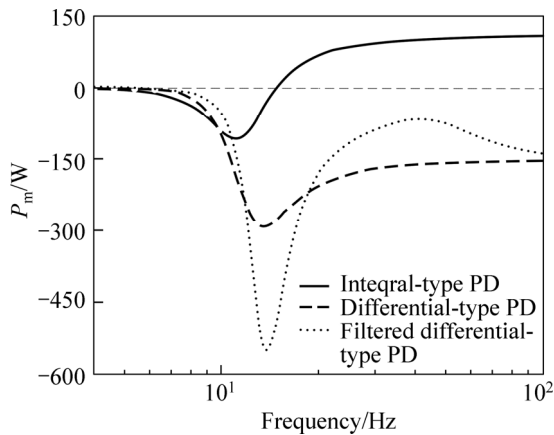
verify the conclusion from the energy point of view. When using a differential-type PD controller, Eq. (17) can be modified as

$$u(s) = (k_p k_c + k_d k_c s) [y_1(s) - y_0(s)] - k_c I(s) + 2RI(s) \quad (35)$$

The transfer function between the electromagnetic force  $F_m(s)$  and the vertical velocity  $v_0(s)$  of vibration is

$$G_0(s) = \frac{F_e(s)}{v_0(s)} = \frac{m_1 F_i k_d k_c s^2 + \eta_0 m_1 s}{m_1 L_0 s^3 + 0.5 m_1 k_c s^2 + F_i k_d k_c s + \eta_0} \quad (36)$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ . The exportation power of levitation system with the differential-type PD controller is shown in Fig. 8.



**Fig. 8** Exportation power versus frequency curves for different type PD controllers

According to Fig. 8, differential-type PD controller can guarantee the levitation system passive at full band, which makes the interaction system stable. The conclusion is consistent with Ref. [36]. Theoretically, the differential-type PD controller can solve the self-excited vibration.

However, the levitation gap sensor operates in a complex electromagnetic environment, the target signal of sensor is weak, and the signal transmission from gap sensor to controller pulls noise. If the noise can not be suppressed effectively during the process of extracting differentiation of levitation gap, the differential signal will be overwhelmed by noise, which results in the unavailability for the maglev engineering. The subtraction method by two different first-order filters proposed in Ref. [2] is given as

$$T(s) = \frac{1}{1+as} - \frac{1}{1+bs} = \frac{(b-a)}{abs^2 + (a+b)s + 1} s, \quad b > a \quad (37)$$

Factually, the subtraction method is equivalent to the direct differential signal filtered by a second-order low-pass filter. Its leading phase is

$$\angle T(j\omega) = 90^\circ - \arctan(a\omega) - \arctan(b\omega) \quad (38)$$

The leading phase is close to  $90^\circ$  if the following requirement is satisfied:

$$a\omega_{vib} < 0.2, \quad b\omega_{vib} < 0.2 \quad (39)$$

where  $\omega_{vib}$  is the frequency of the self-excited vibration. With the increment of the frequency, the filter is equivalent to an integrator, so the noise of differentiation is attenuated, even though the influence of the filter should be further investigated. When using the subtraction method, Eq. (17) can be modified as

$$U(s) = [k_p k_c + k_d k_c T(s)] [y_1(s) - y_0(s)] - k_c I(s) + 2RI(s) \quad (40)$$

The transfer function between the electromagnetic force  $F_m(s)$  and the vertical velocity  $v_0(s)$  of vibration is

$$G_0(s) = \frac{F_e(s)}{v_0(s)} = \frac{m_1 F_i k_d k_c s^2 + \eta_0 m_1 s}{m_1 L_0 s^3 + 0.5 m_1 k_c s^2 + F_i k_d k_c T(s) + \eta_0} \quad (41)$$

where  $\eta_0 = F_i k_c k_p - F_z k_c$ . The exportation power of levitation system with the filtered differential-type PD controller is shown in Fig. 8. With the selected filter, it keeps the levitation system passive still at full band. Meanwhile, it should be pointed out that the absorbed power of levitation system decreases obviously in the neighborhood of 40 Hz. We can predict that if the parameters of filter are unreasonable, it may result in the instability of the coupled system again.

## 4 Conclusions

1) A minimum coupled model containing the quintessential parts with the self-excited vibration occurring is presented, which is composed of a flexible bridge and a single levitation electromagnet.

2) Based on the above model, the mechanism of the self-excited vibration is explained in terms of energy. The energy exportation of the levitation system to the bridge is the root of the self-excited vibration.

3) The influences of the parameters of the integral-type PD controller on the stability of the interaction system are analyzed. Increasing the coefficients of the damping and current can extend the frequency range of stability.

4) The influences of the delay of the three channels on the stability are explored. Generally speaking, the delay of gap channel and damping channel is harmful for the stability of the interaction system, and it should be avoided as much as possible. Inversely, the delay of current channel which can extend the critical frequency of the levitation system is conducive to the stability of the coupled system. Meanwhile, it improves the traceability of levitation system.



5) The differential-type PD control can guarantee the passivity of the levitation system at full band, while the noise of the gap differentiation should be suppressed. The analysis indicates that a well tuned low-pass filter can still keep the coupled system stable.

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