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A new double reduction method for slope stability analysis

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Abstract: The core of strength reduction method (SRM) involves finding a critical strength curve that happens to make the slope globally fail and a definition of factor of safety (FOS). A new double reduction method, including a detailed calculation procedure and a definition of FOS for slope stability was developed based on the understanding of SRM. When constructing the new definition of FOS, efforts were made to make sure that it has concise physical meanings and fully reflects the shear strength of the slope. Two examples, slopes A and B with the slope angles of 63° and 34° respectively, were given to verify the method presented. It is found that, for these two slopes, the FOSs from original strength reduction method are respectively 1.5% and 38% higher than those from double reduction method. It is also found that the double reduction method predicts a deeper potential slide line and a larger slide mass. These results show that on one hand, the double reduction method is comparative to the traditional methods and is reasonable, and on the other hand, the original strength reduction method may overestimate the safety of a slope. The method presented is advised to be considered as an additional option in the practical slope stability evaluations although more useful experience is required.

Key words: slope stability; strength reduction method; double strength reduction method; factor of safety; limit equilibrium method

1 Introduction

Slope stability analysis represents a wide area of geotechnical practices. Of these analyses, factor of safety (FOS), as an index measuring the safety of a slope, has been keeping widely welcomed by many engineers because it supplies with clear physical concept and a comparable and easy-to-use value. Limit equilibrium method (LEM), a traditional and well established approach is widely accepted by engineers and researchers mainly because it is simple to use and able to produce an FOS. Strength reduction method (SRM) is another idea, initiated by ZIENKIEWICZ et al [1] in 1975 and named by MATSUI and SAN [2] in 1992, to calculate the FOS of a slope. This idea combined with finite element method formed the FE-SRM which obtained extensive concerns and application especially in a recent decade because of the improvements of FE package and computer hardware. FE-SRM can not only compute the FOS of a slope but also has several additional advantages over LEM. Systematic comparisons and reviews were conducted by DUNCAN [3]. GRIFFITHS and LANE [4] and ZHENG et al [5] also summarized these advantages.

In strength reduction method, a series of boundary value problems have to be analyzed with corresponding strength parameters reduced by a series of factors until a factor that can just make the slope globally fail is found. For example, if Mohr-Column criterion is used, c and φ need to be reduced. If a group of reduced strength parameters are obtained and just happen to make the slope reach critical failure state, then the corresponding reduction factor will be taken as the FOS of the slope. It is noteworthy that reduction factors for both c and φ are equal in these SRMs. A new idea immediately and naturally comes out, and it is feasible to reduce c and φ with different reduction factors (later it will be called double strength reduction method, D-SRM for short, and to discriminate with SRM, the previous strength reduction method will be later called original strength reduction method (O-SRM)). TANG et al [6] initiated the idea of using two distinct reduction factors for c and φ respectively. However, what relation the two reduction factors should satisfy is not clear. YUAN et al [7] recently implemented a D-SRM where c and φ satisfy a matching principle. SUO [8] also conducted the research on double reduction technique and found that FOS from their D-SRM was smaller than that from the O-SRM. This means that the possibility of overestimating the safety of a slope from O-SRM exists. However, as a result of two distinct reduction factors, there are two

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problems needed to be solved. The first is how to determine the reduction principle for c and φ. Another problem is neither of these two reduction factors could be directly defined as the FOS of the slope as the O-SRM does. In fact, this means that how to define FOS in D-SRM technique becomes a new fundamental problem to be solved. In the previously listed research on double reduction method, the FOSs were just treated as the arithmetic mean or weighted average of the two reduction factors without any validation statements, which obviously lacks concise physical meanings.

This work aims to develop a new double reduction method to obtain the FOS of a slope. In this method, a new double reduction process and a new definition of FOS with concise physical meaning are given. Moreover, examples are given to validate this method.

2 Material point safety factor

Point safety factor is first presented as the double reduction method in this concept.

Factor of safety of a slope is a measure of global safety of the slope. For a slope with given stress distribution, each part of the slope also has the concept of safety. To measure this safety, the concept of local safety factor was proposed by ONO [9], which is usually dependent on the failure criterion. OKUBO et al [10] proposed a modified local safety factor to extend its application to most failure criteria. Material point safety factor is a kind of local safety factor representing the safety degree of a material point in the slope. In numerical analysis programs, if a constant strain element is adopted, then the stress state at a material point will be equal to that at this element. In the numerical examples through this work, constant strain element is adopted, so material point safety is also called element safety factor through this work. Both phrases are treated to be completely equivalent.

The stress state at a material point can be depicted with Mohr circle in σ–τ coordinate system (Fig. 1). The center of the stress circle in Fig. 1 is (σ_0, τ_r). τ_r is the radius of this circle. G is a generic point at the circle whose coordinates are (σ_n, τ_n). Line $L1$ is the Mohr-Coulomb strength line. The shear strength corresponding to point G is

$$\tau_f = c + \sigma_n \tan\phi \tag{1}$$

The ratio of the shear strength on the strength line to the corresponding shear stress on the circle, denoted as S_G, is a measure of the safety degree:

$$S_G = \frac{\tau_f}{\tau_n} = \frac{c+\sigma_n \tan\phi}{\tau_n} = \frac{c+(\sigma_0 - \tau_r \sin\theta)\tan\phi}{\tau_r \cos\theta} \tag{2}$$

As a generic point on the Mohr circle stands for the

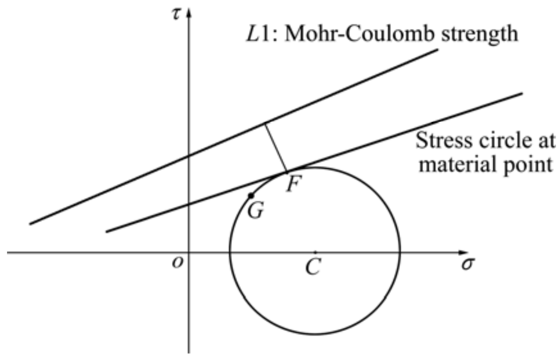

Fig. 1 Stress circle at material point, strength line and material safety factor

stress at a cross section of the material point it belongs to, S_G hence measures the safety on a generic cross section. KOURDEY et al [11] found that there exists a minimum value for Eq. (2) with Mathematica soft. This means that any stress state of a material point has a unique minimum safety factor. This minimum value will be defined as the material point safety (or element safety factor) through this work although there might be kinds of definitions for safety factor on a material point.

Denote the point with the minimum safety factor as F, and then the coordinates of F are

$$\sigma_F = \sigma_0 - \frac{\tau_r^2 \tan\phi}{\tau_s} \tag{3}$$

$$\tau_F = \frac{\tau_r \sqrt{\tau_s^2 - (\tau_r \tan\phi)^2}}{\tau_s} \tag{4}$$

The safety factor at point F is

$$S_F = \frac{\sqrt{(c+\sigma_0 \tan\phi)^2 - (\tau_r \tan\phi)^2}}{\tau_s} = \sqrt{\left(\frac{\tau_s}{\tau_r}\right)^2 - \tan^2\phi} \tag{5}$$

In Eqs. (3)−(5), $\tau_s := c + \sigma_0 \tan\phi$ is the shear strength corresponding to the normal stress at the center of the stress circle.

3 A new double reduction method

3.1 Essential of strength reduction method

From the process of original strength reduction method, the FOS is defined as the reduction factor that can just make the slope globally fail. In other words, the essential of strength reduction method is to find a critical strength curve (surface) that just makes the slope globally fail. For Mohr-Coulomb criterion, this is equivalent to finding critical c and φ. Following this understanding of SRM, this process does not imply that the reduction factors for c and φ are equal.

3.2 Calculation process for D-SRM

As already pointed out, there are two problems to be solved for double reduction methods. One is how to develop a reduction process for c and φ, which is presented in this session. Another is how to define the FOS in the D-SRM technique.

There are theoretically infinite pairs of (c', φ'). Our aim is to develop a definite reduction operation to obtain a pair of (c', φ'). In this specific reduction operation, the (c', φ') is unique. However, the search in two dimensional space of (c, φ) needs too many calculations of boundary problem to solve the slope.

The core idea of the calculation procedure to be developed in geometry is to find a new strength line that just makes the slope fail. The instability criterion to stop the calculation is the same as that of O-SRM. This new strength line is naturally used to obtain the reduced (c', φ') and hence the double reduction factor can be easily calculated. A generalized calculation process of this method is advised as follows:

1) Initialize the stress field of the slope by a real FEM or FLAC solving which adopts the real c and φ of the slope.

2) Compute the element FOSs (defined in Section 2) of all the elements and arrange the elements in FOS' increasing size of order. The ordered element set is denoted as E. We denote the elements of sets from E as S_1, S_2, ⋯. The elements of a generic subset S_i are the 1st−i-th elements of E.

3) Loop the subset SE_i^0 and compute its most common tangent line.

(1) Loop all the elements and calculate the common tangents of any two circles in this subset.

(2) Loop all the common tangents to discard the tangent lines that have minus values of vertical intercepts. The set of these sorted-out tangent lines is denoted as $T1$.

(3) Loop all the tangent lines in $T1$ and pick out the one that has the largest number of tangents with all the other elements in SE_i^0. This line is called most tangency line of subset SE_i^0.

4) Update the Mohr-Column strength criterion with the most tangency line and conduct a new FEM or FLAC solving.

5) Test whether this most tangency line makes the slope fail. If not, go to Step 3) to test the next subset SE_{i+1}^0, else go to Step 6).

6) The most tangency line found in Step 5) is the critical reduction strength line as

$$\tau = c' + \tan\phi'\sigma \tag{6}$$

With this equation, the reduced (c', φ') can be naturally obtained.

7) Compute the two reduction factors.

The two reduction factors are respectively given as

$$k_{\mathrm{c}} = \frac{c}{c'} \tag{7}$$

$$k_{\varphi} = \frac{\tan\varphi}{\tan\varphi'} \tag{8}$$

8) Define and calculate the global FOS of the slope and stop.

A detailed flowchart to obtain the FOS of a slope with double reduction method is presented in Fig. 2.

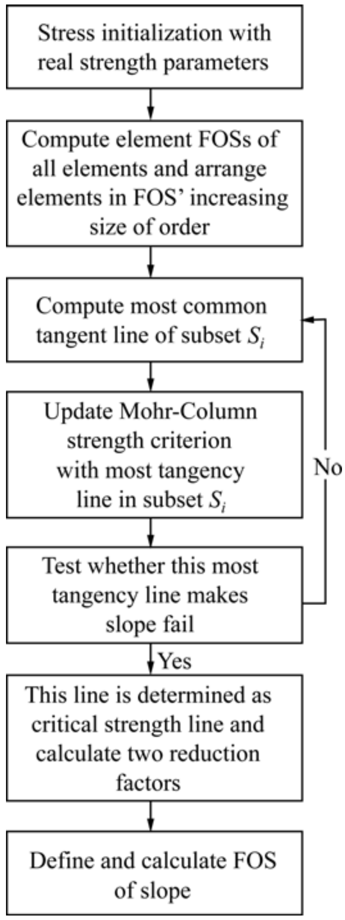

Fig. 2 Calculation flowchart to obtain FOS of slope with double reduction method

According to the above analysis, in D-GSM, global FOS of slope cannot be directly treated as the reduction factors as O-SRM does and hence a specific definition of FOS is required. This will be further discussed in the following section.

3.3 Defining global factor of safety

Since it is impossible to directly use either of the two reduction factors as the global FOS, we have to seek other ways. Defining the global FOS of a slope based on element FOS is a natural idea which has been investigated by several researchers. TAMOTSU and KA-CHING [12] advised that the slope of FOS could be treated as the weighted mean value of the FOSs of all the elements passed through by potential slide line (PSL) of the slope, that is,

$$F_s = \frac{\sum F_{s_i} \Delta s_i}{\sum \Delta s_i} \tag{9}$$

where Δs_i is the length of PSL segment crossing an element i whose element FOS is F_{s_i}. KOURDEY et al [11] even advised that the slope FOS was treated as the average of all the element FOSs passed through by PSL.

When constructing the new definition of FOS, two principles should be followed. The first is that the new FOS should have concise physical meanings and the second is that it can fully reflect the shear strength of the slope. Let's first investigate the case of a slope with two elements. Figure 3 shows the stress circles of these two elements whose radii are respectively r_1 and r_2. σ_{o_1} and σ_{o_2} are respectively the horizontal coordinates of centers of the two circles. The stress circles represent the initialized stress state with real strength parameters of slope. The $L1$ is the Mohr-Coulomb strength line of slope and the other line is the common tangent line and the critical strength reduction line of our double reduction method. P_1 and P_2 are two tangent points of this line whose coordinates are respectively (σ_{1n}, τ_{1n}) and (σ_{2n}, τ_{2n}).

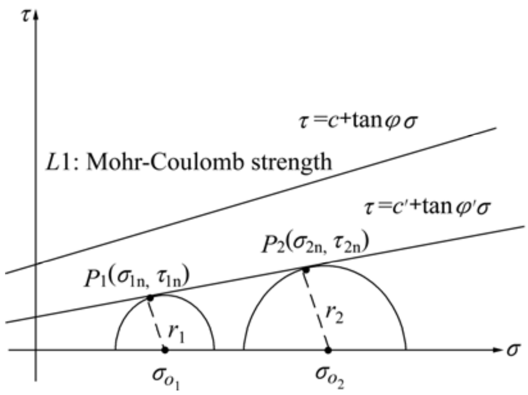

Fig. 3 Definition of slope with two elements

For this ideal case, we define the FOS of the slope as the ratio of the sum of shear strengths to the shear stress of the two tangent points:

$$F_s = \frac{(c + \tan\phi \cdot \sigma_{1n}) + (c + \tan\phi \cdot \sigma_{2n})}{\tau_{1n} + \tau_{2n}} \tag{10}$$

Obviously, this is a generalization of the concept of strength reservation. The following equations hold:

$$\begin{cases} \sigma_{1n} = \sigma_{o_1} - \sin\varphi' \cdot r_1 \\ \sigma_{2n} = \sigma_{o_2} - \sin\varphi' \cdot r_2 \\ \tau_{1n} = c' + \tan\varphi' \cdot (\sigma_{o_1} - \sin\varphi' \cdot r_1) = \cos\varphi' \cdot r_1 \\ \tau_{2n} = c' + \tan\varphi' \cdot (\sigma_{o_2} - \sin\varphi' \cdot r_2) = \cos\varphi' \cdot r_2 \end{cases} \tag{11}$$

Combining Eqs. (10) and (11), we have

$$F_s = \frac{2c + \tan\phi[(\sigma_{o_1} + \sigma_{o_2}) - \sin\phi'(r_1 + r_2)]}{\cos\phi'(r_1 + r_2)} \tag{12}$$

This idea can be directly generalized to the FOS definition of a slope with far more than two elements. When critical strength line common tangent to N elements is found, we still define the FOS of a generic slope as the ratio of the sum of shear strengths to the shear stress of the N tangent points:

$$F_s = \frac{\sum \tau_{f_i}}{\sum \tau'_{f_i}} \tag{13}$$

Equation (13) can be rewritten through some derivations as

$$F_s = \frac{2c + \tan\phi \left[\sum_{i=1}^{N} \sigma_{o_i} - \sin\phi' \sum_{i=1}^{N} r_i \right]}{\cos\phi' \sum_{i=1}^{N} r_i} \tag{14}$$

4 Examples

Two examples with different slope angles (>45° and <45°, respectively) are given as an application of D-SRM and to verify the findings in this work. The results are compared with those from O-SRM and Bishop method (an equilibrium method). In these two examples, any material is assumed to be homogeneous and elastic- perfectly plastic under Mohr-Coulomb yield condition. O-SRM and D-SRM adopt the same meshes. No seepage process and hence no pore pressure are considered. The two numerical calculations are both conducted on FLAC (two dimensional edition, Itasca Ltd.) plus our own second development code. The material properties listed in Table 1 are adopted for both slopes.

Table 1 Material parameters

Unit weight/ (kN·m^{-3})	Bulk modulus/ MPa	Shear modulus/ MPa	Cohesion force/ kPa	Friction angle/ (°)
2500	8.333	4.846	42	27

4.1 Example 1: Slope angle=63°

The initial configuration of the slope is shown in Fig. 4, meshed with uniform quadrilateral elements. The geometries are labeled in Fig. 4. The upper boundary is in zero stress state. Vertical rolling conditions are applied on the left and right sides. Fixed boundary condition is used on the bottom.

Let us first show the results of the D-SRM presented in Section 3. The common tangent element set, defined in Section 3, is shown in Fig. 5. It is these elements' common tangent line that defines the reduced strength line. The greatest element FOS in this common tangent element set is 1.0125. From Eqs. (7)−(8), the two strength reduction factors are respectively k_c=0.808 and $k_\phi=1.414$. Update the Mohr-Coulomb strength criterion with this reduced strength line, conduct a new FLAC solving, and the slope reaches a global failure whose yield elements are shown in Fig. 6. The maximum shear strain is used to identify the potential slide surface shown in Fig. 7. Follow the FOS definition in Eq. (14), the global FOS based on D-SRM is 1.006.

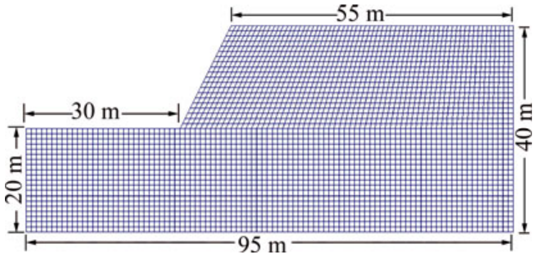

Fig. 4 Initial configuration of slope for Example 1

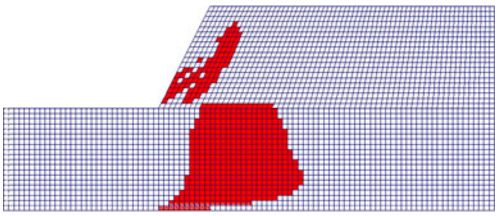

Fig. 5 Common tangent element set corresponding to critical strength line for Example 1

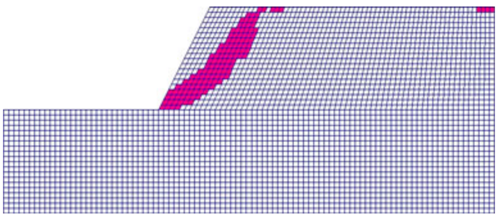

Fig. 6 Yield elements on globally critical failure by G-RSM for Example 1

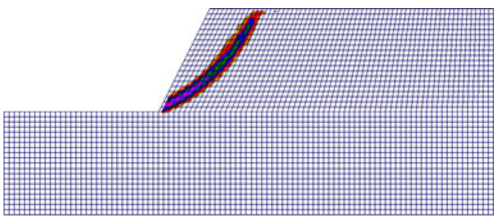

Fig. 7 Most dangerous slide zone by D-SRM for Example 1

The O-SRM is also used to evaluate the stability of this slope. The O-SRM based FOS is 1.021 which is about 1.5% greater than that from D-SRM. The yield elements and the potential slide surface by O-RSM are respectively shown in Figs. 8 and 9. Comparing Figs. 7 and 9, we find that D-SRM predicts 188 yield elements in the potential slide mass while O-SRM predicts 163 yield elements in the potential slide mass.

The FOS of 1.006 from D-SRM is also smaller than 1.012 from the Bishop method whose calculation result is shown in Fig. 10.

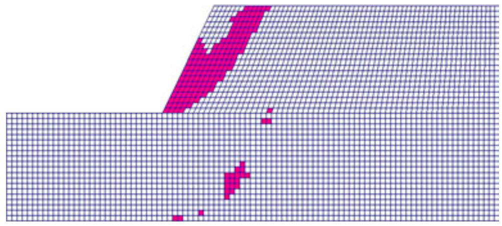

Fig. 8 Yield elements on globally critical failure by O-RSM for Example 1

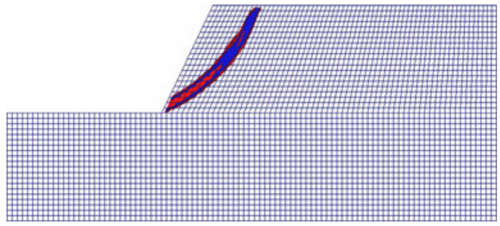

Fig. 9 Potential slide surface by O-SRM for Example 1

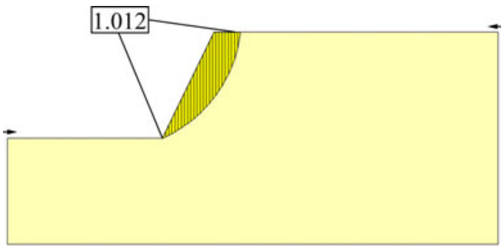

Fig. 10 Most dangerous slide line by LEM (Bishop Method, Example 1)

4.2 Example 2: Slope angle=34°

The configuration of the second example is shown in Fig. 11. The main difference of this example from the first one is that the slope angle is smaller. All the boundary conditions and material properties are the same as those of Example 1. Still the three methods, D-SRM, O-SRM and Bishop LEM, are investigated.

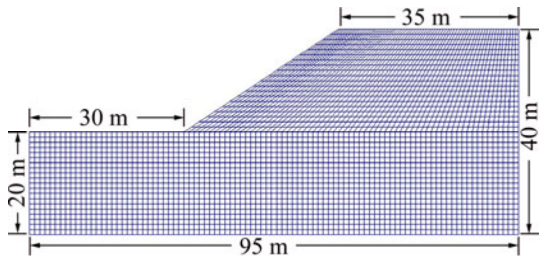

Fig. 11 Initial configuration of slope for Example 2

D-SRM gives the FOS of 1.239 which is higher than that of Example 1. This is obviously reasonable. The common tangent element set corresponding to critical strength line is shown in Fig. 12. It is these elements' common tangent line that defines the reduced strength line. The number of elements in this set is larger than that of Example 1. The greatest element FOS in this common tangent element set is 1.365. From Eqs. (7)−(8), the two strength reduction factors are respectively 1.067 and 2.985. Update the Mohr-Coulomb strength criterion with this reduced strength line, conduct a new FLAC solving, and the slope reaches a global failure whose yield elements are shown in Fig. 13. The maximum shear strain is used to identify the potential slide surface, as shown in Fig. 14.

The O-SRM is also used to evaluate the stability of this slope. The O-SRM based FOS is 1.710 which is 38% greater than the first FOS from D-SRM. The yield elements and the potential slide surface of globally critical failure by O-RSM are respectively shown in Figs. 15 and 16. Comparing Figs. 14 and 16, we find that D-SRM predicts 495 yield elements in the potential slide mass while O-SRM predicts 359 yield elements in the potential slide mass. It again states that D-SRM predicts a larger potential slide range.

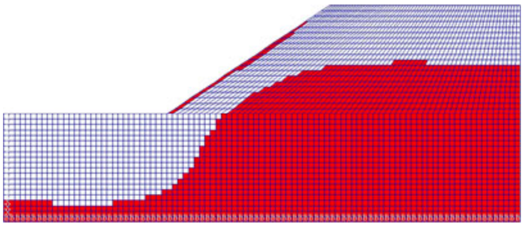

Fig. 12 Common tangent element set corresponding to critical strength line by D-RSM for Example 2

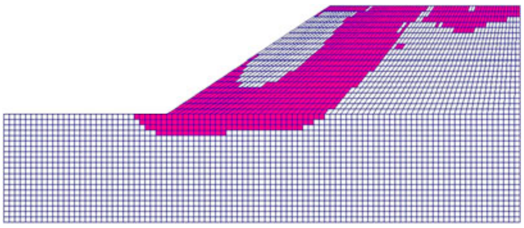

Fig. 13 Yield elements on globally critical failure by D-RSM for Example 2

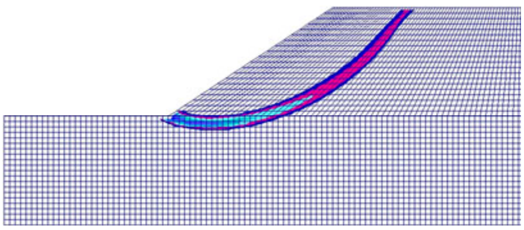

Fig. 14 Potential slide surface by D-SRM for Example 2

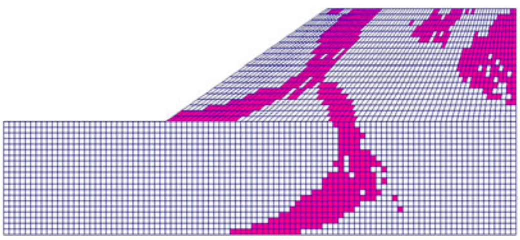

Fig. 15 Yield elements on globally critical failure by O-RSM for Example 2

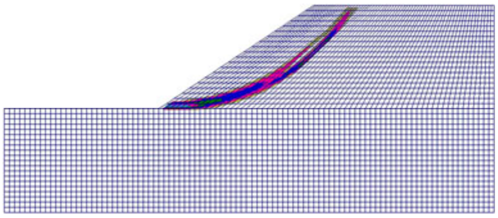

Fig. 16 Most dangerous slide zone by S-SRM for Example 2

The FOSs of 1.239 from D-SRM is 39% smaller than 1.723 from the Bishop method whose calculation result is shown in Fig. 17.

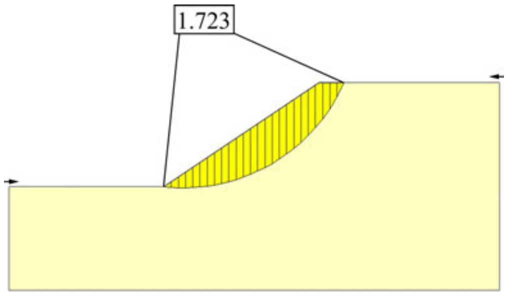

Fig. 17 Most dangerous slide line by LEM (Bishop Method, Example 2)

From the above two examples, the FOS from D-SRM is indeed smaller than that from O-SRM and that of LEM (Bishop method). Moreover, D-SRM predicts a larger potential slide range.

5 Discussion

There are several questions to be discussed. In the previously developed double reduction method, a definition of FOS is additionally needed. This is different from the original strength reduction method in which the only reduction factor is automatically treated as the FOS. This means that the definition of FOS is not unique. This problem may be solved by developing and comparing kinds of definitions of FOS through lots of utilization experience. As the searching process is based on given mesh, this may lead to the mesh dependent problem of searched critical strength line and hence affect the value of FOS.

Compared with traditional calculation process of O-SRM, in D-SRM, an initialization process of slope geo-stress is added before strength reduction calculations. If the FOS of a slope is larger than 1.0, then the elasto-plastic model or pure elastic may be both available for geo-stress initialization. But for slope with FOS smaller than 1.0, if elasto-plastic mode were used to initialize the slope, the slope would fail before subsequent strength reduction calculations and hence no FOS can be acquired. Hence, for this situation, maybe only pure elastic model would be available for geo-stress initialization for O-SRM. Moreover, the common tangent element set searching process in D-SRM will increase the computational time, which may be the main shortcoming of D-SRM.

Through all this work, only the shear strength was mentioned. However, tensile failure mode or tensile-shear mixed failure mode usually exist in engineering practices. For these kinds of failure modes, the specific reduction technique and the definition of FOSs may be different from that of this work and needs further research. Moreover, some other research is advised to be focused on further as follows:

1) A more comprehensive comparisons between D-GSM and other FOS calculations methods.

2) D-SRM method considering tensile-shear combined failure criterion.

3) Nonlinear failure criterion based D-SRM.

4) D-SRM based on other criteria other than Mohr-Coulomb failure criterion.

5) D-SRM on multi-slide surfaces, and extending the D-SRM to three dimensional cases.

6 Conclusions

1) It is believed that the essential of strength reduction method (SRM) is to find a critical strength curve that happens to make the slope globally fail and to determine a definition of factor of safety (FOS). Based on this understanding to SRM. A new double reduction method, which includes a detailed calculation flowchart and a definition of FOS for slope stability was developed. When constructing the new definition of FOS, efforts were made to make sure that it has concise physical meanings and can fully reflect the shear strength of the slope.

2) Two examples, slopes A and B with the slope angles of 63°and 34° respectively, were given to verify the method. These results show that, for these two slopes, D-SRM gives larger FOS than that from O-SRM and Bishop method. And the smaller the slope angle is, the bigger the difference is. It was also found that the double reduction method predicts a deeper potential slide line and a larger slide mass. This means more support measures may be required if this method is adopted.

3) The double reduction method is comparative and reasonable to the traditional methods, and on the other hand, the original strength reduction method may overestimate the safety of a slope. The method presented is advised to be considered as an additional option in the practical slope stability evaluations although more experience is still to be accumulated.

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