J. Cent. South Univ. Technol. (2010) 17: 744−749
DOI: 10.1007/s11771−010−0550−0

Springer

Optimization model of unascertained measurement for underground mining method selection and its application

LIU Ai-hua(刘爱华), DONG Lei(董蕾), DONG Long-jun(董陇军)

School of Resources and Safety Engineering, Central South University, Changsha 410083, China

Abstract: An optimization model of underground mining method selection was established on the basis of the unascertained measurement theory. Considering the geologic conditions, technology, economy and safety production, ten main factors influencing the selection of mining method were taken into account, and the comprehensive evaluation index system of mining method selection was constructed. The unascertained evaluation indices corresponding to the selected factors for the actual situation were solved both qualitatively and quantitatively. New measurement standards were constructed. Then, the unascertained measurement function of each evaluation index was established. The index weights of the factors were calculated by entropy theory, and credible degree recognition criteria were established according to the unascertained measurement theory. The results of mining method evaluation were obtained using the credible degree criteria, thus the best underground mining method was determined. Furthermore, this model was employed for the comprehensive evaluation and selection of the chosen standard mining methods in Xinli Gold Mine in Sanshandao of China. The results show that the relative superiority degrees of mining methods can be calculated using the unascertained measurement optimization model, so the optimal method can be easily determined. Meanwhile, the proposed method can take into account large amount of uncertain information in mining method selection, which can provide an effective way for selecting the optimal underground mining method.

Key words: mining engineering; underground mining method; optimization model; unascertained measurement theory; information entropy

1 Introduction

Underground mining method selection is one of the most important decisions that mining engineers have to make. Choosing a suitable underground mining method to extract a mineral deposit is very important in terms of safety production, increasing ore production, reducing loss rate and ore dilution rate and improving productivity of mining operations [1]. However, underground mining method selection is a complex systematic engineering. The ambiguous factors and lack of geological information make underground mining method selection be of great subjectivity and randomness, which bring complexity and difficulty to the right decision of mining method [2−3].

In fact, traditional underground mining method selection is only determined by a single factor or by the intuitive evaluation of several factors. The evaluation is constrained to human abilities, easily getting subjective and incomplete results. In recent years, many scholars worldwide have introduced many new theories and methods in the selection of underground mining method, which mainly include fuzzy mathematics [4], analytical hierarchy process (AHP) [5−6], gray correlation analysis method [7], gray situation decision making [8], multi-objective decision making [9−10], value engineering [11] as well as artificial intelligence [12]. Fuzzy mathematics and multi-objective decision-making had certain degree of randomness and subjectivity when determining the weight of factors, and gray theory did not take into account the relative important extent of various objectives and factors, while value engineering and artificial intelligence had the disadvantage of narrow knowledge access and poor applicability. The most important thing during the selection process of the best mining method is to take into account large amount of uncertain information. The unascertained mathematics theory can be a new way to do so.

The concept of unascertained information theory was first proposed by WANG [13] in 1990 in the research of architectural engineering, which was different from the fuzzy information, the stochastic information and the gray information. The unascertained measurement optimization model of underground mining method selection was established according to the theory

Foundation item: Project(2007CB209402) supported by the National Basic Research Program of China; Project(SKLGDUEK0906) supported by the Research Fund of State Key Laboratory for Geomechanics and Deep Underground Engineering of China
Received date: 2009−12−10; **Accepted date**: 2010−03−23
Corresponding author: LIU Ai-hua, PhD, Professor; Tel: +86−13755006918; E-mail: alexliu@163.com

of unascertained measurement. The value of the unascertained measurement of each index was calculated, and the indices weights of the factors were calculated using the entropy theory. Finally, credible recognition criteria were established for the evaluation of each mining method. The superiority degrees of mining methods were arranged in order by the credible degree criteria, thus determining the best underground mining method. The proposed model was used in the underground mining method selection of Xinli Gold Mine in Sanshandao of China and obtained ideal results, which can provide a new way for efficient and optimal selection of underground mining method.

2 Unascertained model of underground mining method optimal selection

2.1 Determining classification model system of optimized objects

Suppose R_1, R_2, ⋯, R_n are n objects to be optimized, and the optimization object space is $\boldsymbol{R}=\{R_1, R_2, \cdots, R_n\}$. Each object of R_i (i=1, 2, ⋯, n) has m evaluating indices, so the evaluating index space is $\boldsymbol{X}=\{x^1, x^2, \cdots, x^m\}$. Then, R_i can be denoted as m-dimension $\boldsymbol{R}_i=\{x_i^1, x_i^2, \cdots, x_i^m\}$, where x_i^j is the measured value of optimization object $\boldsymbol{R}_i$ with respect to evaluating index x^j. For different x_i^j, the contribution to the optimized result $\boldsymbol{R}_i$ is different. x_i^j can be divided into two categories of A and B. For category A, the larger the value of x_i^j, the greater the contribution to the superiority degree $\boldsymbol{Q}$; while the smaller the value, the greater the contribution to that of $\boldsymbol{Q}$ for category B. For each x_i^j (i=1, 2, ⋯, n; j=1, 2, ⋯, m), we assume that there are p evaluation grades of C_1, C_2, ⋯, C_p.

The evaluation space is $\boldsymbol{U}$, denoted as $\{C_1, C_2, \cdots, C_p\}$. Suppose C_k (k=1, 2, ⋯, p) is the kth evaluation grade, and the kth grade is higher than the (k+1)th one, denoted as $C_k > C_{k+1}$. If the grading rank $\{C_1, C_2, \cdots, C_p\}$ satisfies $C_1 > C_2 > C_3 > \cdots > C_p$ or $C_1 < C_2 < C_3 < \cdots < C_p$, $\{C_1, C_2, \cdots, C_p\}$ is called the ordered partition class of evaluation space $\boldsymbol{U}$ [14].

2.2 Unascertained measurement of single index

Denote the unascertained measurement as $\mu_{ik}^j = \mu\,(x \in C_k)$, where μ_{ik}^j is the degree of x_i^j belonging to the kth evaluation grade of C_k, which satisfies

$$0 \leqslant \mu(x_i^j \in C_k) \leqslant 1 \tag{1}$$

$$\mu(x_i^j \in \boldsymbol{U}) = 1 \tag{2}$$

$$\mu\left[x_i^j \in \bigcup_{i=1}^{k} C_l\right] = \sum_{i=1}^{k} \mu(x_i^j \in C_l) \quad (k=1, 2, \cdots, p) \tag{3}$$

where Eq.(1) is defined as non-negative bound, Eq.(2) is called normalization and Eq.(3) is called additivity. Then, μ satisfying Eqs.(1)−(3) is called unascertained measurement.

2.3 Determination of index weights

The order degree and effectiveness of the obtained system information can be evaluated using the information entropy [15] when determining the weights of evaluating indices, that is, the weights are determined by the judgment matrix consisting of the value of evaluating indices.

Suppose w_j is the relative important extent of measured index X_j compared with other indices, w_j satisfies $0 \leqslant w_j \leqslant 1$, which is called the weight of x_i. Index weight vector $\boldsymbol{w}$ is characterized by $\boldsymbol{w}=\{w_1, w_2, \cdots, w_m\}$. Then, w_j is given by

$$w_j = v_j \Big/ \sum_{i=1}^{n} v_j \tag{4}$$

$$v_j = 1 + \frac{1}{\lg k} \sum_{i=1}^{k} \mu_{ik}^j \lg \mu_{ik}^j \tag{5}$$

The evaluation matrix of unascertained measurement of single index is known, so w_j can be obtained by Eqs.(4) and (5).

2.4 Composite unascertained measurement of multiple indices

μ_{ik} is denoted as the degree of the optimized object $\boldsymbol{R}_i$ belonging to the kth evaluation grade of C_k. When μ_{ik} is equal to $\mu(R_i \in C_k)$, μ_{ik} is called the composite unascertained measurement of multiple indices.

Based on the unascertained measurement of single index and index weights, the composite unascertained measurement of multiple indices can be worked out as follows:

$$\mu_{ik} = \sum_{j=1}^{m} w_j \mu_{ik}^j \quad (i=1, 2, , n;\ k=1, 2, \cdots, p) \tag{6}$$

where μ_{ik} satisfies $0 \leqslant \mu_{ik} \leqslant 1$ and $\sum_{k=1}^{k} \mu_{ik} = \sum_{k=1}^{k}\sum_{j=1}^{m} w_j \mu_{ik}^j = \sum_{j=1}^{m}\left(\sum_{k=1}^{k} \mu_{ik}^j\right) w_j = 1$.

2.5 Optimization results recognizing and sequencing

It is necessary to adopt the credible degree identification rule, in order to get the final results of the optimized objects. λ ($\lambda \geqslant 0.5$) is denoted as the credible degree. If the evaluation space $\{C_1, C_2, \cdots, C_p\}$ is orderly and meets $C_1 > C_2 > \cdots > C_p$, let

$$k_0 = \min\{k : \sum_{i=1}^{k} \mu_{i1} \geqslant \lambda,\ (k=1, 2, \cdots, p)\} \tag{7}$$

Then, $\boldsymbol{R}_i$ belongs to the k_0th evaluating grade of C_{k0}.

Suppose the score value of C_l is I_l, Q_{R_i} is given by

$$Q_{R_i}=\sum_{i=1}^{p} I_l \cdot \mu_{il} \tag{8}$$

where Q_{R_i} is the unascertained superiority degree of optimization object $\boldsymbol{R}_i$, so $\boldsymbol{Q}=(Q_{R_1}, Q_{R_2}, \cdots, Q_{R_i})$ is called the vector of unascertained superiority degree. The superiority degree of $\boldsymbol{R}_i$ is ordered according to the magnitude of Q_{R_i}.

3 Construction of comprehensive evaluation index system of underground mining method

The optimal mining method should be primarily selected to fit to the geology and occurrence conditions of deposit to ensure safety in production. Besides, it should maximize economic and social benefits as much as possible. Therefore, a lot of factors have to be taken into consideration during the selection process of mining method. The degrees of the influence of the factors are different, so all the factors should be weighted in order to reflect the influencing degrees more precisely. On the basis of the related studies [1−2], ten factors affecting the selection of the best mining method are selected as the evaluation indices, which are stope capacity, mining efficiency, mining cost, ore loss rate, ore dilution rate, mining cutting ratio, operation safety degree, ventilation condition, degree of difficulty in implementation and adaptive degree to the change of orebody, designated as X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 and X_{10}, respectively. Among these indices, stope capacity (X_1), mining efficiency (X_2), mining cost (X_3), ore loss rate (X_4), ore dilution rate (X_5) and mining cutting ratio (X_6) are evaluated using quantitative data. And the classification standards for these indices are determined with reference to similar mines [1, 6], combined with mining methods, as shown in Table 1. The evaluation set is $\{C_1, C_2, C_3\}$, designated classes C_1, C_2 and C_3, which are denoted as poor, medium and superior, respectively. And operation safety degree (X_7), ventilation condition (X_8), degree of difficulty in implementation (X_9) and adaptive degree to the change of orebody (X_{10}) are evaluated by changing non-quantized index into quantized index. In order to avoid one-sidedness and randomness, the classification and value assignment for qualitative indices are decided using expert assessment method. And normalization processing is made for the obtained results, which are given in Table 2. The indices for category A are: stope capacity (X_1), mining efficiency (X_2), operation safety degree (X_7), ventilation condition (X_8), degree of difficulty in implementation (X_9), adaptive degree to the change of orebody (X_{10}). And the indices for category B are: mining cost (X_3), ore loss rate (X_4), ore dilution rate (X_5) and mining cutting ratio (X_6). It should also be noted that the values in Tables 1 and 2 are relative values, and the variation of the specific values does not affect the ranking of the superiority of mining methods though it may affect the level of superiority degree.

4 Unascertained measurement optimization model for underground mining method selection and its engineering application

4.1 Engineering conditions

Sanshandao Gold Mine is located in the special industrial area of Sanshandao in Laizhou, Shandong Province, China. Xinli Mine Field is located in the southwest of Sanshandao Gold Mine, which belongs to submarine deposit. The capacity of Xinli Gold Mine is required to reach 6 000 t/d. The ore body is 1 145 m-long in strike, 135−900 m in dip, with a thickness of 0.48−40.65 m (average thickness of 8.96 m). It presents little undulatory form in strike and dip, the dip of it is southeastward and inclination angle is 33°−67°, with an average dip angle of 46°. The orebody is close to the main controlling fault, which contains fault gouge with a thickness of 5−10 cm. The fractured rock on the hanging wall of the orebody would collapse once exposed. The morphology of orebody is inclined, with the thickness changing greatly, and the boundary between ore body and surrounding rock is unclear. And the ore body has developed joints and fissures. So there is difficulty in maintaining the hanging wall rock and ore body. Therefore, the suitable mining method is extremely important for this complicated ore body.

According to the principle of underground mining method selection, six representative mining methods are selected, combined with the mining technical conditions of Xinli Gold Mine in Sanshandao. The six methods are: in-vein reparatory work and deep and medium hole caving with subsequent filling method (Method Ⅰ),

Table 1 Value assignment of three-level indicators of evaluation indices in underground mining method selection

Classification standard	Stope capacity (X_1)/(t·d^{-1})	Mining efficiency (X_2)/(t·shift^{-1})	Mining cost (X_3)/(RMB¥·t^{-1})	Ore loss rate (X_4)/%	Ore dilution rate (X_5)/%	Mining cutting ratio (X_6)/(m^3·kt^{-1})
C_1	<100	<15	>70	>18	>8	>100
C_2	100−200	15−30	70−60	18−10	8−5	100−50
C_3	>200	>30	<60	<10	<5	<50

mechanized upward horizontal cut and fill stopping (Method Ⅱ), out-vein preparatory work and point-pillar sublevel filling method (Method Ⅲ), room and pillar sublevel filling method (Method Ⅳ), high access back-filling method (Method Ⅴ), and point and pillar individual area filling method (Method Ⅵ). The indicators of the six mining methods are given in Table 3.

4.2 Construction of unascertained measurement function of single index

The unascertained measurement functions of single index were constructed to get the value of the evaluation factors, on the basis of the definition of the unascertained measurement function and the classification in Tables 1 and 2. The unascertained measurement function of each index is illustrated in Fig.1. Then, the evaluation matrix of unascertained measurement of six mining methods could be obtained, according to the functions in Figs.1(a)−(g) and values of the factors given in Table 3. Taking method Ⅰ for example, the values of the ten evaluation indices for method Ⅰ in Table 3 were substituted into the corresponding unascertained measurement functions in Figs.1(a)−(g), respectively. Then, the evaluation matrix of unascertained measurement of method Ⅰ was calculated as

$$(\mu_{1jk})_{10\times3}=\begin{bmatrix}0.00 & 0.00 & 1.00\\0.00 & 0.00 & 1.00\\0.00 & 0.00 & 1.00\\1.00 & 0.00 & 0.00\\1.00 & 0.00 & 0.00\\0.00 & 0.12 & 0.88\\0.00 & 1.00 & 0.00\\0.00 & 1.00 & 0.00\\0.00 & 1.00 & 0.00\\1.00 & 0.00 & 0.00\end{bmatrix} \tag{9}$$

4.3 Composite unascertained measurement of multiple indices of mining method

The weights of the indices were determined by Eqs.(1)−(6). So the weights of $\boldsymbol{R}_1$ denoted as $\{w_1, w_2, \cdots, w_{10}\}$ were {0.103 4, 0.103 4, 0.103 4, 0.103 4, 0.103 4, 0.069 4, 0.103 4, 0.103 4, 0.103 4, 0.103 4}. Then, the composite unascertained measurement of multiple indices of $\boldsymbol{R}_1$ were calculated as {0.310 2, 0.318 5, 0.371 3}.

Table 2 Classification and value of qualitative indices in underground mining method selection

Classification standard	Value	Qualitative indices			
		Operation safety degree (X_7)	Ventilation condition (X_8)	Degree of difficulty in implementation (X_9)	Adaptive degree to change of orebody (X_{10})
Class C_1	0.17	Support cost is high, safety and stability are worse	Ventilation condition is poor, more air flow regulating facilities are required, and ventilation cost is high	Filling work intensity is great, operation links and production management work are complicated	It is suitable for exploitation of regular deposit with single occurrence state
Class C_2	0.50	Roof-contacted filling is of high difficulty, support treatments are needed, and pillars are designed to ensure safety of stope	Ventilation network is simple, but auxiliary fans are required in local area to improve ventilation condition	Stope should be exploited in certain order, and difficulty of production management is increased	It can be applied to exploitation of irregular deposit, but dilution ratio and loss ratio are great
Class C_3	0.83	Ground pressure is controlled well, mining operation is in good safety and stability	Distribution of air flow in mine is uniform, ventilation condition is good and reliable, and ventilation cost is low with small engineering quantity	Stope mining is not interfered by others, and production management and intensified mining can be carried out	It is flexible and suitable for irregular deposit

Table 3 Measured data of mining methods evaluation indices

Method No.	Measured data of evaluation indices									
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Method Ⅰ	400.0	42.7	52.97	19.0	8.5	52.97	0.50	0.50	0.50	0.17
Method Ⅱ	105.0	16.6	60.22	17.5	6.0	105.80	0.50	0.83	0.50	0.83
Method Ⅲ	108.7	17.2	57.72	19.3	6.0	82.60	0.50	0.50	0.17	0.50
Method Ⅳ	93.2	15.4	72.61	13.3	6.0	115.90	0.50	0.50	0.17	0.17
Method Ⅴ	73.4	13.6	67.96	9.0	5.0	25.00	0.83	0.17	0.50	0.50
Method Ⅵ	103.5	17.4	61.61	16.2	6.0	48.70	0.83	0.50	0.17	0.50

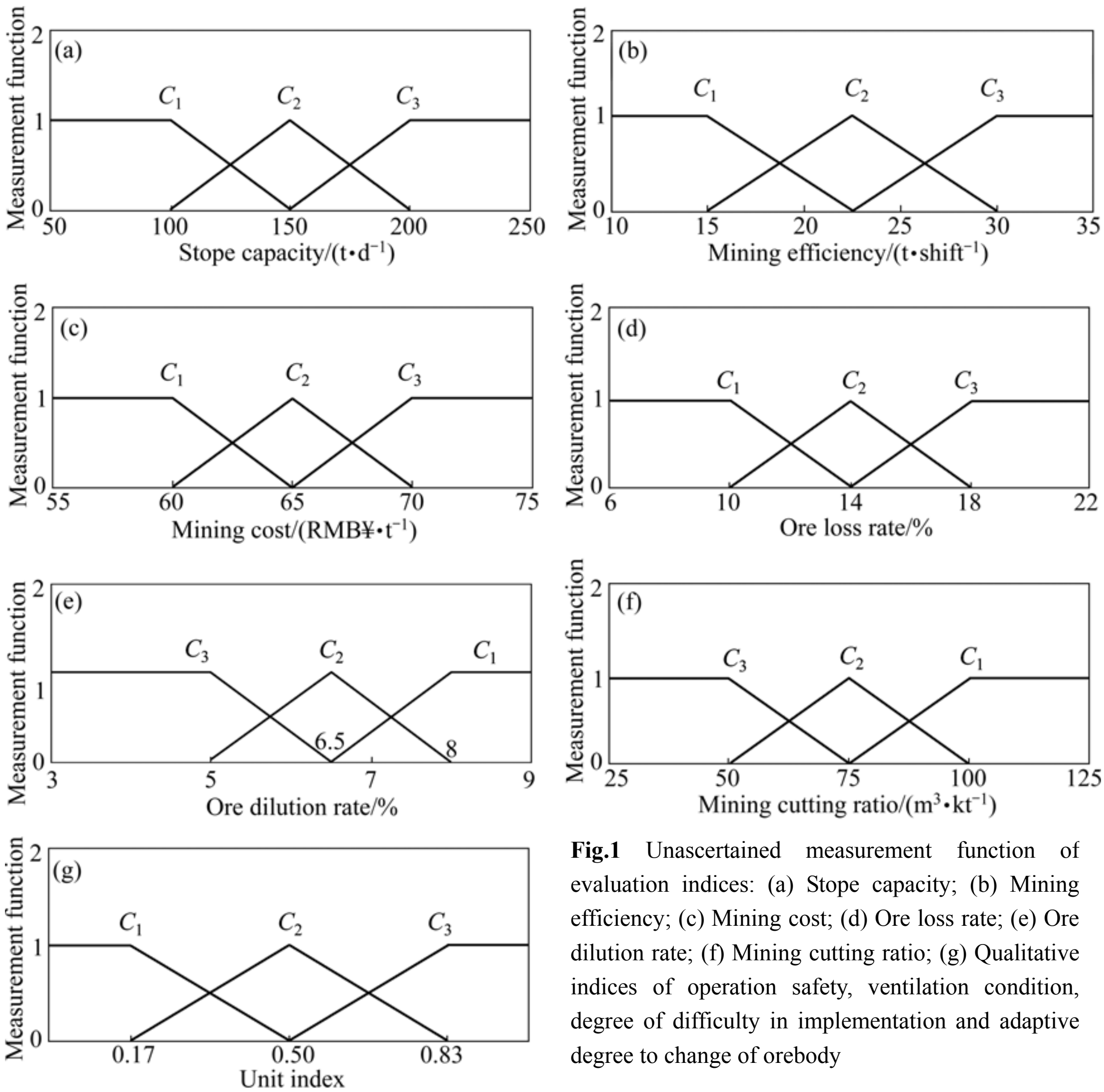

Fig.1 Unascertained measurement function of evaluation indices: (a) Stope capacity; (b) Mining efficiency; (c) Mining cost; (d) Ore loss rate; (e) Ore dilution rate; (f) Mining cutting ratio; (g) Qualitative indices of operation safety, ventilation condition, degree of difficulty in implementation and adaptive degree to change of orebody

4.4 Optimization results recognizing

The credible recognition rule was adopted to judge the grade of X_i instead of the maximum measurement identification rule, due to the sequence of the evaluating grade $\{C_1, C_2, C_3\}$.

The confidence level was taken as 0.6. According to Eq.(7) of composite unascertained measurement vector of multiple indices and Eq.(8) of credible recognition rule, k_0=0.628 7, which was larger than λ in ascending order, thus the superiority degree of $\boldsymbol{R}_1$ belonged to grade C_2; when in descending order, k_0=0.689 8$>\lambda$, the same result could be obtained.

From the above, we can get that the two identified results are in accordance. Therefore, the superiority degree of $\boldsymbol{R}_1$ is determined as grade C_2, that is, method Ⅰ is not the most suitable. In the same way, the other mining methods are evaluated. The composite unascertained measurement of multiple indices and the optimized results are listed in Table 4. It shows that the superiority degrees of methods Ⅰ−Ⅵ are 2.061 1, 2.038 6, 1.784 8, 1.377 1, 2.082 4 and 2.029 4, respectively.

Table 4 Results of unascertained measurement evaluation

Method No.	C_1	C_2	C_3	Result	Superiority degree
Method Ⅰ	0.310 2	0.318 5	0.371 3	Medium	2.061 1
Method Ⅱ	0.322 2	0.317 0	0.360 8	Medium	2.038 6
Method Ⅲ	0.361 1	0.493 0	0.145 9	Medium	1.784 8
Method Ⅳ	0.650 4	0.322 1	0.027 5	Poor	1.377 1
Method Ⅴ	0.343 9	0.229 8	0.426 3	Superior	2.082 4
Method Ⅵ	0.298 1	0.374 4	0.327 5	Medium	2.029 4

4.5 Analysis of evaluation results

The conclusions can be drawn from Table 4 that the superiority degrees of the mining methods are decreasingly ordered as follows: method Ⅴ, method Ⅰ, method Ⅱ, method Ⅵ, method Ⅲ and method Ⅳ. Therefore, method Ⅴ is superior to other methods, which is selected as the best mining method. The practice shows that the chosen mining method is applicable to Xinli Gold Mine and gets high technical and economic benefits.

5 Conclusions

(1) Large numbers of factors are involved in the selection of underground mining method. In this work, ten evaluation indices including stope capacity, mining efficiency, mining cost, ore loss rate, ore dilution rate, mining cutting ratio, operation safety degree, ventilation condition, degree of difficulty in implementation and adaptive degree to the change of orebody are taken into consideration. The comprehensive evaluation index system of mining method selection is constructed. And new measurement standards are constructed. Then the unascertained measurement functions of ten evaluation indices are established.

(2) The optimal selection model of underground mining method is established based on the unascertained measurement theory. During the selecting process, the indices weights of the factors are comparatively determined using the entropy theory, which avoids the disadvantage of difficulty in the weight distribution of so many factors. And then the evaluation results are obtained according to the magnitudes of unascertained superiority degrees.

(3) The unascertained measurement optimization model is applied to the comprehensive evaluation and selection of the preselected standard mining methods in Xinli Gold Mine in Sanshandao. According to the evaluation results of the unascertained measurement optimization model, method Ⅴ is selected as the best mining method. And the result is compared with the practical situation, which shows the high access filling mining method has already made some favorable effects. The model is proved to be reasonable and effective for underground mining method selection, which enriches the methods of mining method selection. Furthermore, the comprehensive evaluation model can also be used for the multi-scheme optimal selection of other system engineering.

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(Edited by YANG You-ping)