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Modified image analytical solutions for ground displacement using nonuniform convergence model

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Abstract: Based on the image theory, the analytical solutions of tunneling-induced ground displacement were derived in conjunction with the nonuniform convergence model. The reasonable value of Poisson ratio in the analytical solution was discussed. The ground settlement width parameter which could reflect the ground condition was introduced to modify the analytical solutions proposed above, and new analytical solutions were presented. To evaluate the validity of the present solutions using the nonuniform convergence model, the results were compared with the observed values for four engineering projects, including 38 measured data of ground settlement. The agreement shows that the present solutions using the nonuniform convergence model are effective for evaluating the tunneling-induced ground displacements.

Key words: subway tunnel; ground displacement; image method; ground settlement width parameter

1 Introduction

In the process of urban tunnel construction, ground movement is caused by underground excavation. When the ground movement develops to a certain value, the safety of ground buildings and the normal use of underground pipeline would be greatly influenced. As a result, it is significant to study the ground movement caused by tunnel construction. There are several approaches to predict the ground movement associated with tunneling, which mainly include the empirical method, the analytical method, the finite difference method and the finite element method.

According to the measured results of surface settlement for tunnel excavation in coal area, the tunneling-induced ground settling tank is proposed, and could be approximately represented by error function. At present, the empirical formula of Peck is widely used. According to the formula, the volume of ground settling tank caused by tunnel excavation is equal to the ground loss under the condition of no drainage [1−2]. Based on the summarization of a large quantities of data for the ground settlement due to tunneling, MAIR et al [3] presented that the ground loss is between 1% and 2% for the tunnel which was excavated by open shield in stiff clay. Though these methods have been applied widely in engineering practice, there are some important limitations. For instance, due to the complicated conditions in practical engineering, the information obtained by empirical method could not satisfy different ground conditions and construction techniques. Consequently, the analytical method which can estimate the tunneling-induced ground movement needs to be developed. By using the analytical method, the preliminary field information of ground movement and complete expression of ground displacement and stress can be easily obtained, so this method has been widely used in the field of ground displacement prediction.

In order to derive a closed form analytical solution, some attempts have been made. For instance, SAGASETA [4] presented the image method to solve the problem of displacement field caused by void in a linear elastic semi-infinite body. However, SAGASETA's method is only applicable to the homogeneous incompressible soil whose Poisson ratio is equal to 0.5. Based on the image method principle proposed by SAGASETA and assuming the soil is linear elastic material, VERRUIJT and BOOKER [5] derived the formulation of the vertical ground displacement and the horizontal ground displacement subjected to elastic half-plane solutions. Furthermore, VERRUIJT and BOOKER [5] considered that the tunnel transformation mechanism was mainly caused by the uniform radial displacement of surface soil for tunnel, and the elliptical deformation of the tunnel liner. Therefore, this formula is not only applicable to the volume incompressible soil, but also suitable for the case of arbitrary Poisson ratio.

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However, it is apparent that the width of settling tank and horizontal displacement obtained by this formula are larger than the observed values. There are two main reasons for this phenomenon. First, VERRUIJT and BOOKER's solution [5] is based on the assumption that the soil is linear elastic material, but actually the strength envelopes of materials are nonlinear. Second, the assumption that deformation between tunnel and soil interface is uniform radial displacement does not correspond with the practical situation.

For the purpose of refining the solution of VERRUIJT and BOOKER [5], based on the equivalent ground loss parameter proposed by LEE et al [6], LOGANATHAN and POULOS [7] modified VERRUIJT and BOOKER's formula [5] in conjunction with oval non-equivalent movement model, and derived the ground displacement formula. Though the prediction values obtained by this formula well agree with the measured values in stiff clay, the settlement is overestimated in soft clay and the predicted width of settling tank is larger than the observed value.

According to the nonuniform convergence model, the image analytical solution of ground displacement caused by metro tunnel construction is derived. Moreover, the reasonable value of Poisson ratio in the analytical solution is discussed and the analytical solution is modified by introducing a surface settlement width parameter which can reflect the ground condition. In order to evaluate the validity of the present solution, the results calculated by the formula are compared with the observed values for some engineering projects.

2 Principle of image method

The two convergence displacement modes due to the tunnel excavation are uniform convergence and non-uniform convergence, respectively. The specific analysis steps of VERRUIJT and BOOKER [5] analytical solutions are as follows:

1) Without considering the existence of the ground, transform the semi-infinite body problem into an infinite body void problem. As a result, additional normal stress and additional shear stress would be generated on the original location.

2) It is assumed that there is equivalent volume shrinkage in original void mirror position in the infinite body, which would generate additional normal stress and additional shear stress on the original location.

3) The sum of the additional shear stress on the original location generated in first two steps is zero, and the sum of the additional normal stress is $2\sigma_0$. In order to satisfy the boundary conditions, the additional normal stress $2\sigma_0$ is reversely applied on the ground of the semi-infinite body.

The ground displacement produced in the first two steps can be solved by the classical elastic theory, and the calculating diagram is shown in Fig.1.

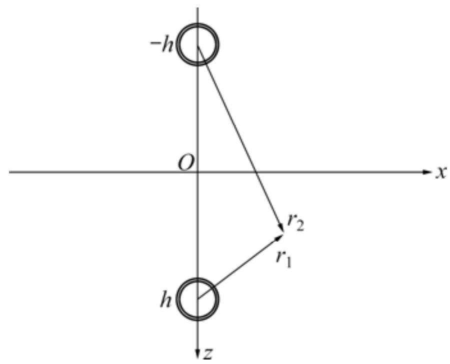

Fig.1 Diagram solution of VERRUIJT and BOOKER's formula [5]

The formulas of the horizontal ground displacement u_x and vertical ground displacement u_z can be expressed as

$$u_x=-\varepsilon R^2\left(\frac{x}{r_1^2}+\frac{x}{r_2^2}\right)+\delta R^2\left[\frac{x(x^2-kz_1^2)}{r_1^4}+\frac{x(x^2-kz_2^2)}{r_2^4}\right] \tag{1}$$

$$u_z=-\varepsilon R^2\left(\frac{z_1}{r_1^2}+\frac{z_2}{r_2^2}\right)+\delta R^2\left[\frac{z_1(kx^2-z_1^2)}{r_1^4}+\frac{z_2(kx^2-z_2^2)}{r_2^4}\right] \tag{2}$$

where ε is the uniform radial displacement, R is the radius of shield tunnels, and δ is the ovalization coefficient. In these solutions, $z_1=z-h$, $z_2=z+h$. r_1 is the distance from the centre of the excavation face to the calculation point; r_2 is the distance between the image centre of the excavation face and the calculation point.

The ground settlement width parameter k is determined by

$$k=\mu/(1-\mu) \tag{3}$$

where μ is the Poisson ratio of soil.

Based on the solutions of image problem and using Fourier transforms, VERRUIJT and BOOKER [5] derived the solutions of the ground displacement which is associated with the third step. The resulting equations are

$$u_x=-\frac{2\varepsilon R^2 x}{m}\left(\frac{1}{r_2^2}-\frac{2mzz_2}{r_2^4}\right)-\frac{4\delta R^2 xh}{m+1}\left(\frac{z_2}{r_2^4}+\frac{mz(x^2-3z_2^2)}{r_2^6}\right) \tag{4}$$

$$u_z = \frac{2\varepsilon R^2}{m}\left(\frac{(m+1)z_2}{r_2^2} - \frac{mz(x^2 - z_2^2)}{r_2^4}\right) - 2\delta R^2 h\left(\frac{x^2 - z_2^2}{r_2^4} + \frac{m}{m+1}\frac{2zz_2(3x^2 - z_2^2)}{r_2^6}\right) \tag{5}$$

where the parameter m is determined by $m=1/(1-2\mu)$.

By overlapping the solutions of three steps, the final solution of the ground displacement can be determined from the following expression:

$$u_0 = 2\varepsilon R^2 \frac{m}{m+1}\frac{h}{x^2+h^2} - 2\delta R^2 \frac{h(x^2-h^2)}{(x^2+h^2)^2} \tag{6}$$

3 Deformation of nonuniform convergence model

According to the solution of VERRUIJT and BOOKER [5], the ground convergence model is assumed to be uniform. However, the nonuniform convergence model is more proximate to the media property of practical engineering, and there is a certain difference of the ground settlement curve between the present calculation results and the observed values. Adopting the image theory, the ground displacement solution is derived using the nonuniform convergence model. The calculation diagram is shown in Fig.2. The image solutions in Fig.2 are divided into two parts. One is the ground displacement part induced by complete collapse of the tunnel initial excavation face, and the other is the ground displacement part caused by complete collapse of final profile. The difference of the two parts is the ground displacement in nonuniform convergence model, and each part of the ground displacement can be calculated by the image theory.

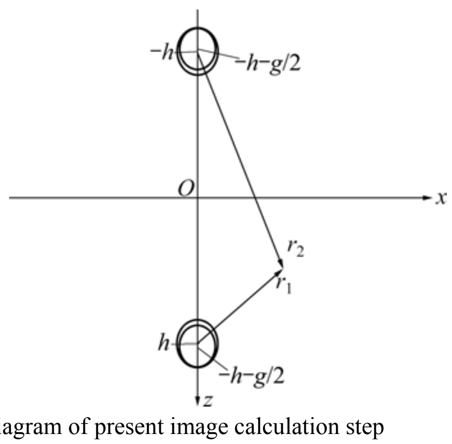

Fig.2 Diagram of present image calculation step

3.1 Complete collapse deformation

Without considering the influence of liner deformation, the calculating process of the ground displacement for complete collapse of the tunnel initial excavation face is as follows. By assuming that the initial excavation face collapses totally, the stratum displacement is calculated when the radial displacement of excavation face ε in VERRUIJT and BOOKER's solution is equal to 1. That is to set the radial displacement of excavation face u_0 equal to the radius of excavation face, thus the expression of stratum displacement may be written as

$$u_x = -R^2\left(\frac{x}{r_1^2} + \frac{x}{r_2^2}\right) - \frac{2R^2 x}{m}\left(\frac{1}{r_2^2} - \frac{2mzz_2}{r_2^4}\right) \tag{7}$$

$$u_z = -R^2\left(\frac{z_1}{r_1^2} + \frac{z_2}{r_2^2}\right) + \frac{2R^2}{m}\left(\frac{(m+1)z_2}{r_2^2} - \frac{mz(x^2 - z_2^2)}{r_2^4}\right) \tag{8}$$

where $r_1 = \sqrt{x^2 + (z-h)^2}$, $z_1=z-h$, $r_2 = \sqrt{x^2+(z+h)^2}$, and $z_2=z+h$.

In a similar way, the ground displacement formulas induced by complete collapse of the tunnel final profile can be expressed as

$$u'_x = -(R-g/2)^2\left(\frac{x}{r_1'^2} + \frac{x}{r_2'^2}\right) - \frac{2(R-g/2)^2 x}{m}\left(\frac{1}{r_2'^2} - \frac{2mzz'_2}{r_2'^4}\right) \tag{9}$$

$$u'_z = -(R-g/2)^2\left(\frac{z'_1}{r_1'^2} + \frac{z'_2}{r_2'^2}\right) + \frac{2(R-g/2)^2}{m}\left(\frac{(m+1)z'_2}{r_2'^2} - \frac{mz(x^2 - z_2'^2)}{r_2'^4}\right) \tag{10}$$

where $r'_1 = \sqrt{x^2+(z-h-g/2)^2}$, $z'_1 = z-h-g/2$, $r'_2 = \sqrt{x^2+(z+h+g/2)^2}$, $z'_2 = z+h+g/2$, g is the gap parameter, that is, the settlement of tunnel crown. Due to $h >> g/2$, it is obvious that $r'_1 \approx \sqrt{x^2+(z-h)^2}$, $z'_1 \approx z-h$, $r'_2 \approx \sqrt{x^2+(z+h)^2}$, and $z'_2 \approx z+h$, $h+g/2 \approx h$.

Therefore, the ground displacement of tunnel subjected to non-uniform convergence model is equal to the difference of Eqs.(9) and Eq.(10). The expression of final ground displacement is

$$U_x = u_x - u'_x \tag{11}$$

$$U_z = u_z - u'_z \tag{12}$$

3.2 Liner deformation influence

Taking into account of the influence of liner deformation, the formulas of the vertical ground displacement and horizontal ground displacement for the ground (z=0) can be written as

$$U_{x0} = -4gR(1-\mu)\frac{x}{x^2+h^2} - \frac{4\delta R^2 h^2(1-2\mu)}{(2-2\mu)(x^2+h^2)^2} \tag{13}$$

$$U_{z0} = 4gR(1-\mu)\frac{h}{x^2+h^2} - 2\delta R^2 h\frac{x^2-h^2}{(x^2+h^2)^2} \quad (14)$$

Due to $\varepsilon R^2=u_0R=gR/2$ and without considering the influence of the liner deformation, the vertical displacement at the ground surface subjected to non-uniform convergence model is twice as much as the value of vertical displacement in the uniform convergence model. As a result, the maximum surface settlement obtained by the solution of VERRUIJT and BOOKER [5] is smaller than the observed value. It is obvious that the non-uniform convergence model correspond to engineering projects more precisely than the uniform convergence model.

4 Modified image solutions

Although the maximum surface settlement subjected to the non-uniform convergence model is much closer to the actual value than the solution of VERRUIJT and BOOKER [5], the width of ground settling tank is also wider than the actual value. This may be explained by the fact that the analytical solution does not consider the influence of soil conditions on the width of ground settling tank. Based on the oval soil movement model and the non-equivalent ground loss parameter, LOGANATHAN and POULOS [7] modified the formula of VERRUIJT and BOOKER [5]. LOGANATHAN and POULOS [7] considered that the ground settlement occurred predominantly within β wedge between the ground surface and the tunnel, and the β value is about 45° in clays (β is the ground influence angle). In the present analysis, it is found that the settlement influence boundary, assumed by LOGANATHAN and POULOS [7], has error. For example, CORDING and HANSMIRE presented that β is smaller than 45° in soft clay while larger than 45° in sandy clay. As a result, the ground movement derived by LOGANATHAN and POULOS [7] is only suitable for the stiff clay, and error is made when the formula is employed in soft clay and sandy clay.

4.1 Effect of Poisson ratio

The only parameter related to the formation condition in the formula of VERRUIJT and BOOKER [5] is the Poisson ratio. Moreover, the magnitude of Poisson ratio has direct effect on the value of the maximum ground settlement. Since there is no one-to-one relationship between the stratum Poisson ratio and the formation condition [8−9], it is possible to get similar prediction result when other conditions (ground loss, tunnel depth and tunnel radius) are the same, which does not accord with the engineering projects.

As the soil Poisson ratio is less than 0.5, the volume of ground settling tank obtained by the solution of VERRUIJT and BOOKER [5] is larger than the ground loss volume. Besides, the drainage and influence of soil consolidation are not considered in the construction process, and the volume of ground settling tank should be equal to the ground loss volume. Thus, there are certain errors in the prediction result derived by VERRUIJT and BOOKER [5] and LOGANATHAN and POULOS [7].

As the modification of the solution of VERRUIJT and BOOKER [5], the solution of LOGANATHAN and POULOS [7] has a better estimated ground movement in the stiff clay. However, by analyzing the numerical results [7], it is found that the ground settlement curves of the Thunder Bay Tunnel, the Barcelona Subway Tunnel and the Bangkok Sewer Tunnel are all obtained when the Poisson ratio is equal to 0.5, and the difference between the result and the observed value would be significant if the Poisson ratio is set to the actual value.

Therefore, it is found that the influence of soil volume compression should not be considered because of the stratum displacement caused by the ground loss. Instead, the Poisson ratio should be set to 0.5, which means that the volume of ground settling tank is equal to the ground loss volume. VERRUIJT and BOOKER [5] presented that the volume of ground settling tank is larger than the ground loss volume when considering the effect of drainage and consolidation and the SAGASETA's solution would underestimate the ground settlement. However, it is found that the effect of these factors on the ground displacement cannot be determined only by the Poisson ratio. In addition, other methods should be employed to investigate this problem. The strength reduction theory is used to study the ground settlement and deformation caused by tunnel excavation and dewatering, and the long-term consolidation settlement [10−15].

4.2 Improvement of analytical solution

In order to consider the influence of stratum conditions comprehensively, the Poisson ratio is needed to be modified by introducing a parameter related to soil types, when the Poisson ratio is set to 0.5. In this work, the above analytical solution in the non-uniform convergence model is modified. The process is as follows.

The equivalent ground loss $\varepsilon_{x,z}$ is employed instead of the uniform radial displacement ε in the solution of VERRUIJT and BOOKER [5] proposed by LOGANATHAN and POULOS [7], and the expression of $\varepsilon_{x,z}$ can be written as

$$\varepsilon_{x,z}=\varepsilon_0 A\exp\left[-(Bx^2+Cz^2)\right] \tag{15}$$

$$\varepsilon_0=\left[\pi\left(R+g/2\right)^2-\pi R^2\right]/\pi R^2\approx g/R \tag{16}$$

where parameters A, B and C can be determined by the stratum settlement boundary condition which were assumed by LOGANATHAN and POULOS [7]. Based on the modified solution of LOGANATHAN and POULOS [7], ignoring the influence of liner deformation, and setting the Poisson ratio to be 0.5, Eqs.(13) and (14) are modified as

$$U_{x0}=2gR\frac{x}{x^2+h^2}A\exp(-Bx^2) \tag{17}$$

$$U_{z0}=2gR\frac{h}{x^2+h^2}A\exp(-Bx^2) \tag{18}$$

Parameter A is determined by the volume incompressible hypothesis, and can be expressed as

$$\frac{2gRA}{h}=\frac{V_s}{2.5i} \tag{19}$$

where $V_s=\pi gR$, $i=Kh$, K is the width parameter of settlement tank (Ref.[12] gave its initial propositional values under different soil types), substituting them into Eq.(19), we have

$$A=\pi/5K \tag{20}$$

Parameter B is determined by the ground settlement trough width. In the Peck formula, when $x=i$, we can obtain

$$S(x)=0.606\,5\,S_{max} \tag{21}$$

By substituting Eq.(21) into Eq.(15), the expression of B can be written as

$$B=\frac{\ln[0.606\,5(i^2+h^2)/h^2]}{-i^2}=\frac{\ln[(K^2+1)0.606\,5]}{-K^2h^2} \tag{22}$$

After substituting Eq.(20) and Eq.(22) into Eq.(17) and Eq.(18), we can get the modified vertical ground displacement and horizontal ground displacement.

Similarly, based on the modified solution of LOGANATHAN and POULOS [7], the formulas of the vertical displacement and horizontal displacement at any point are refined as

$$u_x=-\frac{\pi}{5K}Rgx\left\{\frac{1}{x^2+(z-h)^2}+\frac{1}{x^2+(z+h)^2}-\frac{4z(z+h)}{[x^2+(z+h)^2]^2}\right\}\exp[-(Bx^2+Cz^2)] \tag{23}$$

$$u_z=\frac{\pi}{5K}Rg\left\{-\frac{z-h}{x^2+(z-h)^2}+\frac{z+h}{x^2+(z+h)^2}-\frac{2z[x^2-(z+h)^2]}{[x^2+(z+h)^2]^2}\right\}\exp[-(Bx^2+Cz^2)] \tag{24}$$

where parameter C is determined by the condition that $u_z=g$ when $x=0$ and $z=h-g$:

$$C\approx-\ln\left[\frac{5K}{\pi}\frac{(2h-R)^2}{4h^2-2R^2}\right]/(h-R)^2 \tag{25}$$

4.3 Engineering example analysis

1) Example 1: Heathrow Express Trail Tunnel

The tunnel connects the center of London and the Heathrow airport. The tunnel diameter, length and depth are 8.5 m, 8 km and 19 m, respectively. The stratum conditions are as follows: 0−2 m in fill ground, 2−4 m in terrace gravel, below 4 m for the stiff London clay. Based on Refs.[7, 13], the ground settlement width parameter and the gap parameter are set to be 0.43 and 58 mm, respectively. According to Eqs.(18), (23) and (24), the ground settlement curve, horizontal displacement 9 m away from the tunnel centerline and the settlement above the centerline are shown in Fig.3.

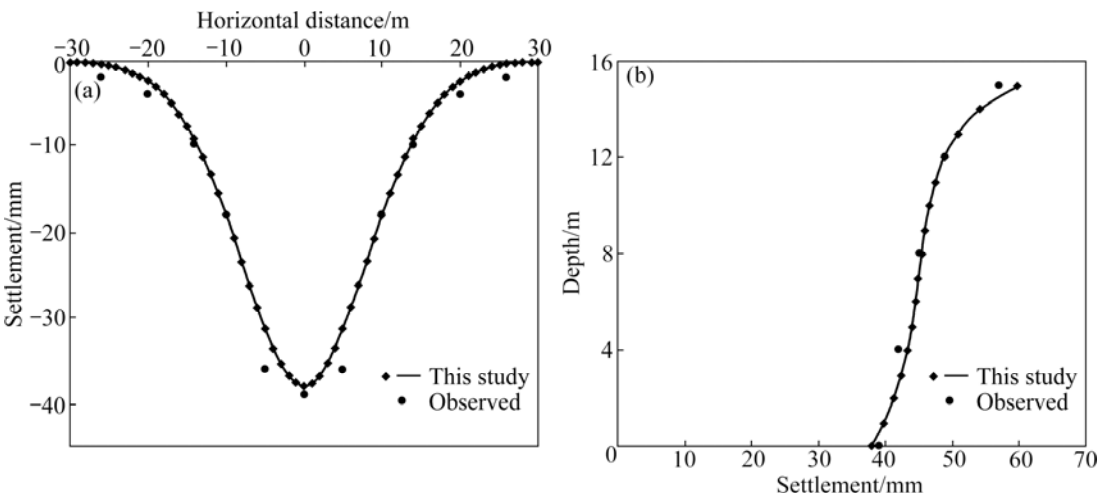

Fig.3 Contrast between prediction value and field data in Heathrow Subway Tunnel: (a) Ground settlement curve; (b) Horizontal displacement 9 m away from tunnel centerline

2) Example 2: Canada Thunder Bay Tunnel

The tunnel diameter, length and depth are 2.47 m, 3.3 km and 10.5 m, respectively. The stratum conditions are as follows: 0−8 m in silty sand, and 8−13 m from soft to firm clay. Based on Refs.[12−13], the ground settlement width parameter and the gap parameter are set to be 0.28 and 95.6 mm, respectively. Figure 4 presents the ground settlement curve, the horizontal displacement 2.2 m away from the tunnel centerline and the settlement above the centerline.

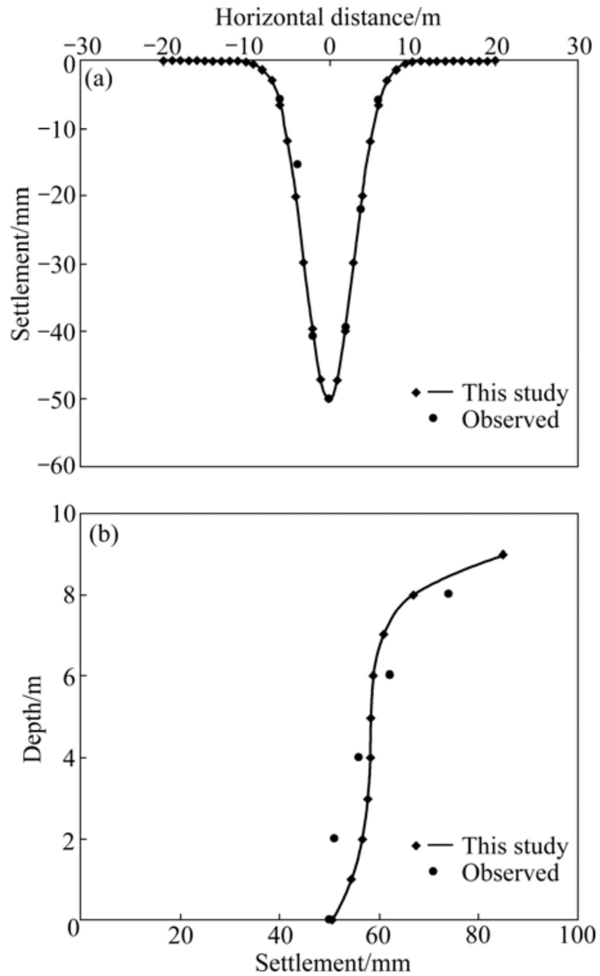

Fig.4 Contrast between prediction value and field data in Thunder Bay Tunnel: (a) Ground settlement curve; (b) Horizontal displacement 2.2 m away from tunnel centerline

3) Example 3: UK Green Park Tunnel

The tunnel was excavated by hand circular shield. The formation in 0−2 m below the ground surface is sand and gravel, and the formation below 2 m is stiff heavily London clay. Based on Refs.[7, 13], the centre of tunnel is 29.4 m below the ground surface, and the tunnel diameter is 1.14 m. Furthermore, the ground settlement width parameter and the gap parameter is 0.4 and 27 mm respectively. Figure 5 presents the ground settlement curve, and the settlement above the centerline.

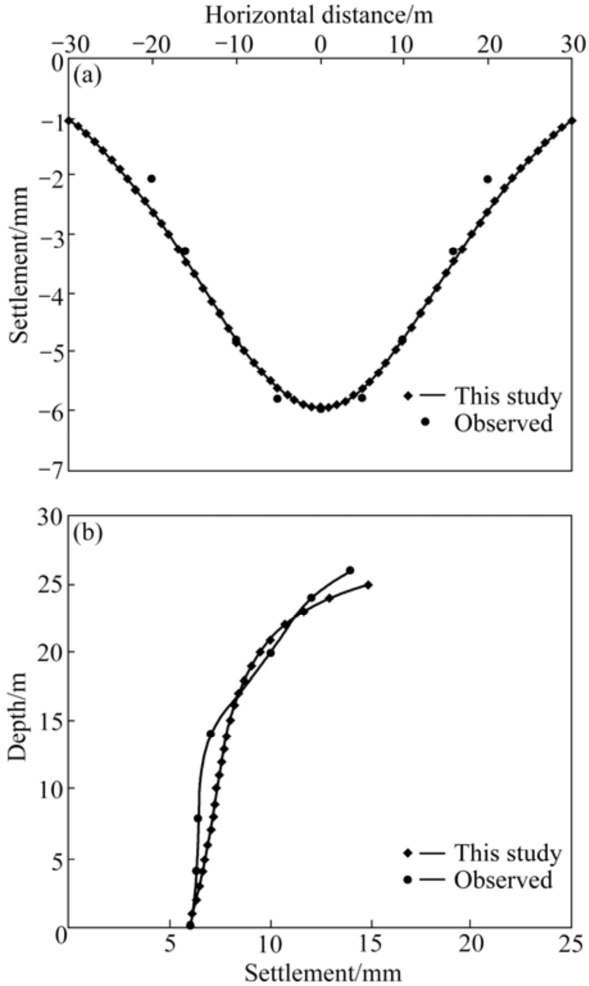

Fig.5 Contrast between prediction value and field data in Green Park Tunnel: (a) Ground settlement curve; (b) Settlement above tunnel centerline

4) Example 4: Thailand Bangkok Sewer Tunnel

The tunnel was excavated by hand circular shield. Based on Refs.[7, 13], the formation in 0−12 m below the ground surface is soft clay, the formation below 12−25 m is cracked soil, and the stratum below 25−35 m is fine sand. The centre of tunnel is 18.5 m below the ground surface, and the tunnel diameter is 2.66 m. Furthermore, the ground settlement width parameter and the gap parameter are 0.47 and 62 mm, respectively. The ground settlement curve and the settlement above centerline are shown in Fig 6.

By comparing the predicted values and the observed values among Thunder Bay Tunnel, Heathrow Subway Tunnel, Green Park Tunnel, Barcelona Subway Network Extension, and Bangkok Sewer Tunnel which are shown in Figs.3−6, it can be found that the modified image analytical solutions can provide better prediction of the ground settlement, stratum horizontal displacement and stratum vertical displacement.

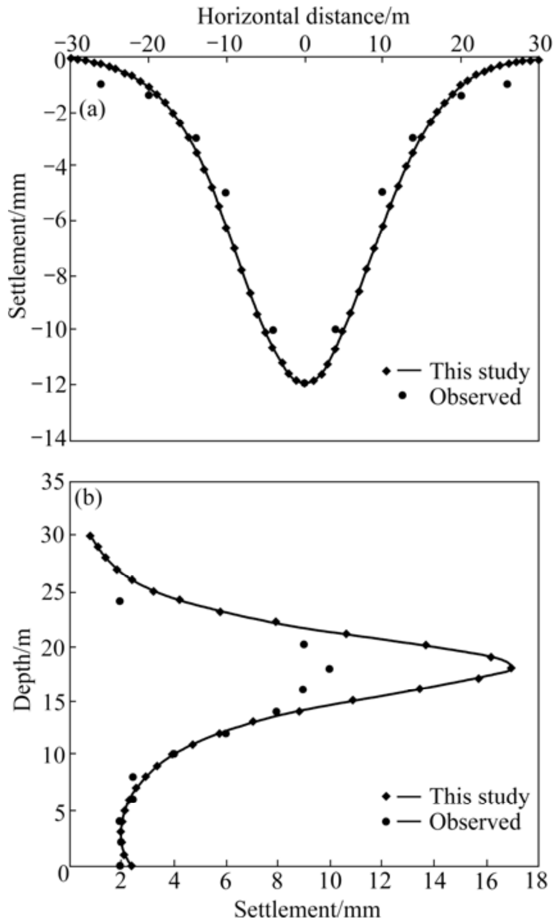

Fig.6 Contrast between prediction value and field data in Bangkok Sewer Tunnel: (a) Ground settlement curve; (b) Settlement above tunnel centerline

5 Conclusions

1) Based on the image theory and nonuniform convergence model, the analytical solution of tunneling-induced ground displacement is derived. The reasonable value of Poisson ratio in the analytical solution is discussed. From the results, it is found that the analytical solution matches the observed value very well when the Poisson ratio is equal to 0.5. The ground settlement width parameter which can reflect the ground condition is introduced into the present solutions to calculate the ground settlement, the stratum horizontal displacement and the stratum vertical displacement.

2) By comparing the prediction values and the observed values among example engineering, it can be found that the modified image solution derived in the present discussion is an effective technique for predicting the ground movement caused by the underground excavation.

3) Owing to the use of the ground settlement width parameter, the analytical solution in the framework of the image theory is modified. By analyzing the engineering projects which are employed in the modified image theory, it is found that the modified solution is suitable for a variety of stratum situations and matches the observed value better than the image analytical solution using the uniform convergence model.

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