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Back analysis of general slope under earthquake forces using upper bound theorem

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Abstract: Long time monitoring is acquired to obtain the displacement data for displacement-based geotechnical material back analysis, and these data are hard to be measured under some special condition, such as earthquake. For a simple homogeneous slope, the position of a critical failure surface is determined by value of $c/\tan\phi$. Utilizing upper bound theorem of limit analysis, the external work rate and internal energy for normal slope under earthquake forces are given, and the formula for minimum safety factor is derived. On this basis, the equation of slip surface and the surface depth of a given position are solved. In this way, the strength parameter can be analyzed by known slip surface depth. For practical use, the surface depth for a given slope under varying strength parameter is presented. Finally, two examples are given to show its simplicity and effectiveness.

Key words: back analysis; limit analysis; critical slip surface; earthquake force

1 Introduction

General geotechnical engineering stability analysis techniques, such as limit equilibrium [1], numerical simulation [2], and limit analysis [3−5] were developed and proliferated to become powerful tools for geotechnical engineering design and construction procedure. They have, as in other engineering fields, been applied in these slope problems mostly to calculate the safe factor for design and construction purposes.

However, difficulties in using these methods were soon experienced by geotechnical engineers who tried to analyze the stability or to predict structures behaviors by limited or incomplete strength parameters. It is rightful, thus, that the focus is shifted towards finding ways to determine the missing parameters or those cannot be obtained by routine test. The procedure of using field measurements in order to obtain input material parameters is called back analysis technique [6−7].

Since this method was first proposed by KAVANAGH and CLOUGH [8], the deep development and wide usage, rapid advances in back analysis technology brought qualities of new approaches, by which engineers solved plenty of parameter obtained problems successfully. The new approaches of back analysis can be divided into two groups grandly: the inverse method and optimal method. The former, given by SAKURAI et al [9], is based on the system equations, by which the numerical solution of material parameters or loading condition can be derived by the observed displacements. However, as the equations are established on some impractical assumptions, the inverse method is difficult to apply to practical engineering [10−12]. Another popular method in back analysis is optimization method, in which, the sum of error square between calculated displacements and observed ones is often treated as the optimization objective. The system equations here are only used as constraint conditions and free of converse illation, so the optimization method is more applicable for practice. Extensive studies have been conducted to develop different models of displacement-based back analysis. What's more, some back analyses also have been utilized based on field measurements of strains and stresses.

Some slopes are instable under earthquake. For these slope failures, the conventional method for evaluating the effect of an earthquake on the slope stability is the so-called pseudo-static method. Due to the abruptness of the earthquake, in most seismic slope cases, the field measurements (displacement, strain and stress) are unavailable. Lock of these data increases the difficultness of back analysis. Consequently, a question arising in practice is how to determine the strength parameters of the slope under earthquake forces. Furthermore, the multi-step slope is generally applied in

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practice. However, the stability and failure research of such slope under earthquake is too few to satisfy the demands of construction [13−17].

In fact, compared with the displacement, strain and stress, the location of slip surface is easy to measure. In the present work, a new back analysis method is proposed for the two-step soil slope under earthquake forces based on the slip surface depth, which is obtained by limit analysis. Earthquake forces, regarded as external forces, are calculated using a seismic coefficient. In order to see the validity of the present approach, back analysis result is compared with the conventional result.

2 Relationship between slip surface and strength parameters

The factor of safety (F) of slope engineering is defined as a ratio of the actual shear strength of the soil to the shear strength making the slope into the limit state. In Mohr-Coulomb failure criterion, the soil strength is described as two parameters: the cohesion c and internal friction φ. Thus, F can be expressed as

$$F=\frac{c}{c'}=\frac{\tan\phi}{\tan\phi'} \tag{1}$$

where c is the actual cohesion, ϕ is the internal friction angle, and c' and $\tan\phi'$ are shear strength making the slope into the limit state.

For a homogeneous slope with a certain geometry, unit weight and pore water pressure distribution, the location of a critical slip surface is related only to $c'/\tan\phi'$. In order to demonstrate this, it is assumed that the soil strengthen parameters of a simple homogeneous slope are c_0 and $\tan\phi_0$, and the minimum safety factor F_0 is given. When the strength reduction method is applied, the strength parameter is changed to

$$c_1=c_0/F_0 \tag{2}$$

$$\phi_1=\tan^{-1}(\tan\phi_0/F_0) \tag{3}$$

The slope is in the limit state, denoted as state A. The slope slip surface under c_1, $\tan\phi_1$, is the critical slip surface. If the initial shear strength is (c'_0, ϕ'_0), there are

$$c'_0=\omega c_0 \tag{4}$$

$$\phi'_0=\tan^{-1}(\omega\tan\phi_0) \tag{5}$$

Thus, it can be obtained:

$$\frac{c_0}{\tan\phi_0}=\frac{c'_0}{\tan\phi'_0} \tag{6}$$

The reduced shear strength also is

$$c_1=c'_0/\omega F_0 \tag{7}$$

$$\phi_1=\tan^{-1}(\tan\phi'_0/\omega F_0) \tag{8}$$

The slope is in the limit state too, denoted as state B. In states A and B, the slope geometry, unit weight, and pore water pressure distribution are the same, so the location of a critical slip surface remains at same position.

Noting the fact that $c_0/\tan\phi_0=c'_0/\tan\phi'_0$, it is concluded that if other conditions except strength parameter are the same, the position of slip surface depends only on the magnitude of $c_0/\tan\phi_0$ of that slope because of the same reduced strength parameter, but with different safety factor. For a certain slope, if the location of slip surface is given, $c/\tan\phi$ can be also determined. For convenience, the following dimensionless parameter for $c/\tan\phi$ is introduced:

$$\lambda_{c,\phi}=\frac{c}{\gamma H\tan\phi} \tag{9}$$

where γ is unit weight of soil, and H is reference height, which is taken as vertical distance between two end points of a given failure surface.

The critical slip surface under different $\lambda_{c,\phi}$ is shown in Fig. 1. Same $\lambda_{c,\phi}$ makes the same critical slip surface, and the slip surface becomes deeper as the magnitude of $\lambda_{c,\phi}$ increases.

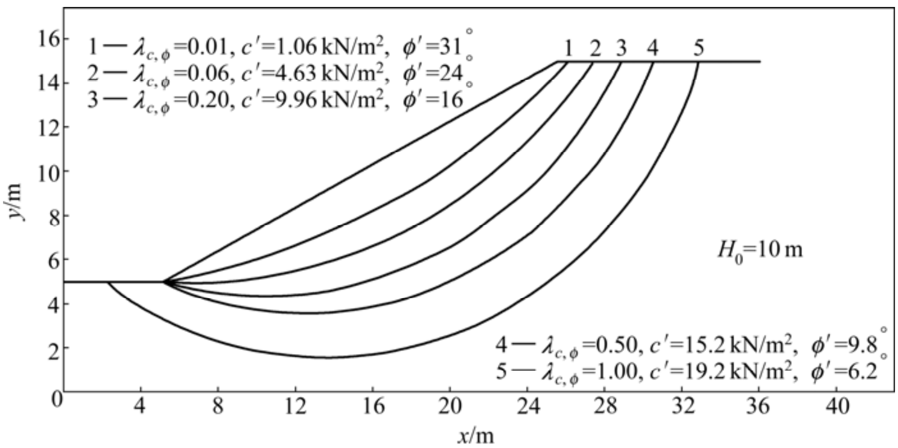


Fig. 1 Critical slip surface with different $\lambda_{c,\phi}$ in simple slope

3 General slope safety factor under earthquake forces

The pseudo-static method is generally applied in solving the safety factor of slope under earthquake forces, by which the dynamic effect of earthquake is considered as the horizontal and vertical static forces [18−20]. Although as an approximate method, the pseudo-static method performs well in earthen structure design and construction, the seismic stability of general slope is analyzed by this method in the present work.

An earthquake has two possible effects on the seismic stability of slope. One is to increase the driving forces, and the other is to decrease the shearing resistance of the soil. In the present analysis, only the increase of the driving forces is investigated under an earthquake, and the shearing strength is assumed to be unaffected. The horizontal and vertical driving forces are equivalent to static forces acting on the rigid body in the pseudo-static method, expressed as coefficients k_h and k_v. In conventional research of geotechnical structure seismic stability, the dynamic effect in vertical direction is less considered based on the cognition that the two direction accelerations cannot attain the peak simultaneously. What's more, the maximum vertical acceleration effect is only 40%−50% that of the horizontal one [21−23], so only the horizontal seismic coefficient k_h is considered; the vertical seismic coefficient is disregarded.

Combining the strength reduction method and the upper bound theorem, slope safety factor F_s, being the evaluation index for slope stability, can be solved. After F_s is determined, the velocity discontinuity line, obtained with reduction strength soil, is critical slip surface of the slope. For the velocity discontinuity line is closed to logarithm-spiral [24−25], we employ the rotational log-spiral discontinuity mechanism for the present analysis.

Utilizing the linear Mohr-Coulomb failure criterion, the rotational log-spiral discontinuity mechanism is employed for the present analysis, as shown in Fig. 2. The region *OBAO* rotates as a rigid body about the centre of rotation *O*, with the material below the velocity discontinuity line remaining at rest. It will lead to a limit load or stability factor which is not less than or equal to the actual one, if the energy dissipation rate along the velocity discontinuity line is equal to the work rate of the external forces in any kinematically admissible velocity [26−30].

For the homogeneous soil slope, the external rate of work is done by the soil weight *W* bounded by the boundary line *BB'CA'A* and the sliding surface, the surcharge *q* on the top surface and earthquake forces expressed as the inertia forces. Thus, the rate of work due

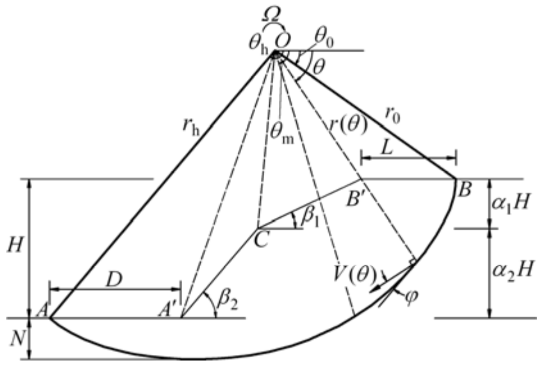


Fig. 2 Failure mechanism for a homogeneous slope

to the soil weight *W* and the horizontal inertia force k_h can be expressed as

$$W_{soil}=\gamma r_0^3\Omega[f_1-f_2-f_3-f_4-f_5]+\gamma r_0^3\Omega k_h[\bar{f}_1-\bar{f}_2-\bar{f}_3-\bar{f}_4-\bar{f}_5] \quad (10)$$

where k_h is the horizontal seismic coefficient, defined as the ratio of the horizontal inertia force k_hW to soil weight W above the logarithmic spiral surface, γ is the total unit self-weight of the soil, r_0 is the initial radius of the log-spiral, and Ω is the angular velocity. The expressions f_1–f_5 are the rates of work due to the rock weight W, which can be expressed as

$$f_1=\{(3\tan\phi\cos\theta_h+\sin\theta_h)\exp[3(\theta_h-\theta_0)\tan\phi]-(3\tan\phi\cos\theta_0+\sin\theta_0)\}/3(1+9\tan^2\phi) \quad (11)$$

$$f_2=\frac{1}{6}(L/r_0)\sin\theta_0(2\cos\theta_0-L/r_0) \quad (12)$$

$$f_3=\frac{\alpha_1}{3}(\frac{H}{r_0})[\cos^2\theta_0+\frac{L}{r_0}(\frac{L}{r_0}-2\cos\theta_0)+\sin\theta_0\cot\beta_1(\cot\theta_0-\frac{L}{r_0})-\frac{\alpha_1}{2}\frac{H}{r_0}\cot\beta_1(\cos\theta_0+\frac{L}{r_0}-\sin\theta_0\cot\beta_1)] \quad (13)$$

$$f_4=\frac{\alpha_2}{3}(\frac{H}{r_0})\{(\cos^2\theta_h+\sin\theta_h\cos\theta_h\cot\beta_2)\cdot\exp[2(\theta_h-\theta_0)\tan\phi]+(2\frac{D}{r_0}\cos\theta_h+\frac{D}{r_0}\sin\theta_h\cot\beta_2+\frac{\alpha_2}{2}\frac{H}{r_0}\cos\theta_h\cot\beta_2+\frac{\alpha_2}{2}\frac{H}{r_0}\sin\theta_h\cot^2\beta_2)\cdot\exp[(\theta_h-\theta_0)\tan\phi]+(\frac{D}{r_0})^2\} \quad (14)$$

$$f_5=\frac{1}{6}(\frac{D}{r_0})\sin\theta_h\{2\cos\theta_h\exp[(\theta_h-\theta_0)\tan\phi]+\frac{D}{r_0}\}\cdot\exp[(\theta_h-\theta_0)\tan\phi] \quad (15)$$

$$\frac{H}{r_0}=\sin\theta_h\exp[(\theta_h-\theta_0)\tan\varphi]-\sin\theta_0 \quad (16)$$

$$\frac{L}{r_0}=\cos\theta_0-\cos\theta_h\exp[(\theta_h-\theta_0)\tan\phi]-\frac{D}{r_0}-(\frac{H}{r_0})(\alpha_1\cot\beta_1+\alpha_2\cot\beta_2) \tag{17}$$

The rate of work due to the surcharge q and horizontal inertia force k_hq can be expressed as

$$Q_q=qr_0^2\Omega f_6+qr_0^2\Omega k_h f_7 \tag{18}$$

$$f_6=\frac{1}{2}\frac{L}{r_0}(2\cos\theta-\frac{L}{r_0}) \tag{19}$$

$$f_7=\frac{L}{r_0}\sin\theta \tag{20}$$

where q is the applied vertical surcharge. The coefficients related to the rate of work due to the horizontal inertia force k_hW can be expressed as

$$\bar{f}_1=\{(3\tan\phi\sin\theta_h-\cos\theta_h)\exp[3(\theta_h-\theta_0)\tan\phi]-(3\tan\phi\cos\theta_0-\cos\theta_0)\}/3(1+9\tan^2\phi) \tag{21}$$

$$\bar{f}_2=\frac{1}{3}\frac{L}{r_0}\sin^2\theta_0 \tag{22}$$

$$\bar{f}_3=\frac{\alpha_1}{3}(\frac{H}{r_0})[\cos\theta_0\sin\theta_0+\sin^2\theta_0\cot\beta_1+\frac{\alpha_1}{2}\frac{H}{r_0}(\cos\theta_0+\sin\theta_0\cot\beta-\frac{L}{r_0})-\frac{L}{r_0}\sin\theta_0) \tag{23}$$

$$\bar{f}_4=\frac{\alpha_2}{3}(\frac{H}{r_0})\{\exp[2(\theta_h-\theta_0)\tan\phi](\cos\theta_h\sin\theta_h+\sin^2\theta_h\cot\beta_2)-(\frac{\alpha_2}{2}\frac{H}{r_0}\cos\theta_h+\frac{\alpha_2}{2}\frac{H}{r_0}\sin\theta_h\cot\beta_2-\frac{D}{r_0}\sin\theta_h)\exp[(\theta_h-\theta_0)\tan\phi]-\frac{\alpha_2}{2}(\frac{H}{r_0})(\frac{D}{r_0})\} \tag{24}$$

$$\bar{f}_5=\frac{1}{3}\frac{D}{r_0}\exp[2(\theta_h-\theta_0)\tan\phi]\sin^2\theta_h \tag{25}$$

For the rigid material considered, the internal energy is dissipated only along the sliding surface. The rate of energy dissipation can be expressed as

$$W_{int}=\frac{cr_0^2\Omega}{2\tan\phi}\{\exp[2(\theta_h-\theta_0)\tan\phi]-1\} \tag{26}$$

Equating the work rate of external forces to the internal energy dissipation rate, we obtain $W_{soil}+Q_q=W_{int}$. Substituting the expressions for W_{soil}, Q_q and W_{int} into this equation, we obtain

$$H=\frac{1}{2\gamma\tan\phi}\times\frac{c\{\exp[2(\theta_h-\theta_0)\tan\phi]-1\}-2\tan\phi(f_6+k_hf_7)q}{(f_1-f_2-f_3-f_4-f_5)+k_h(\bar{f}_1-\bar{f}_2-\bar{f}_3-\bar{f}_4-\bar{f}_5)}\times\{\sin\theta_h\exp[(\theta_h-\theta_0)\tan\phi]-\sin\theta_0\} \tag{27}$$

where location of the log-spiral is controlled by three parameters, θ_h, θ_0 and D, which are regarded as variables. The optimization method is often used to optimize the objective function Eq. (27) with respect to θ_h, θ_0 and D, to get a least upper bound for the critical height H_c of the inclined soil slope.

When the soil strength parameter changes to (c_f, $\tan\varphi_f$), the critical height H_c of soil equals actual height H, which bring the slope into limit state. The strength parameter (c_f, $\tan\varphi_f$) is obtained by

$$c_f=c/F_s \tag{28}$$

$$\tan\phi_f=(\tan\phi)/F_s \tag{29}$$

Substituting Eqs. (28) and (29) into Eq. (30) and making H_c=H, we obtain

$$F_s=c\{\exp[2(\theta_h-\theta_0)\tan\phi_f]-1\}-2\tan\varphi_f(f_6(\phi_f)+k_hf_7(\phi_f))q\times\frac{1}{2\gamma H\tan\phi_f}\times\{\sin\theta_h\exp[(\theta_h-\theta_0)\tan\phi_f]-\sin\theta_0\}\times\{1/(f_1(\phi_f)-f_2(\phi_f)-f_3(\phi_f)-f_4(\phi_f)-f_5(\phi_f))+k_h(\bar{f}_1(\phi_f)-\bar{f}_2(\phi_f)-\bar{f}_3(\phi_f)-\bar{f}_4(\phi_f)-\bar{f}_5(\phi_f))\} \tag{30}$$

The extreme value of F_s is minimum upper bound solution of slope safety factor. The problem actually is mathematical programming as follows:

$$\min\quad F_s=F_s(\theta_0,\theta_h,D/r_0)$$

$$\text{s.t}\quad\begin{cases}0<\theta_0<\frac{\pi}{2},\ \theta_0<\theta_h<\pi,\\ \frac{H}{r_0}>0,\ \frac{L}{r_0}>0\\ f_1-f_2-f_3-f_4-f_5>0\\ \frac{D}{r_0}\geq 0\end{cases}$$

When parameters θ_0, θ_h and D are obtained by optimization algorithm, the slip surface of slope in limit state can be determined.

4 Back analysis based on slip surface

As mentioned above, the dimensionless parameter $\lambda_{c,\phi}$ determines the location of potential slip surface. It has been shown that the relations between c and φ can be identified from slips in homogeneous slope by considering the condition: the theoretical critical slip surface is consistent with the actual one. A straightforward back analysis technique that also meets this condition is presented. In this method, the magnitude of $c/\tan\phi$ or $\lambda_{c,\phi}$ can be solved by the location of failure surface, namely the slip surface depth of a known position r_0 can be determined by

$$r = r_0 \exp[(\theta - \theta_0)\tan\phi] \tag{31}$$

The above formula in Cartesian coordinate system is

$$\sqrt{(y-y_0)^2 + (x-x_0)^2} = r_0 \exp[-\arctan(\frac{y-y_0}{x-x_0}) - \theta_0]\tan\phi \tag{32}$$

where x_0 and y_0 are the coordinates of logarithm-spiral original point and determined by the following formula:

$$x_0 = \frac{\alpha_1 H}{\tan\beta_1} + \frac{\alpha_2 H}{\tan\beta_2} + L - r_0\cos\theta_0 \tag{33}$$

$$y_0 = H + r_0 \sin\theta_0 \tag{34}$$

Substituting Eqs. (33) and (34) into Eq. (32), when x coordinate of a position is given, the magnitude of y, namely slip surface depth, can also be gained as shown in Fig. 3.

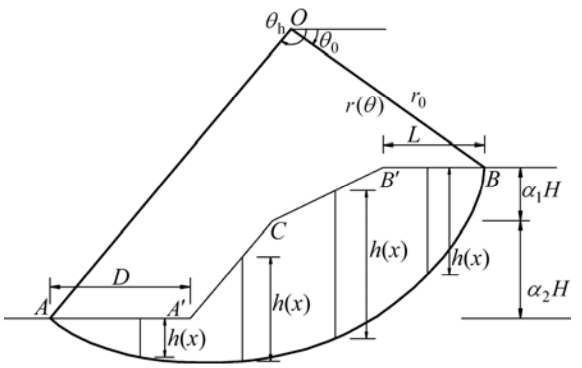


Fig. 3 Depth of potential sliding surface at different positions

$$h(x) = \begin{cases} H - y, \\ \quad \dfrac{\alpha_1 H}{\tan\beta_1} + \dfrac{\alpha_2 H}{\tan\beta_2} \le x < \dfrac{\alpha_1 H}{\tan\beta_1} + \dfrac{\alpha_2 H}{\tan\beta_2} + L \\ \tan\beta_2 (x - \dfrac{\alpha_1 H}{\tan\beta_1}) - y, \\ \quad \dfrac{\alpha_2 H}{\tan\beta_2} \le x < \dfrac{\alpha_1 H}{\tan\beta_1} + \dfrac{\alpha_2 H}{\tan\beta_2} \\ x\tan\beta_2 - y, \ 0 \le x < \dfrac{\alpha_2 H}{\tan\beta_2} \\ -y, \ -D \le x < 0 \end{cases}$$

When the geometry, unit weight and pore water pressure distribution of a homogeneous slope are given, the location of slip surface will be determined by the magnitudes of $\lambda_{c,\phi}$. Thus, the slip surface depth can be obtained by the equation of logarithm-spiral. In this way, the relationship between slip surface and $\lambda_{c,\phi}$ is built, confirming the possibility of back analysis by slip surface depth.

For practice use in geotechnical engineering, the magnitudes of $\lambda_{c,\phi}$ and slip surface depth at slope crest of different β_1 and β_2 are presented in Table 1 and Table 2, with the parameter q=0 kN/m^2, γ=20 kN/m^3, H=20 m, $\alpha_1 H/\tan\beta_1=\alpha_2 H/\tan\beta_2$, and k_h varying from 0 to 0.20.

Table 1 Slip surface depth at slope changing points for β_2=80° (Unit: m)

k_h	β_1/(°)	$\lambda_{c,\phi}$				
		0.1	0.3	0.5	0.7	1
0	10	13.30	15.60	16.29	16.93	17.62
	20	13.74	15.81	16.51	17.19	17.83
	30	14.10	15.97	16.75	17.30	18.13
	40	14.36	15.95	16.94	17.47	18.32
	50	14.67	16.22	17.13	17.68	18.68
0.05	10	13.97	16.04	16.66	17.24	17.86
	20	14.36	16.23	16.86	17.47	18.05
	30	14.69	16.37	17.08	17.57	18.32
	40	14.93	16.35	17.25	17.73	18.49
	50	15.20	16.60	17.42	17.92	18.82
0.10	10	14.57	16.44	17.00	17.51	18.07
	20	14.93	16.61	17.17	17.73	18.24
	30	15.22	16.73	17.37	17.81	18.49
	40	15.43	16.72	17.52	17.95	18.64
	50	15.68	16.94	17.68	18.12	18.93
0.15	10	15.12	16.80	17.30	17.76	18.27
	20	15.43	16.95	17.46	17.95	18.42
	30	15.70	17.06	17.63	18.03	18.64
	40	15.89	17.04	17.77	18.16	18.78
	50	16.11	17.24	17.91	18.31	19.04
0.20	10	15.61	17.12	17.57	17.98	18.44
	20	15.89	17.25	17.71	18.16	18.58
	30	16.13	17.35	17.87	18.23	18.77
	40	16.30	17.34	17.99	18.34	18.90
	50	16.50	17.52	18.12	18.48	19.14

Based on the field measurements, the magnitudes of $\lambda_{c,\phi}$ can be determined by the slip surface depth, also the relationship between c and φ. For the concrete value of them, another relationship is needed often by the two following methods:

1) Assuming the magnitudes of one parameter, the other can be determined. The assumption of the parameter, with the less influence on the safety factor or easily determined by engineering experience, is made in most cases (Fig.4(a)).

2) The safety factor of failure section is supposed to be 1. Thus, the relationship of the two parameters with F_s=1 is obtained. Based on the curve of c−φ and the magnitudes of $\lambda_{c,\phi}$, the back analysis result can be gained, as shown in Fig. 4(b).

Table 2 Slip surface depth at slope changing points for β_1=60° (Unit: m)

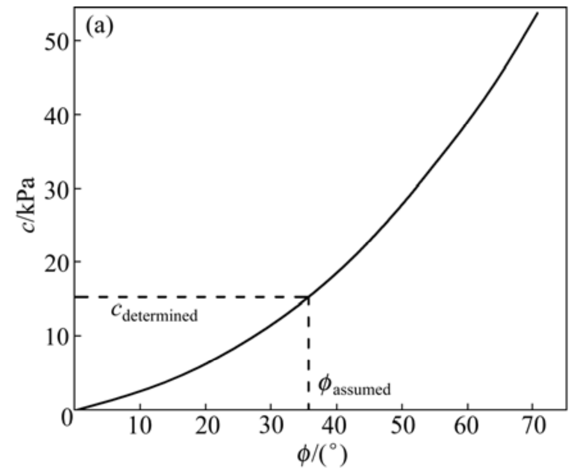
k_h	β_2/(°)	$\lambda_{c,\phi}$ 0.1	0.3	0.5	0.7	1
0	10	0.01	0.52	5.95	7.62	10.75
	20	0.90	2.47	9.50	12.00	10.00
	30	4.32	7.74	10.24	12.81	13.34
	40	5.66	9.80	11.75	13.15	14.95
	50	8.71	13.30	14.52	15.70	16.02
0.05	10	1.61	2.08	7.07	8.61	11.49
	20	2.43	3.88	10.34	12.64	10.80
	30	5.58	8.72	11.02	13.39	13.88
	40	6.80	10.62	12.41	13.70	15.35
	50	9.62	13.83	14.96	16.04	16.34
0.10	10	3.08	3.51	8.11	9.52	12.17
	20	3.84	5.17	11.11	13.23	11.54
	30	6.73	9.62	11.74	13.92	14.37
	40	7.86	11.37	13.02	14.21	15.72
	50	10.45	14.33	15.36	16.36	16.63
0.15	10	4.44	4.83	9.06	10.36	12.79
	20	5.13	6.35	11.82	13.77	12.22
	30	7.79	10.45	12.40	14.40	14.82
	40	8.83	12.06	13.58	14.67	16.06
	50	11.21	14.78	15.73	16.65	16.90
0.20	10	5.68	6.04	9.93	11.13	13.37
	20	6.32	7.44	12.48	14.27	12.84
	30	8.77	11.22	13.01	14.85	15.23
	40	9.73	12.69	14.09	15.10	16.38
	50	11.91	15.20	16.07	16.92	17.15

5 Comparisons

5.1 Example 1

Considering the earthquake forces, WANG et al [31] conducted numerical simulation to explore the process and mechanism of formation of the sliding surface using dynamic-strength-reduction method, with other quantified information. The sliding surface solved by pseudo-static method is shown in Fig. 5. The unit weight of the slope is 20.0 kN/m^3. Utilizing the method in presented work, the magnitude of $\lambda_{c,\phi}$ is calculated by the depth of slip surface in the middle point of the slope. The calculating parameter in back analysis is shown in Table 3, and the variable D is fixed to be zero with the velocity discontinuity surfaces by the slope toe.

From the calculation results, it is found that the magnitude of $c/\tan\phi$ is 0.29, which is similar with the



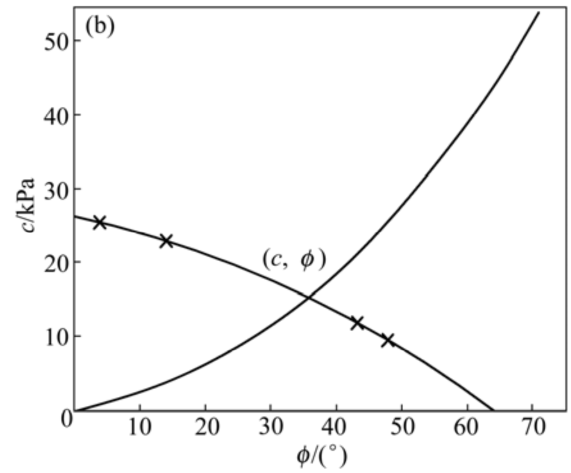


Fig. 4 Basic back analysis approaches applied for slope: (a) Assumed method; (b) Safety factor method

magnitude of $c/\tan\phi$=0.27 (c=40 kPa, ϕ =20°) obtained by the provided strength parameter. The error is about 6%.

5.2 Example 2

DENG and LI [32] proposed a searching method to determine the most probable slip surface under earthquake forces. In this work, based on limit equilibrium method, considering the effect of earthquake forces, the seismic safety factor and critical slip surface of soil slope are solved under the horizontal and vertical earthquake forces. Figure 6 shows the failure surface with different k_h.

Appling the method that the present research proposed, data of failure surface are used for strength parameter back analysis, thus the calculating parameters are shown in Table 4.

The $\lambda_{c,\phi}$ back analyzed by the slip surface with different k_h is shown in Table 5. The maximum error is less than 7.5%, indicating that the present technique is an effective technique for evaluating strength parameter of soil slope utilizing the slip surface under earthquake forces.

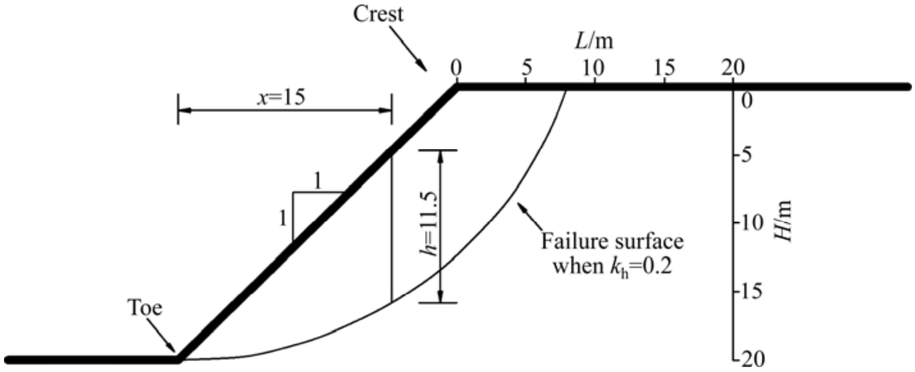


Fig. 5 Critical slip surface of soil slope when k_h=0.2 [31]

Table 3 Calculating parameters in Example 1

Parameter	α_1	α_2	H/m	β_1/(°)	β_2/(°)	q/(kN·m^{-2})	D/m	k_h
Value	0.5	0.5	20	45	45	0	0	0.2

Table 4 Calculating parameters in Example 2

Parameter	α_1	α_2	H/m	β_1/(°)	β_2/(°)	q/(kN·m^{-2})	D/m
Value	0.5	0.5	20	30	30	0	0

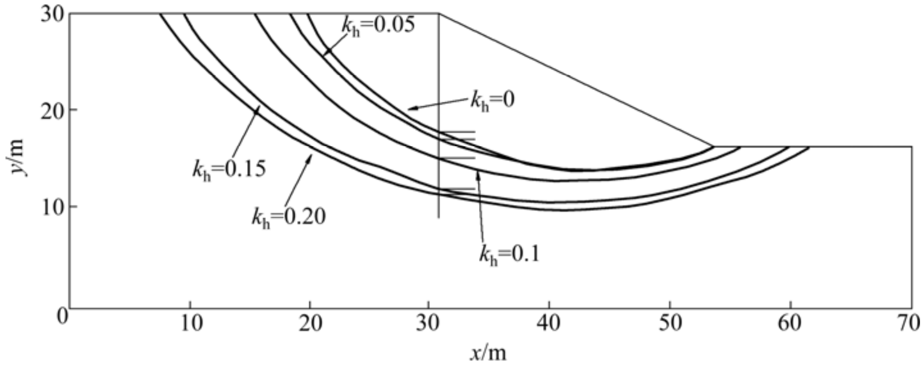


Fig. 6 Seismic critical slip surface under different k_h by DENG

Table 5 $\lambda_{c,\phi}$ and errors by back analysis

k_h	Slip surface depth/m	$\lambda_{c,\phi}$	Error/%
0	7.25	0.516	2.38
0.05	7.74	0.531	5.36
0.10	8.95	0.489	2.98
0.15	10.74	0.479	4.96
0.20	11.21	0.467	7.34

6 Conclusions

1) In Mohr-Coulomb criterion, the formula of critical slip surface is determined by the magnitude of $c/\tan\phi$, while not by c or $\tan\phi$ respectively, when other conditions are determined. For conveniency, the dimensionless parameter $\lambda_{c,\phi}=c/\gamma H\tan\phi$ is introduced and the magnitude of $\lambda_{c,\phi}$ is back analyzed by the depth of slip surface.

2) The work rate of external forces and the internal energy of the two-steps slope under earthquake forces based on the upper bound theorem is solved and the formula is built for slope safety factor as well.

3) The formula velocity discontinuity line, namely the critical slip surface, is solved by optimization. On this basis, the slip surface depth of different positions is obtained. What's more, the magnitude of $\lambda_{c,\phi}$ and slip surface depth with different β_1, β_2 and k_h is listed.

4) By the slip surface depth of a given position, the magnitude of $\lambda_{c,\phi}$ is back analyzed in two examples by the presented method. The results coincide well with the model strength parameters, indicating the effectiveness of this method.

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