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Determination of minimum cover depth for shallow tunnel subjected to water pressure

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Abstract: Prediction of the state of roof collapse is a big challenge in tunnel engineering, while the limit analysis theory makes it possible to derive the analytical solutions of the collapse mechanisms. In this work, an exact solution of collapsing shape in shallow underwater tunnel is obtained by using the variation principle and the upper bound theorem based on nonlinear failure criterion. Numerical results under the effect of river water and supporting pressure are derived and discussed. The maximum water depth above the river bottom surface is determined under a given buried depth of shallow cavities and the critical depth of roof collapse is obtained under a constant river depth. In comparison with the previous results, the present solution shows a good agreement with the practical results.

Key words: shallow underwater subway; roof collapse; nonlinear failure criterion

1 Introduction

It remains a big challenge to predict the potential collapse of a cavity roof correctly in geotechnical and civil engineering. Considering the random variability of the mechanical properties of the rock both in situ and from the presence of cracks and fractures in the rock mass, it is presented as a rather complex problem. If more factors are taken into consideration, i.e., the effect of river water, temperature, adjacent load as well as coupled field, it will become more and more complicated. Though it sounds very intricate, more and more scholars have devoted themselves to researching the stability of cavity, tunnel face, and other similar engineering since limit analysis theory was applied to derive the lower and upper bound solutions by DAVIS et al [1]. Meanwhile, after comparing with limit equilibrium method and slice method, it can be found that the solutions obtained from limit analysis are more rigorous and there is no assumption made on considering the corresponding forces in the work [2−5]. Therefore, limit analysis method has been widely used to analyze the stability and limit state of cavities excavated both in deep and shallow strata based on its advantages.

At present, the limit analysis method is often taken as a supplementary method in geotechnical engineering and it has been mainly used to evaluate the stability of tunnel face excavated in shallow strata with linear Mohr–Coulomb criterion. However, attention should be highly paid to the collapse mechanism of tunnel that it is a complicated nonlinear evolution process and the characteristic properties of rock and soil material are also nonlinear which have been investigated in many tests. Consequently, the nonlinear failure criterion, such as Hoek–Brown failure criterion, and Power-law criterion, is the better choice in solving the problem of roof collapse for shallow-buried tunnels. Due to the advantage of Hoek–Brown failure criterion over reflecting the actual situation, it has been widely applied into geotechnical engineering [6−10]. MERIFIELD et al [11] obtained the ultimate bearing capacity of a surface footing on a rock mass with limit analysis theorems according to the generalized Hoek–Brown failure criterion, and the results of the bearing capacities show that the upper bound solutions are the same with the lower ones, which means that they are the real solutions. For the purpose of discussing the potential roof collapse of a deep rectangular cavity and tunnel with arbitrary sections, on the basis of Hoek–Brown failure criterion

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and the upper bound theorem of limit analysis, FRALDI and GUARRACINO [12−13] calculated the exact solution of detaching profile.

It is known to all that water as well as other fluid often has an effect on the stability of underwater structure to varying degrees, so it is of great significance to study its failure mechanism and corresponding control method. In previous study, HUANG et al [14] used the conformal mapping of the complex variable methods to calculate the analytical solutions of steady ground water flowing into a horizontal tunnel. FENG et al [15] investigated the effect of surrounding rock deterioration on segmental lining structure for underwater shield tunnel with large cross-section by means of model test and the result shows that the crown and bottom of tunnel are liable to collapse. WANG et al [16] proposed the theoretical and experimental study of external water pressure on tunnel lining in controlled drainage under high water level and the results indicate that there exists an optimum size for grouting zone to exert supporting pressure. According to the study done by above authors, the effect of water must be taken into account and it should be incorporated into the expression of upper bound theorem when using the method of limit analysis. Meanwhile, supporting structure which can provide supporting pressure should also be used reasonably to maintain the cavity being stable, especially for shallow-buried cavity and tunnel.

2 Upper bound theorem

According to the upper bound theorem, the load obtained by equating the rate of the energy dissipation to the external rate of work in any kinematically admissible velocity field is no less than the actual collapse load when the velocity boundary condition is met [17]. In this work, in order to take the effects of river water into consideration in the realm of the upper bound theorem of limit analysis for shallow underwater cavity, an assumption that the effect of river water is equal to the water pressure acting on the detaching curve is made, that is, the effect of water pressure can be seen as surface forces. Moreover, the supporting pressure is also regarded as surface forces acting on lining or other supporting structure, thus the upper bound theorem can be expressed as

$$\int_{\Omega}\sigma_{ij}\,\dot{\varepsilon}_{ij}\mathrm{d}\Omega \ge \int_{s}T_i\,v_i\mathrm{d}s+\int_{\Omega}X_i\,v_i\mathrm{d}\Omega \tag{1}$$

where σ_{ij} and ε_{ij} are the stress and strain rate in the kinematically admissible velocity field respectively; T_i means surface forces on boundary s, X_i is the body force; Ω represents the volume of the collapse mechanism; v_i is the velocity along the velocity discontinuity surface.

In this work, some other assumptions should be made when using the upper bound theorem to analyze the stability problem in geotechnical engineering:

1) The material is perfectly plastic and follows an associated flow rule;

2) The blocks bounded by the detaching curve and boundary surface are regarded as rigid materials [18−20];

3) The soil or rock mass is totally impermeable, i.e., rigid body;

4) The river bottom surface is horizontal.

3 Curved failure mode of roof collapse

In the upper bound theorem of limit analysis, it is of great significance to construct a kinematically admissible failure mechanism. Considering the actual mechanical characteristics of rock mass over the roof of a shallow-buried cavity and that the river water doesn't flow into the cavity so as to make sure that the bottom of river is stable, certainly choosing a curved velocity discontinuity line, $f(x)$, to describe the detaching surface is made coincident with the reality well, as shown in Fig. 1. Due to the presence of velocity discontinuity surface, the plastic flow occurs along the detaching curve. Therefore, on the basis of associated flow rule and Hoek–Brown failure criterion, the energy dissipation rate along the velocity discontinuity surface can be computed. Then through equating the energy dissipation rate to the rate of external work, the virtual work equation meeting the velocity boundary condition is obtained. Moreover, for the purpose of deriving the exact shape of the collapsing block in a limit state, it is necessary to use the variational calculation method to minimize the objective function.

4 Upper limit analysis

Hoek–Brown failure criterion is widely used with two forms of expressions which are represented by major and minor principal stresses and normal and shear stresses, respectively [21]. Considering the fact that the energy dissipation along the detaching curve is caused by normal and shear stresses, the form of the later is of great convenience, i.e.,

$$\tau = A\sigma_{\mathrm{c}}\left[(\sigma_{\mathrm{n}}+\sigma_{\mathrm{t}})\sigma_{\mathrm{c}}^{-1}\right]^{B} \tag{2}$$

where σ_{n} means the normal stress; τ is the shear stress; A and B are physical parameters characterizing the soil or rock mass; σ_{c} and σ_{t} represent the uniaxial compressive strength and the tensile strength of the soil or rock mass, respectively.

Based on the Hoek–Brown failure criterion expressed in terms of normal and shear stresses and associated flow rule, the normal and shear stresses and

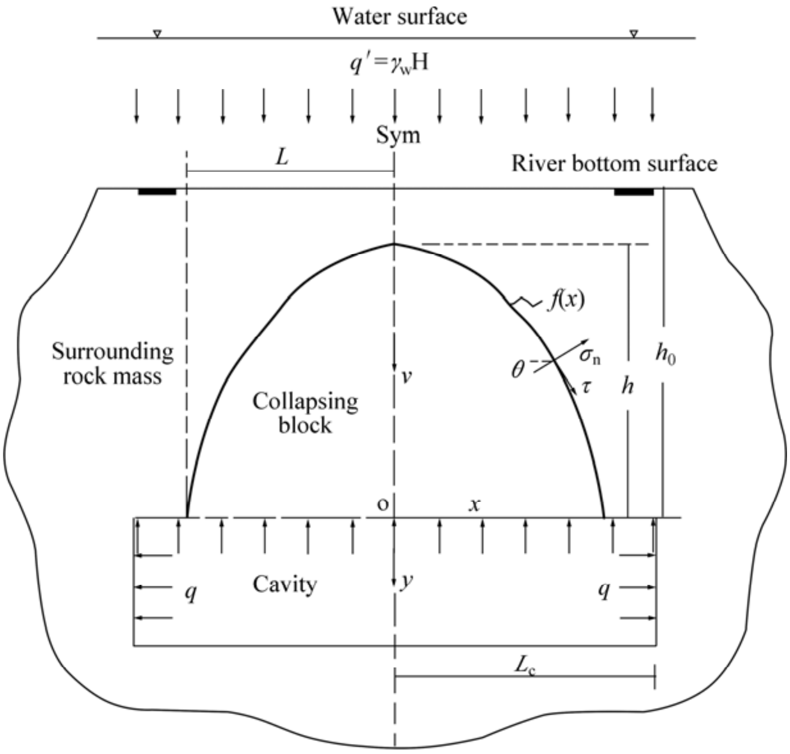

Fig. 1 Dormant collapsing mechanism for rectangular underwater cavity

strain on the detaching curve can be worked out immediately. Therefore, the energy dissipation rate determined by the internal forces on the detaching curve can be expressed as [12]

$$D_i=\sigma_n\dot{\varepsilon}_n+\tau_n\gamma_n=\left\{\left[-\sigma_t+\sigma_c(AB)^{1/(1-B)}(1-B^{-1})f'(x)^{1/(1-B)}\right]/\left[t\sqrt{1+f'(x)^2}\right]\right\}v \tag{3}$$

where ε_n and γ_n are normal and shear plastic strain rate, respectively, $f'(x)$ is the first derivative of $f(x)$, and t is the thickness of the detaching surface. Thus, the total energy dissipation rate determined by the internal forces along the detaching curve is

$$D=\int_s D_i\, t\mathrm{d}s=\int_0^L\left[-\sigma_t+\sigma_c(AB)^{1/(1-B)}(1-B^{-1})f'(x)^{1/(1-B)}\right]v\,\mathrm{d}x \tag{4}$$

where $\mathrm{d}s=\sqrt{1+f'(x)^2}\,\mathrm{d}x$ is the elementary length of the detaching curve, $f(x)$. Considering the symmetric characteristic of the collapsing block with regard to the y-axis, the corresponding calculation of the rate of work can choose a half and then the work rate of failure block produced by mass can be written as

$$P_\gamma=\int_0^L \gamma f(x)v\,\mathrm{d}x \tag{5}$$

where γ is the soil weight per volume, and L means the half width of the failure block. The power of supporting pressure of shallow cavity can be written as

$$P_q=-qLv \tag{6}$$

As the effect of river water is regarded as surface distribution force acting on the detaching curve, the power of water effect is expressed as

$$P_s=\int_s \gamma_w H f(x)v\,\mathrm{d}s=\gamma_w HLv \tag{7}$$

where γ_w presents the water mass per volume and H is the depth of river water above the river bottom surface.

Due to the result derived by virtue of virtual work equation which is one of the upper bound solutions rather than the real solution, it is necessary to find the solution most close to the real solution through optimization calculation method. Thus, an objective function which is made up of external rate of work and the internal energy rate of dissipation can be constructed as

$$\zeta[f(x),f'(x),x]=D-P_\gamma-P_q-P_s=\int_0^L\left[-\sigma_t+\sigma_c(AB)^{1/(1-B)}(1-B^{-1})f'(x)^{1/(1-B)}-\gamma f(x)\right]v\,\mathrm{d}x+qLv-\gamma_w HLv=\int_0^L\psi\left[f(x),f'(x),x\right]v\,\mathrm{d}x+qLv-\gamma_w HLv \tag{8}$$

where $\psi[f(x),f'(x),x]$ is a functional which can be written as

$$\psi\left[f(x),f'(x),x\right]=-\sigma_t+\sigma_c(AB)^{1/(1-B)}(1-B^{-1})f'(x)^{1/(1-B)}-\gamma f(x) \tag{9}$$

On the basis of upper bound theorem, the analytical solution of detaching curve is derived by optimizing the objective function. For the purpose of making the integral ψ over the interval [0, L] to reach an extreme, obviously, it is a typical problem of the calculus of variations, i.e., there exists a function, $y=f(x)$, which satisfies the customary regularity conditions and then makes Eq. (8) reach a stationary value, thus the expression of ψ is functional which can be turned into an Euler equation according to variational principle. The first variation of the total dissipation ζ can be expressed as

$$\delta\psi\left[f(x), f'(x), x\right]=0 \quad\Rightarrow\quad \frac{\partial\psi}{\partial f(x)}-\frac{\partial}{\partial x}\left[\frac{\partial\psi}{\partial f'(x)}\right]=0 \tag{10}$$

and from Eq. (9), the corresponding expressions can be easily computed as

$$\begin{cases}\partial\psi/\partial f(x)=-\gamma\\ \partial\psi/\partial f'(x)=-\sigma_{\rm c}(AB)^{1/(1-B)}B^{-1}f'(x)^{B/(1-B)}\end{cases} \tag{11}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial\psi}{\partial f'(x)}\right]= -\sigma_{\rm c}(AB)^{1/(1-B)}(1-B)^{-1}f'(x)^{(2B-1)/(1-B)}f''(x) \tag{12}$$

By substituting Eqs. (11) and (12) into Eq. (10), the Euler equation of the problem presents

$$\eta f'(x)^{(2B-1)/(1-B)}f''(x)-\gamma=0 \tag{13}$$

where

$$\eta=\sigma_{\rm c}(AB)^{1/(1-B)}(1-B)^{-1} \tag{14}$$

It is a nonlinear second-order homogeneous differential equation in Eq. (13) which can be worked out by virtue of integral calculation. After the first integration, it can be written as

$$\eta(1-B)B^{-1}f'(x)^{B/(1-B)}-\gamma x-\tau_0=0 \tag{15}$$

where τ_0 is a integral constant which can be calculated by the geometrical condition. On the basis of symmetrical characteristics of the detaching curve, $f(x)$, it is indicated that it should satisfy $f'(x=0)=0$. Thus, the value of τ_0 is equal to zero and the expression of detaching curve $f(x)$ can be derived through integral calculation as

$$f(x)=B\left[\frac{\gamma B}{\eta(1-B)}\right]^{(1-B)/B}x^{1/B}-h = A^{-1/B}\left(\frac{\gamma}{\sigma_c}\right)^{(1-B)/B}x^{1/B}-h \tag{16}$$

where h stands for an integration constant which can be calculated when the detaching curve $f(x)$ has been obtained.

By substituting Eq. (16) into Eq. (9), the expression of ψ is worked out:

$$\psi=-\sigma_{\rm t}+\sigma_{\rm c}(AB)^{1/(1-B)}(1-B^{-1})\left[\frac{B}{\eta(1-B)}\right]^{1/B}\cdot x^{1/B}-\gamma A^{-1/B}\left(\frac{\gamma}{\sigma_{\rm c}}\right)^{(1-B)/B}x^{1/B}+\gamma h= \gamma h-\sigma_{\rm t}-B^{-1}A^{-1/B}\sigma_{\rm c}{}^{(B-1)/B}\gamma^{1/B}x^{1/B} \tag{17}$$

By combining Eq. (17) with Eq. (8) through calculating the integral of ψ over the interval [0, L], the exact form of ζ shows

$$\zeta[f(x), f'(x), x]=\int_0^L\psi[f(x), f'(x), x]\,v\,{\rm d}x+qLv-\gamma_{\rm w}HLv=(\gamma h-\sigma_{\rm t})Lv-A^{-1/B}(1+B)^{-1}\cdot \sigma_{\rm c}{}^{(B-1)/B}\gamma^{1/B}L^{(1+B)/B}v+qLv-\gamma_{\rm w}HLv \tag{18}$$

Moreover, there is another implicit condition of $f(x=L)=0$, i.e.,

$$f(x=L)=0 \quad\Rightarrow\quad h=A^{-1/B}(\gamma/\sigma_{\rm c})^{(1-B)/B}L^{1/B} \tag{19}$$

At last, in order to get the analytical solution, there exits another equation which includes the relationship between h and L based on upper bound theorem of limit analysis. Thus, it can be worked out easily by means of equating the rate of the energy dissipation to the external rate of work, i.e.,

$$\zeta[f(x), f'(x), x]=0 \quad\Rightarrow(\gamma h-\sigma_t)L-A^{-1/B}(1+B)^{-1}\cdot \sigma_{\rm c}{}^{(B-1)/B}\gamma^{1/B}L^{(1+B)/B}+qL-\gamma_{\rm w}HL=0 \tag{20}$$

Combining Eq. (19) and Eq. (20), the exact expression of h and L can be obtained as

$$h=(1+B)(\gamma B)^{-1}(\sigma_{\rm t}+\gamma_{\rm w}H-q) \tag{21}$$

$$L=A(\gamma/\sigma_{\rm c})^{(B-1)}h^B=A\left(\frac{1+B}{B}\right)^B\frac{\sigma_{\rm c}}{\gamma}\left(\frac{\sigma_{\rm t}+\gamma_{\rm w}H-q}{\sigma_{\rm c}}\right)^B \tag{22}$$

As a result, the expression of $f(x)$ is

$$f(x)=A^{-1/B}(\gamma/\sigma_{\rm c})^{(1-B)/B}x^{1/B}-(1+B)(\gamma B)^{-1}(\sigma_{\rm t}+\gamma_{\rm w}H-q) \tag{23}$$

and the overall mass of roof collapse per unit length, G, can be calculated as

$$G=2A(1+B)^B\gamma^{-1}\sigma_{\rm c}{}^{1-B}\left[B^{-1}(\sigma_{\rm t}+\gamma_{\rm w}H-q)\right]^{1+B} \tag{24}$$

5 Discussion of analytical results

In order to evaluate the validity of the analytical solutions obtained in this work, comparisons between the results derived in this work and the previous work are made. Compared with the work of Ref. [12], it is obvious that the former is coincident with the later absolutely when q=0 and H=0, which indicates the validity of the method.

6 Numerical results of collapse shape

For the purpose of investigating the effect of parameters, including A, B, $\sigma_{\rm c}$, $\sigma_{\rm t}$, γ, H and q on the shape of collapsing block, the numerical results are derived by virtue of calculating software. According to the analytical solution of detaching curve, $f(x)$, expressed in Eq. (23), the exact detaching curve of shallow underwater cavity or tunnel can be easily drawn with regard to different parameters in Fig. 2, where different parameters range as follows: A=0.15−0.75, B=0.5−0.9, σ_c/σ_t=1/300−1/50, σ_c=10 MPa, γ=(15−25) kN/m^3, H=(0−20) m, q=(0−0.2) MPa.

It can be immediately seen from Fig. 2 that, A, B, σ_c, σ_t, γ, H and q all have an impact on the collapsing scale to some degree. With the increase of B, γ and q, the width and height of the collapsing block both tend to decrease. However, with increasing the values of σ_t and H, the width and height of the collapsing block have a increasing tendency. Moreover, the effect of parameter A is different from others, i.e., when A increases, the width of the collapsing block becomes larger, but the corresponding height remains the same.

It is obvious that the depth of river water has adverse influence on the scale of roof collapse, so it should be controlled effectively by discharging the water

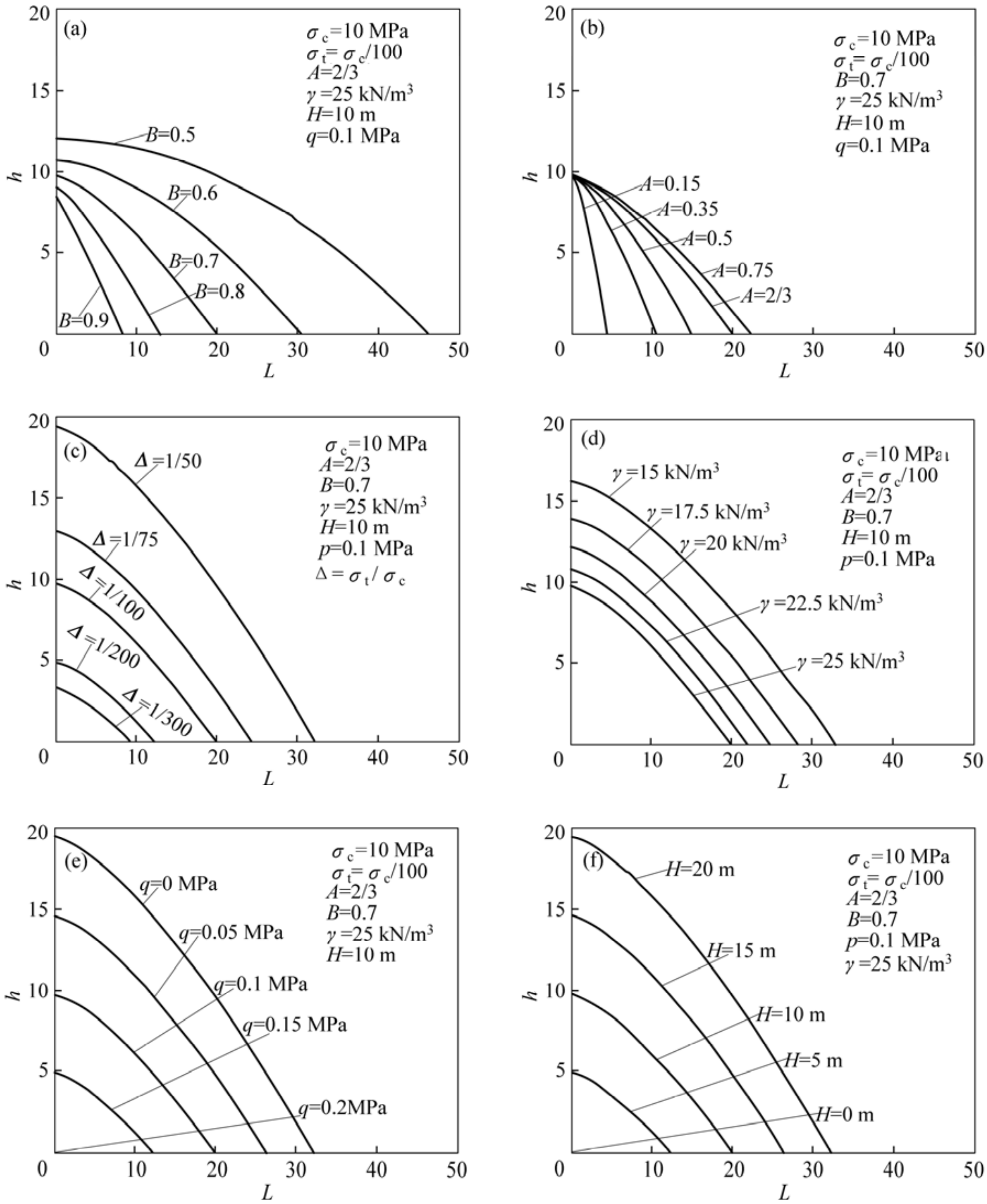

Fig. 2 Shapes of collapsing block with regard to different parameters: (a) Effect of strength parameter B; (b) Effect of strength parameter A; (c) Effect of tensile strength; (d) Effect of unit weight; (e) Effect of supporting pressure; (f) Effect of H

flow in good time when rainy season is coming. Meanwhile, it can be found that the maximum depth of river water can be calculated when the burial depth of cavity h_0 is given and the expression is

$$H_{\max} = \gamma B h_0 \left[(1+B)\gamma_{\mathrm{w}} \right]^{-1} - (\sigma_{\mathrm{t}} - q)\gamma_{\mathrm{w}}^{-1} \tag{25}$$

If the depth of river water is given as a constant and the supporting pressure hasn't been exerted in the early stage, according to Fig. 2, there must be a critical height of collapsing block, which means the detaching curve can't extend to river bottom surface, i.e., the water can't flow into the cavity when h reaches h_0, so the expression of h_{critical} can be written as

$$h_{\mathrm{critical}} = h_0 = (1+B)(\gamma B)^{-1}(\sigma_{\mathrm{t}} + \gamma_{\mathrm{w}} H) \tag{26}$$

What's more, in the view of engineering, safety stock must be considered reasonably. In this situation, the ratio $r=h_0/h$ is a relatively effective parameter to describe the safety stock, so the value of ratio r can be used to evaluate the stability of the roof of cavities and it should be determined cautiously and reasonably specific to actual engineering situation.

Generally, the excavated cavities or tunnels are under the complicated situation where many factors have different impacts on the stability of roof collapse to varying degrees, so primary lining or even twice lining which means exerting supporting pressure is widely used to protect the roof of cavities from collapsing [22−27]. Different values of supporting pressure can be chosen under different geological environment so as to guarantee that h=0, i.e., no roof collapse happens. In this case, the minimal supporting pressure can also be derived as

$$q_{\min} = \sigma_{\mathrm{t}} + \gamma_{\mathrm{w}} H \tag{27}$$

From Fig. 2(e) and Fig. 2(f), it can be found obviously that h=0 when q=0.2 MPa and H=0 m, so choosing a reasonable value of q and keeping a proper water level H can make the roof of cavity much more stable.

7 Conclusions

1) By virtue of applying the upper bound theorem and the tool of calculus of variations, the analytical solutions describing the shape of roof collapse in underwater tunnel or cavity have been obtained according to Hoek–Brown failure criterion. The effect of river water is seen as work rate of distribution surface forces is incorporated into upper bound theorem and the result shows that it has an adverse impact on the scale of collapsing block to a large degree. The supporting pressure is also regarded as distribution force acting on the cavity structures whose work rate is incorporated into upper bound theorem. The more the supporting pressure exerts under certain situation, the more stable it will be. The solutions derived under the condition of river water agree with the work of FRALDI and GUARRACINO very well when q=0 and H=0, which proves the validity of the method proposed in this work by considering the effect of river water.

2) In shallow-buried underwater tunnel or cavity, the supporting pressure exerting timely and reasonably has a positive effect on the stability of roof collapse, and it has a minimal value which can protect the cavity from water inflowing. There exists a maximum depth of river water when the buried depth is given so as to make sure that the detaching curve doesn't extend to the river bottom surface. The method proposed in this work to evaluate the roof collapse can be regarded as a reference and a supplementary tool for stability analysis.

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