

A fast explicit finite difference method for determination of wellhead injection pressure

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Abstract: A fast explicit finite difference method (FEFDM), derived from the differential equations of one-dimensional steady pipe flow, was presented for calculation of wellhead injection pressure. Recalculation with a traditional numerical method of the same equations corroborates well the reliability and rate of FEFDM. Moreover, a flow rate estimate method was developed for the project whose injection rate has not been clearly determined. A wellhead pressure regime determined by this method was successfully applied to the trial injection operations in Shihezi formation of Shenhua CCS Project, which is a good practice verification of FEFDM. At last, this method was used to evaluate the effect of friction and acceleration terms on the flow equation on the wellhead pressure. The result shows that for deep wellbore, the friction term can be omitted when flow rate is low and in a wide range of velocity the acceleration term can always be deleted. It is also shown that with flow rate increasing, the friction term can no longer be neglected.

Key words: wellhead pressure; injection pressure; bottom-hole pressure; fast explicit finite difference method

1 Introduction

In recent years, the number of gas injection or exploitation projects increases fast with growing energy needs and severe environmental problems. These projects include not only traditional EOR and natural gas exploitations, but also the enhancement recovery of CBM, shale gas, EGS and CO₂ storage or sour gas re-injections [1–3]. Of these projects, wellbore stability and some other engineering targets claim proper design of wellhead pressure which is a key operation parameter in wellbore injection control. So, a rational and systematic determination methodology of wellhead injection pressure is very important for promoting the implementations of these projects.

In many respects, determination of wellhead pressure is the same as that of bottom-hole pressure because they need to solve the same equation system, namely the equation of continuity, the equation of motion and the energy conservation equation. As a result of the complexity of this equation system, no widely accepted analytical solution has been gained up to now. In the practice of wellbore and pipeline transportations, the equation system is usually simplified by considering specific engineering conditions and hypothesis. Furthermore, different solving methods for these

different simplified equations bring forth various kinds of determination methods.

As for the determination of bottom-hole pressure of oil and gas wells, many attempts have been made, to improve or present more accurate prediction methods which as a whole mainly include two classes. One is based on an integration expression of pressure derived from the fundamental equation system. As no direct integration solution has been gained, many specific calculation methods try to give its approximate solution, of which “average temperature and deviation factor method” is the most widely known. It was first presented by RZASA and KATZ [4] who replaced gas temperature and deviation factor with their averages, respectively. This calculation method was later revised by SUKKAR and CORNELL [5], CULLENDER and SMITH [6], YANG and HUANG [7], who canceled the assumption that temperature and deviation factor were constant. KATZ et al [8], MESSER et al [9] and others improved the numerical accuracy of this method. As the thermo-physical parameters, such as viscosity, density, temperature, friction factor and deviation factor, are important in calculation, many researchers paid their attention to the better determinations of these parameters [10–11]. This method has been used widely by many researchers [12–16]. However, this method needs many numerical iterative corrections [17]. Moreover, the

kinetic energy term which was considered to be important [18–19] in low pressure and shallow wellbore was ignored. Another class of bottom-hole pressure determination method is based on the numerical method of initial value problems of differential equation(s) [17] which is an important progress. In this method, the fundamental differential equations are treated as differential equation(s) about pressure and/or other variables which will be then solved with numerical techniques such as Runge-Kuta method. However, in this work, the acceleration term is ignored considering its very weak effect on pressure. This may produce a limitation of generalization to other fields in which acceleration term is not negligible.

Studies on the determination of bottom-hole pressure supply great references for that of wellhead pressure. Very similar method was used in determination of wellhead pressure in sour gas injection [20–22]. However, both determination processes are different in boundary conditions. For the former, the wellhead pressure is usually easy to measure, while the latter has difficulty in giving the bottom-hole pressure especially when the injection is still in design stage.

This work aims to present a fast explicit method for the determination of wellhead pressure from the pressure equation that does not ignore any term. It is expected that the effect of friction and acceleration on wellhead pressure is naturally obtained from calculation rather than prior assumptions. Moreover, a flow rate estimation is advised for injections whose bottom-hole condition is not completely defined.

2 Calculation method for wellhead pressure

2.1 Model and assumptions

To make the analysis and statements hereafter clearer and more intuitive, a wellbore structure model is given in Fig. 1, the left of which is the one dimensional discretized mesh used in Section 2.2.

The mathematical model of pipe flow discussed in this work should follow some basic assumptions as follows:

- 1) Single phase gas flow in pipe;
- 2) One dimensional steady flow in pipe;
- 3) Injection to a single formation;
- 4) The wellbore is vertical and only one casing is considered.

2.2 Main equations and calculation method

A coordinate system is set as Fig. 1 whose positive direction is the same as that of the gas injection. Based on the basic principle of flow dynamics, the equation of continuity for one-dimensional steady flow is

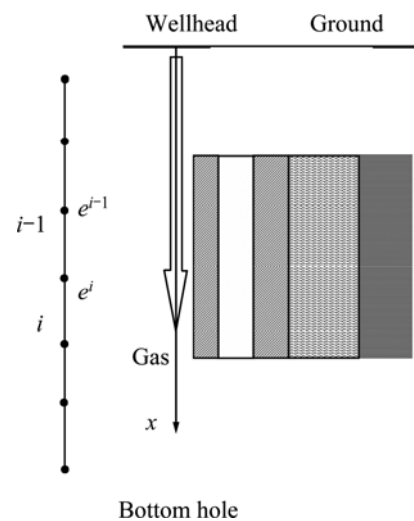


Fig. 1 Wellbore structure model and discretized mesh

$$\frac{d(\rho v)}{dx} = 0 \tag{1}$$

where ρ is the density of gas (kg/m^3), and v is the velocity (m/s).

The equation of motion in vertical wellbore is [23]

$$\frac{dp}{dx} = \rho g - \rho v \frac{dv}{dx} - \lambda v^2 \frac{\rho}{2D} \tag{2}$$

where p is the pressure of gas (Pa), g is acceleration of gravity (m/s^2), λ is the friction coefficient and D is the interior diameter of injection tube (m).

On the right side of Eq. (2), the first term reflects gas gravity on the pressure gradient, so-called gravity term for short. Similarly, the second and the third terms are respectively called acceleration term and friction term.

The energy conservation equation which will supply the gas temperature will not be coupled with the above two equations as an explicit temperature formula will be directly used in Section 4.

The equation of state (EOS) of gas is [17]

$$\rho = \frac{pM}{ZRT} \tag{3}$$

where Z is the deviation factor or compression factor, R is universal gas constant, T is thermodynamic temperature, and M is gas molar mass.

Equation (1) implies that

$$\rho v = \bar{C} \quad \text{or} \quad v = \bar{C}/\rho \tag{4}$$

where \bar{C} is constant independent of wellbore coordinate. Its physical meaning is the mass flow rate of the wellbore.

Eliminating ρ from Eqs. (3) and (4) produces

$$v = \frac{\bar{C}ZRT}{pM} \tag{5}$$

Substituting Eqs. (3) and (5) into Eq. (2) yields

$$\frac{dp}{dx} = g \frac{pM}{RTZ} - \frac{\bar{C}^2 R}{M} \frac{d}{dx} \left(\frac{ZT}{p} \right) - \frac{\lambda \bar{C}^2 RT}{2D} \frac{Z}{pM} \quad (6)$$

Equation (6) is a differential equation about pressure p and deviation factor Z . To transform Eq. (6) to an ODE about p , Z has to be replaced with a function dependent on p . However, there are several ways to acquire Z such as empirical formulas, table or chart-looking up methods and EOS method and there is not a widely accepted formula about Z . This makes it difficult to discuss the analytical solution.

Still, the approximate solution of Eq. (6) will be studied. For this aim, the one-dimensional definitive range of wellbore is meshed into line segments—elements and their linkers—nodes. The nodes are numbered increasingly from the head to bottom of well. Obviously, Eq. (6) is true strictly in each element. The thermo-physical parameters (viscosity, density, temperature, friction factor and deviation factor etc.) in each element are assumed to be constant provided its size is not so large that cannot be accepted. We call this process physical approximation. Thus, an ODE about pressure p is obtained as

$$\frac{dp}{dx} = g \frac{pM}{RTZ} - \frac{\bar{C}^2 RT}{M} Z \frac{d}{dx} \left(\frac{1}{p} \right) - \frac{\lambda \bar{C}^2 RT}{2D} \frac{Z}{pM} \quad (7)$$

Expanding the derivative term in the right side and rearranging Eq. (7) yield the following equation in e_i :

$$\left(1 - \frac{\bar{C}^2 RT}{Mp^2} Z \right) \frac{dp}{dx} = g \frac{pM}{RTZ} - \frac{\lambda \bar{C}^2 RT}{2D} \frac{Z}{pM} \quad (8)$$

Equation (8) is a linear ODE about p in the i -th line element e_i . There are two ways to study its numerical solution as its analytical solution cannot be obtained. The first is to use the classical numerical methods of ODE such as Runge-Kuta method. The other way is to use finite difference method. Here, the latter will be discussed as it will give a fast and explicit solution. The solutions from both ways for the case study will be compared in Section 5.

The next step is to discretize Eq. (8) in generic segment e_i . As the segment is already efficiently small, the subdivision number of generic segment e_i will be 1. So, we have

$$\left(1 - \frac{\bar{C}^2 RT}{Mp_i^2} Z \right) \frac{p_i - p_{i-1}}{\Delta x} = g \frac{p_i M}{RTZ} - \frac{\lambda \bar{C}^2 RT}{2D} \frac{Z}{p_i M} \quad (9)$$

Rearranging Eq. (9) yields

$$p_{i-1} = p_i + \frac{-\Delta x g \frac{p_i^3 M}{RTZ} + \frac{\Delta x \lambda \bar{C}^2 RT}{2D} \frac{Z p_i}{M}}{\left(p_i^2 - \frac{\bar{C}^2 RT}{M} Z \right)} \quad (10)$$

Equation (10) is an explicit solution of node pressure in each line element. Looping all the line segments from well bottom, the pressure profile and the wellhead pressure can be obtained.

If $\bar{C}=0$, Eq. (9) will be degenerated into

$$p_{i-1} = p_i - \Delta x g \frac{p_i M}{RTZ} = p_i - \Delta x g \rho_i \quad (11)$$

This is just the static pressure of gas, which means that Eqs. (9) or (10) can be used to calculate both static and flow pressures.

Gas temperature will be calculated using Ramy's formula, Z will be calculated based on Peng–Robinson's equation and viscosity will be calculated with GUO's method [24–25]. The bottom-hole pressure which is the initial value of the pressure equation will be given through the product of formation fracture pressure and a reduction factor belonging to (0, 1). This is to ensure the wellbore formation stability.

3 Wellbore heat transmission

During injection, the temperature of gas in the tubing at any depth depends on the heat transmission process because of temperature difference between the gas and the formation. There are several factors influencing this process including gas velocity, injection duration, thermal conductivity of the media outside the tube and the initial temperatures of gas and the surrounding formations. RAMY [26] presented an explicit temperature prediction function:

$$T(H, t) = aH + b - aA + (T_0 + aA - b)e^{(-H/A)} \quad (12)$$

where

$$A = 11.593 \frac{G_t C [K + f(t) r_{to} U_t]}{2\pi r_{to} U_t K} \quad (13)$$

$$f(t) = \ln \left[\frac{2\sqrt{\alpha t}}{r_h} \right] - 0.29 \quad (14)$$

where a is the geothermal gradient, °C/m; b is the surface average temperature, °C; T_0 is the temperature of injected gas, °C; H is the depth, m; t is the injection time, h; G_t is the injection rate, t/d; K is the coefficient of thermal conductivity of rock, W/(m·K); r_{to} is the tubing outer radius, m; α is the average heat release coefficient of formation, m²/h; r_h is the inner radius of borehole, m; U_t is the total heat transfer coefficient from gas to formation, W/(m²·K).

The following will discuss the calculation of key parameters and how the temperature is calculated [27–28].

1) Total heat transfer coefficient, U_t

If water or gas exists in the annulus, the total heat

transfer coefficient is

$$U_{to} = \left[\frac{1}{h_c + h_r} + \frac{r_{to} \ln \frac{r_h}{r_{co}}}{\lambda_{cem}} \right]^{-1} \quad (15)$$

where h_c is the heat transfer coefficient of medium in annulus during natural convection, W/(m·K); h_r is the radiation heat transfer coefficient, W/(m²·K); r_{to} is the outer radius of tubing, m; r_h is the radius of borehole, m; r_{co} is the outer radius of casing, m; λ_{cem} is the heat transfer coefficient of cement ring, W/(m·K).

2) Radiation heat transfer coefficient, h_r

$$h_r = \sigma F_{tci} (T_{to}^{*2} + T_{ci}^{*2}) (T_{to}^* + T_{ci}^*) \quad (16)$$

$$F_{tci} = \left(\frac{1}{\epsilon_{to}} + \frac{r_{to}}{r_{ci}} \left(\frac{1}{\epsilon_{ci}} - 1 \right) \right)^{-1} \quad (17)$$

where σ is the Stefan-Boltzmann constant, 5.673×10^{-8} W/(m²·K⁴); F_{tci} is the effective heat release coefficient from outer surface of tube to the inner wall surface of casing; T_{to} is the temperature of outer wall surface of tubing, °C; T_{ci} is the temperature of inner wall surface of casing, °C; $T^* (= T + 273)$ is the thermodynamic temperature, K; ϵ_{to} is the radiation coefficient of outer wall surface of tube, 0.9; r_{ci} is the inner radius, m; ϵ_{ci} is the radiation coefficient of inner wall surface of casing, 0.9.

3) Natural convection heat transfer coefficient, h_c

$$h_c = \lambda_{hc} / [r_{to} \ln(r_{ci}/r_{to})] \quad (18)$$

4) Temperature at interface between cement ring and formation

$$T_h = \left[T_0 f(t) + \frac{\lambda_e}{r_{to} U_{to}} T_e \right] / \left[f(t) + \frac{\lambda_e}{r_{to} U_{to}} \right] \quad (19)$$

5) Temperature at inner wall surface of casing

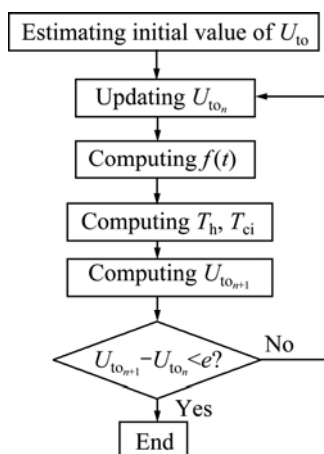


Fig. 2 Iteration calculation steps of U_i (e : error bound)

$$T_e = b + \frac{1}{2} (H \times a) \quad (20)$$

$$T_{ci} = T_h + r_{to} U_{to} \ln \frac{r_h}{r_{co}} (T_{to} - T_h) / \lambda_{cem} \quad (21)$$

where T_0 is the temperature of injection gas, °C; T_e is the average temperature, °C; λ_e is the heat transfer coefficient of rock, W/(m·K); λ_{hc} is the heat transfer coefficient of medium in annulus, W/(m·K).

6) Iteration calculation steps of U_i

When the moment U_i is calculated, substitute it to Eq. (20) and the temperature of gas in the tubing at any depth can be easily calculated.

4 Application

4.1 Project overview

Shenhua CCS demo project is the first large-scale CO₂ saline aquifer storage project of whole process in the world. The injection site is located in Erdos, Inner Mongolia Autonomous Region. There are two three-opening vertical wells, of which one is for injection named as Zhongshenzhu 1# and the other is for monitoring named as Zhongshenjian 1#. The depths of both wells are about 3 000 m. Well is completed with tail pipe which is tied back to wellhead. In March 9th, 2011, another monitoring well about 2 500 m deep started to drill supported by Ministry of Land and Resources to strengthen the monitoring work. It planned to inject about 0.1 Mt CO₂ each year. The trial injection work was successfully finished recently.

As the first large-large scale injection of CO₂, there are few experiences on the stability and safety control of wellbore and formation. It is widely accepted that the wellhead pressure and its design are key problems during injection. We advise that the whole injection operations should be divided to several stages, including trial injection stage, formal injection stage and exploratory stage. During the trial injection stage, one representative formation should be first chosen and injected with increased wellhead pressure from relatively low level. Most important of all, an upper wellhead pressure limit should be determined as safety control. This upper limit could be adjusted according to the actual injection effect, project aim changes or reservoir reconstruction.

Shihezi group formation was chosen for the first-time trial injection. At that time, this formation was not reconstructed. The basic information about this formation is given in Table 1.

Figure 3 shows the geothermal curve, in which the segment above 1 599 m is given with 2.93/100 °C/m [29], while the segment below 1 600 m is drawn directly from log data.

Table 1 Basic information of injection formation

No.	Item	Value	Remark
1	Formation depth/m	2 105.8–2 208.2	Shihezi
2	Formation pressure/MPa	23.02	Well logging data
3	Fracture pressure/MPa	47.36	Well logging data

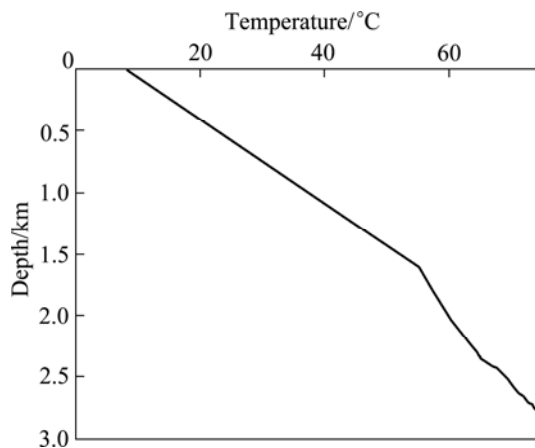


Fig. 3 Geothermal curve

For convenience of usage, this curve is fit with a linear function:

$$T=0.023x+13.13 \tag{22}$$

The $R^2=0.98$ shows an excellent linearity of these two variables and hence this equation is reliable.

4.2 Wellhead pressure and pressure profile

All the calculation values of main parameters are listed in Table 2. Although the project’s target is to inject at least 0.1 Mt CO₂ each year, the annual injection amount for this single formation is set to 45 kt each year considering that there are many formations available. Assume that injection takes place 300 d each year and one day include 24 h, then $\bar{C}=575 \text{ kg}/(\text{m}^2 \cdot \text{s}^{-1})$.

To validate the accuracy and numerical stability of FEFDM, Runge-Kutta method is used for comparison. The solutions from both numerical methods are listed in Table 3. For every flow rate of injection gas, the relative deviations from both outcomes are so small that they can be neglected. This implies that the new explicit method is reliable in accuracy and numerical stability.

Figure 4 shows the CO₂ pressure profile along the wellbore and the wellhead pressure is 19.5 MPa. Figure 5 shows the calculated temperature profile in the tubing. For comparison, the formation temperature profile is also shown.

Considering that there exists many uncertainties in engineering, an upper limit of wellhead pressure of 18 MPa is advised from these calculation results. In January, 2011, 25.2 t CO₂ was successfully injected to Shihezi

Table 2 Main calculation parameters

Parameter	Value	Reference
$A/(\text{°C} \cdot \text{m}^{-1})$	0.023	
$C/(\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1})$	2.04	[26]
$b/\text{°C}$	13.13	
$T_0/\text{°C}$	20	
t/h	240	
$G_t/(\text{t} \cdot \text{d}^{-1})$		
$K/(\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1})$	2.09	[26]
r_{to}/mm	73	
r_h/mm	215.9	
r_{co}/mm	139.7	
r_{ci}/mm	124.26	
$\alpha/(\text{m}^2 \cdot \text{h}^{-1})$	0.003 7	[26]
$\lambda_{cem}/(\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1})$	0.35	[26]
$\sigma/(\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})$	5.673×10^{-8}	[28]
ε_{to}	0.9	[28]
ε_{ci}	0.9	[28]
$\lambda_{hc}/(\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1})$	0.6	
$\lambda_w/(\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1})$	2.25	
D/mm	134.98	
$\Delta x_i/\text{m}$	1	
r_o/mm	31	
l/m	102.4	
r_h/mm	107.95	
k	0.8	
p_f/MPa	47.1	
$K/10^{-3} \mu\text{m}^2$	0.79–5.99	
p_e/MPa	23.02	
H/m	2156	

Table 3 Results comparisons of FEFDM with ODE numerical method

No.	Flow rate/(kg·s ⁻¹)	Wellhead pressure/MPa		Relative deviation/%
		FEFDM	ODE method	
1	1.736	19.507 1	19.515 3	0.042
2	5.787	19.936 0	19.942 9	0.035
3	7.716	20.642 0	20.647 1	0.025
4	13.5	24.118 4	24.125 6	0.030
5	19.29	29.474 2	29.481 5	0.025

Line element size: 1 m; four segments in each line element; radius of tubing: 31 mm.

formation. The effective injection rate is about 0.44 kg/s, and the wellhead pressure increases from 11.25 to 18.92 MPa and is stopped there. The successful trial injection shows that upper limit wellhead pressure of 18 MPa from the determination method in this work is reliable. It is noteworthy that in the subsequent injections, the upper limit wellhead pressure is adjusted because the fracturing and perforating are adopted to improve the permeability of the formation.

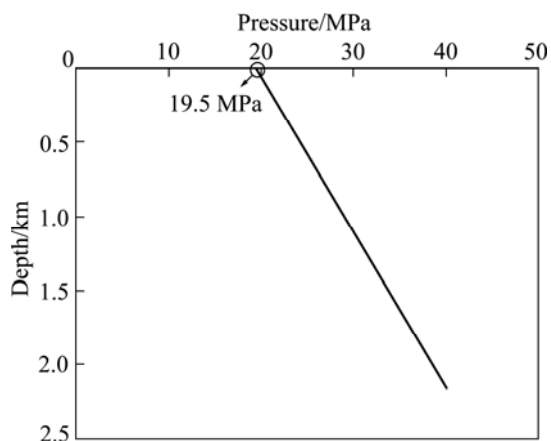


Fig. 4 Wellbore pressure profile

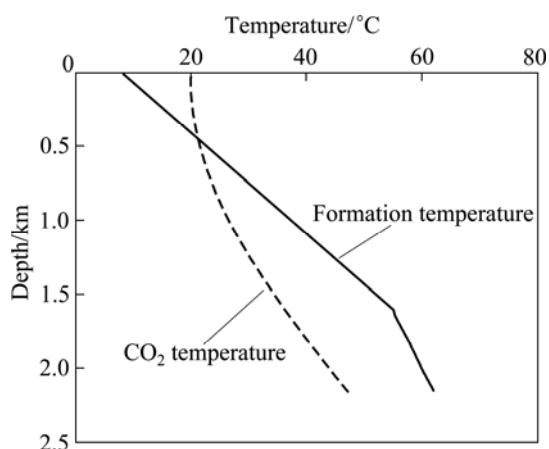


Fig. 5 Temperature profiles of tubing and formation

5 Effect of friction term and acceleration term on wellhead pressure

It is meaningful to investigate the effect of bottom-hole pressure and friction on the wellhead pressure directly from the equation. All calculations are divided into four groups of A–D, as listed in Table 4. Each group contains 12 flow rate cases from 0.77 to 38.58 kg/s.

It is shown that when flow velocity is very small, the wellhead pressure almost keeps the same value. While the flow rate is significantly increased, the wellhead pressure also increases rapidly, as shown in Fig. 6. The reason is that, when flow velocity is low, it is in the state of laminar flow and the friction coefficient is very small. When flow rate increases to turbulent flow, the friction coefficient increases and makes a considerable friction pressure drop. The difference values (Curve 3 in Fig. 6) just reflect the effect of friction term. So at higher velocity, friction term should not be ignored. By comparing the values in B, C and D, some characteristics are found: 1) With the flow rate increasing, values in B almost stay the same although the flow rate is high;

Table 4 Influence of friction and acceleration terms on wellhead pressure (MPa)

Flow rate/ (kg·s ⁻¹)	A	B	C	D
0.77	19.957 9	19.936 5	19.957 9	19.936 5
1.736	19.507 3	19.408 6	19.507 3	19.408 6
2.315	19.427 0	19.255 5	19.427 0	19.255 5
2.894	19.417 0	19.153 7	19.417 1	19.153 7
5.787	19.938 3	18.921 8	19.938 4	18.922 0
7.716	20.644 0	18.857 6	20.644 2	18.857 8
11.574	22.749 7	18.790 4	22.750 1	18.790 9
13.5	24.127 2	18.770 6	24.127 6	18.771 2
17.361	27.497 9	18.743 7	27.498 4	18.744 7
19.29	29.484 8	18.734 0	29.485 3	18.735 3
28.935	42.277 7	18.703 9	42.277 5	18.706 8
38.58	59.565 5	18.687 2	59.563 2	18.692 4

A–Friction and acceleration terms retained; B–Friction term omitted, acceleration term retained; C–Friction term retained, acceleration term omitted; D–both terms omitted (radius of tubing of 31 mm).

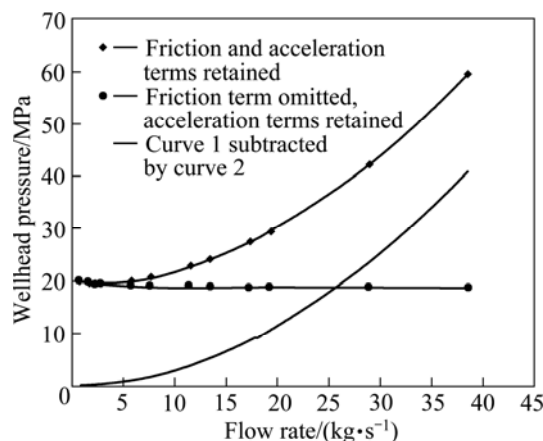


Fig. 6 Effect of flow rate on wellhead pressure

2) Values in C are almost the same as those in A; 3) Values in C are almost the same as those in B, hence the same as those in A. This fully demonstrates that the acceleration term has very limited influence on the wellhead pressure at least in the flow rate range of this calculation. This implies that for the deep well transportation can be neglected in a very wide range of flow rate.

It is noteworthy that the aim of the above calculations is just to study the wellhead pressures from the bottom-hole conditions. So, the injectivity of the formations is not considered which is another key problem in injection practice. This will be studied in future work.

6 Conclusions

1) The fundamental differential equations of one

dimensional pipe flow are studied in depth, based on which the equation of motion of gas is transformed to an ODE about pressure considering that the flow rate at any position of pipe is constant. A fast explicit finite difference method (FEFDM) is developed to solve this ODE. A comparison with the traditional numerical method of solving ODE is conducted, which validates the reliability of this new method. For the injection design project whose injection rate has not been clearly determined a flow rate estimate method is developed. These altogether form a simple but systematic methodology for wellhead injection pressure design.

2) For deep wellbore, the friction term can be omitted when flow rate is low (for the case <5 kg/s) and in a wide range of velocity the acceleration term can always be deleted. With the flow rate increasing, the friction term can no longer be neglected.

3) For next step, it will be useful to search the analytical solution of equation and extend this method to multiphase flow and injection. Another difficult but important work is to develop the Wellhead pressure determination method for Multiaquifer well.

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