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Stability analysis of subgrade cave roofs in karst region

JIANG Chong(蒋 冲)[1, 2], ZHAO Ming-hua(赵明华)[1], CAO Wen-gui(曹文贵)[1]

(1. Geotechnical Engineering Institute, Hunan University, Changsha 410082, China;
2. School of Resources and Safety Engineering, Central South University, Changsha 410083, China)

Abstract: According to the engineering features of subgrade cave roof in karst region, the clamped beam model of subgrade cave roof in karst region was set up. Based on the catastrophe theory, the cusp catastrophe model for bearing capacity of subgrade cave roof and safe thickness of subgrade cave roof in karst region was established. The necessary instability conditions of subgrade cave roof were deduced, and then the methods to determine safe thickness of cave roofs under piles and bearing capacity of subgrade cave roof were proposed. At the same time, a practical engineering project was applied to verifying this method, which has been proved successfu1ly. At last, the major factors that affect the stability on cave roof under pile in karst region were deeply discussed and some results in quality were acquired.

Key words: pile foundation; karst; subgrade cave roof; cusp catastrophe model; stability

1 Introduction

The determination method about the bearing capacity of subgrade cave roof and the safe thickness of its lower karst cave roof is the key content of karst area subgrade cave roof design. Therefore, the research on related problems is of important theoretical and practical engineering value. So far, the determination method of the bearing capacity of subgrade cave roof and the safe thickness of the karst subgrade cave roof is traditional empirical semiquantitative analytical technique. Lots of empirical semiquantitative analytical techniques about the safe thickness of karst cave roof such as the method based on the beam slab force mode, pressure arch theory and roof fall and blocking conception can be found in Ref.[1], but the calculation results are rough, because all the methods cannot reflect the force condition of the pile and the roof rock mass accurately. Study on the strata safe thickness of karst area bearing course at pile tip can be found in some literatures, which are based on the limit equilibrium analysis theory[2−7]. However, because the limited knowledge of karst area rock-socketed pile bearing capacity and the force condition and damage mechanism of the karst cave roof under the pile foundation, there is distance between the calculation result and the practical engineering. Using nonlinear finite element and numerical manifold method respectively, combining with strength reduction technique and enumeration, LI et al[8] and CAO et al[9] set up numerical method for roof safe thickness determination of highway roadbed limestone cave, which is a beneficial attempt. But the force condition of the karst area bridge pile foundation is complex and the implementation process is very complicated, which limits the engineering application. So the further research on the determination method of the bearing capacity of subgrade cave roof and the safe thickness of its lower karst cave roof is urgent need. The purpose of this study is to simplify the determination method and rationalize the calculation result.

Therefore, firstly, according to the sudden features of subgrade cave roof, such as discontinuity, irregularity and non-uniform, in the destroy of cavern under the pile foundation of bridge in karst area, in this work, catastrophe theory was introduced[10] and the cusp catastrophe model about the bearing capacity of subgrade cave roof and the safe thickness of its lower karst cave roof was set up. Then, the failure and instability conditions of the subgrade cave roof in karst area were set up based on the model above. Finally, a new determination method about the bearing capacity of subgrade cave roof and the safe thickness of its lower karst cave roof was established.

2 Common way of karst cave roof stability analysis

There are many factors about karst cave roof, which has no calculation method now. Engineering analogy

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Corresponding author: JIANG Chong, PhD; Tel: +86-13808459476; E-mail: jiang4107@mail.csu.edu.cn

and calculation method are used in engineering usually.

2.1 Engineering analogy

According to the engineering experience and predecessor's research data, many factors such as slab thickness, roof shape (level, vaulted), roof integrity, length of buildings striding karst cave, which affect karst cave top stability. According to the different factors, they are divided into complete roof thick span ratio method and incomplete roof thickness and height ratio method.

2.2 Calculation method

Calculation method is based on engineering common theory or empirical formula to approximately calculate safe depth of karst cave top, so to evaluate karst cave top stability. Now engineering common calculation methods include PU rupture arch method, empirical formula method, roof collapse blocking method, collapse balance method, beam plate force model estimation method and shear concept estimation method.

Obviously, these methods belong to half theoretical and semi-empirical structural mechanical analysis method, and they are degree of limitation. So it is necessary to build a new method of safe depth of karst cave top determination.

3 Cusp catastrophe model for bearing capacity of subgrade cave roof and safe thickness of cave roofs in karst region

Catastrophe theory is a branch of non-linear science, and catastrophe theory was created by the French mathematician Rene Thom (1972). There are two ways to apply it to the research on discontinuity phenomenon. One based on the research system jump features is more used for the catastrophic phenomenon of the psychology and social sciences by selecting appropriate control variables and state variables. The other is establishing the system, such as in the mutant phenomenon in physics and mechanics[4, 11−13].

There are seven kinds of basic catastrophic models when the control variable less than or equal to 4 and state variable less than or equal to 2 were given in Ref.[10]. Among them, Cusp catastrophe model was most widely used to determine the bearing capacity of rock-socked piles and safe thickness of cave roofs in karst region by using Cusp catastrophe model accord to the appropriate mechanical model and establish a potential function and Cusp catastrophe model and its bifurcation set equation. The following will be introduced separately.

3.1 Mechanical model

The roof must first be reasonable simplification to the interaction system of cave roof and piles of rocks, now let us make the following assumptions.

1) The cave roof is complete and the occurrence texture is level and the cave roof is as beams to analyze the rock-socked pile as a direct role on the cave roof to the interaction system of the cave roof and rock-socked pile.

2) Considering rock-socked pile as simple end-bearing pile, not considering the lateral friction of upper soil around the pile and while ignoring the impact of stress.

3) Not considering the dead weight of rock roof and the force of the casing layer, the pile-resistance is simplified as an uniform load.

In this work, the clamped-clamped beam was only considered. The cave roof is simplified as a clamped-clamped beam, and the load on the cave roof is simplified as an uniform load (simple beam and the cantilever and concentrated load will be discussed in another paper), the simplified mechanical model is shown in Fig.1. Cave roof span is L, the level width is unit length, thickness is H, the rock elastic modulus is E, pile-resistance is simplified as uniform load q.

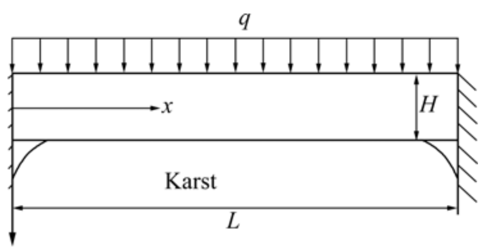

Fig.1 Simplified mechanical model

3.2 Determination of potential function

After the establishment of the mechanical model, the next step is critical to obtain the total potential function of the mechanical model system. After the establishment of potential function, it is translated into standard form of Cusp catastrophe model by mathematical method. And the standard form of potential function for Cusp catastrophe model is

$$f(x) = x^4 + ux^2 + vx \quad (1)$$

In this case, x is the state variable of system, u and v are control variables of the system. The three-dimensional space is set-up by x, u and v, as shown in Fig.2.

Supposing the axial line deflection curve of beam as

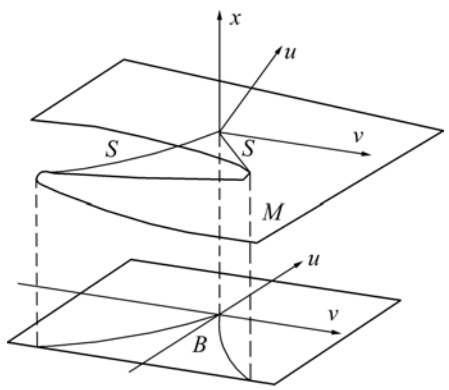

Fig.2 Cusp catastrophe model of cave roofs

$$\omega = A\left(1-\cos\left(\frac{2\pi x}{L}\right)\right) \tag{2}$$

where A is the axial line deflection of neutral point; x is the arc-length, ω is the deflection of beam.

The total potential function of the mechanical model system, is made up of bending strain energy of beam and power done by the external forces in the corresponding displacements, and the total potential function is

$$f(x) = U_1 + U_2 - W_1 \tag{3}$$

where U_1 is the bending strain energy of beam; U_2 is the potential energy increment; W_1 is the power by uniform load q in the corresponding displacements.

Based on the elastic mechanics, the bending strain energy of beam is

$$U_1 = \frac{1}{2}\int_0^l EI\left(\frac{\mathrm{d}^2\omega}{\mathrm{d}x^2}\right)^2 \sqrt{1+\left(\frac{\mathrm{d}\omega}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \tag{4}$$

where E is the rock elastic modulus, I is the moment of inertia of beam.

The potential energy increment is

$$U_2 = \int_0^L q\omega(x)\mathrm{d}x \tag{5}$$

The power done by uniform load q in the corresponding displacements is

$$W_1 = \int_0^l q(L-x)\left(\frac{\mathrm{d}\omega}{\mathrm{d}x}\right)^2 \mathrm{d}x \tag{6}$$

Substituting formula (4) and (6) into Eqn.(3):

$$f(x) = \int_0^L \frac{EI}{2}\left(\frac{\mathrm{d}^2\omega}{\mathrm{d}x^2}\right)^2 \sqrt{1+\left(\frac{\mathrm{d}\omega}{\mathrm{d}x}\right)^2}\,\mathrm{d}x + \int_0^L q\omega(x)\mathrm{d}x - \int_0^L \frac{q}{2}(L-x)\left(\frac{\mathrm{d}\omega}{\mathrm{d}x}\right)^2 \mathrm{d}x \tag{7}$$

To expansion Eqn.(7) as Taylor Series, the similar expressions as the potential function of the beam structure is

$$f(x) = \frac{EI\pi^6 A^4}{16L^5} + \left(\frac{2EI}{L^8\pi^2} - \frac{q\pi^2}{L^2}\right)A^2 - qL^6 A \tag{8}$$

Then, let

$$\begin{cases} A = \sqrt[4]{\dfrac{16L^5}{EI\pi^6 x}} \\ u = \left(\dfrac{2EI}{L^8\pi^2} - \dfrac{q\pi^2}{L^2}\right)\sqrt{\dfrac{16L^5}{EI\pi^6}} \\ v = -qL^6\sqrt[4]{\dfrac{16L^5}{EI\pi^6}} \end{cases} \tag{9}$$

Eqn.(1), the standard form of potential function of subgrade cave roof in karst region, is obtained.

3.3 Establishment of bifurcation set equation

According to the catastrophe theory, after derivation calculus to potential function $f(x)$, the system surface equilibrium equation is

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = 4x^3 + 2ux + v = 0 \tag{10}$$

After derivation calculus to system surface equilibrium Eqn.(10), the singular set of the structure system is as follows:

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} = 12x^2 + 2u = 0 \tag{11}$$

From Fig.2, it is a folded surface. When $u<0$, the folded surface is divided into three layers, which are upper layer, middle layer and lower layer.

In the middle layer:

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} < 0 \tag{12}$$

And in the upper and lower layer:

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} > 0 \tag{13}$$

On the surface, the point set of all the vertical tangent line meets Eqn.(11). Uniting Eqns.(10) and (11), deleting x, the bifurcation set equation of structure system cusp catastrophe model is as follows:

$$8u^3 + 27v^2 = 0 \tag{14}$$

It expresses the projection which is on plane of control variables u and v in geometry singular set.

Because it has the possibility of span bifurcation set when $u \leqslant 0$, so necessary condition of system catastrophe is $u \leqslant 0$. When u and v satisfy Eqn.(14), the system is in the critical balance status, which gets the critical condition of system catastrophe.

The bifurcation set equation of cusp catastrophe model of subgrade cave roof was obtained by substituting formula (9) into formula (14).

$$8\left[\left(\frac{2EI}{L^8\pi^2}-\frac{q\pi^2}{L^2}\right)\sqrt{\frac{16L^5}{EI\pi^6}}\right]^3+27\left[qL^6\sqrt[4]{\frac{16L^5}{EI\pi^6}}\right]^2=0 \quad (15)$$

4 Determination of subgrade cave roof stability condition

For the equilibrium surface equation of karst cave roof cusp catastrophe model(10), if $\Phi = 8u^3 + 27v^2$, when $u>0$, $\Phi>0$, Eqn.(10) has only one real root and corresponds to a stable equilibrium. In this case, the system deformation will be continuous and the karst cave roof has no catastrophe occurrences.

When $\Phi=0$, Eqn.(10) has three real roofs, among them two real roofs are equal. That is:

$$\begin{cases} x_1 = -v^{1/3} \\ x_2 = x_3 = v^{1/3}/2 \end{cases} \quad (16)$$

They correspond to the singular set of the equilibrium surface. In this case, the system is on two critical states of the stability. The system will leap from critical stable state to stable state with weak perturbation and the state variable leaps to:

$$\Delta x = (x_1 - x_2) = (x_1 - x_3) = \frac{3}{2}(-v)^{1/3} \quad (17)$$

When $\Phi=0$, Eqn.(10) has three unequal real roofs, two of them express stable state and one express is unstable state. The system changes smoothly under these states, which belongs to slow deformation process. The mutation occurs only when the system arrives the equilibrium boundary (singular set). Therefore, when $\Phi=0$, divergence occurs in the system topology (number and stability of equilibrium state). u and v constitute the system of bifurcation set, when the control variable meets $\Phi=0$.

Besides, when the control point leaps over the bifurcation point, divergence occurs in the structure system. In this case, two situations happen, that is, $v \leqslant 0$ or $v>0$. When $v \leqslant 0$, $qL^6 \leqslant 0$, which doesn't match the engineering practice. When $v>0$, $qL^6>0$, which matches the practice.

From the analysis above, only when $u \leqslant 0$, Eqn.(11) has real roofs and the system can leap over the bifurcation set while abrupt change occurs. Therefore, the necessary conditions for catastrophic instability of karst cave roof are

$$\begin{cases} 8\left[\left(\frac{2EI}{L^8\pi^2}-\frac{q\pi^2}{L^2}\right)\sqrt{\frac{16L^5}{EI\pi^6}}\right]^3+27\left[qL^6\sqrt[4]{\frac{16L^5}{EI\pi^6}}\right]^2=0 \\ \left(\frac{2EI}{L^8\pi^2}-\frac{q\pi^2}{L^2}\right)\sqrt{\frac{16L^5}{EI\pi^6}} \leqslant 0 \\ (qL^6>0) \end{cases} \quad (18)$$

Thus bearing capacity of subgrade cave roof and safe thickness of cave roof in karst region can be further defined.

5 Determination method for bearing capacity subgrade cave roof and safe thickness of cave roofs in karst region

Through the analysis of subgrade cave roof stability and instability, we can get the necessary condition of cave roof catastrophe, and calculate the micro-safe depth of subgrade cave roof and bearing capacity of subgrade cave roof.

5.1 Calculating methods of ultimate bearing capacity

The subgrade cave roof does not catastrophe instabilities, which asks $u \geqslant 0$:

$$\frac{2EI}{L^8\pi^2}-\frac{q\pi^2}{L^2} \geqslant 0 \quad (19)$$

So,

$$q \leqslant \frac{EH^3}{6L^6\pi^4} \quad (20)$$

5.2 Calculating method of safety thickness

Simplifying formula (18) and solving the equation get the micro-safe depth of the subgrade cave roof:

$$H \geqslant \sqrt[3]{\frac{6L^6 q\pi^4}{E}} \quad (21)$$

6 Analysis of engineering examples

6.1 Example 1

Canal bridges of a certain expressway in Hunan province lie in the river alluvial landscape area between the hills. The top-down of site formation was filled with fine sand, fine sand, clay, gravelly sand, dolomite limestone and so on. The bedrock karst caves developed. The artificial digging hole perfusion was used, and the total was 20. The construction is porous, when digging hole to 9.0−11.0 m (about half of designed pile depth), channel in canal and surface cross strait appear a large of subsidence. The ultimate subsidence reaches 4−5 m, resulting in that digging hole is no way, and need for governance.

Such as pile 8 (Fig.3), there are caves under 15.3 m, primary designed pile diameter is d=1.2 m, design load is 2 600 kN, pile passes through thick dolomite limestone (10.70−15.30 m), slice thickness (4.6 m) depth is under 17.00 m. According to the experimental results, physical and mechanical indexes of dolomite limestone are severer, 27.3 kN/m^3, internal friction angle φ=35°, rock block unitarian compressive strength R=120 MPa, top plate span L=5 m, top plate elastic modulus E=120 GPa.

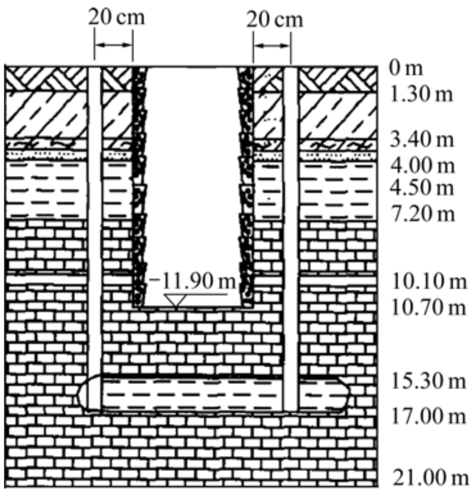

Fig.3 Construction design of pile 8

With the works, through self-compiled program in the work, the minimum safe thickness of this pile is 2.705 m, so the pile is steady.

The project has opened to traffic and run well for many years.

6.2 Example 2

The main building of the project is layers, skirt building is 4 layers, including basement with 2 layers. The settlement joint does not set between the main building and the skirt building. Engineering geological condition and geotechnical parameters are in Table 1[14−15]. In this work, pile 40 was discussed.

According to the data of exploration, the roof depth of the pile is 8.0 m, roof elastic modulus is E=55−76 GPa, long axis direction of karst cave span is 3.2 m, and the pile is filled by gravel clay, as shown in Fig.4.

Table 1 Parameters of rock and soil

No.	Soil layer	Thickness/ m	Natural unit weight/ (kN·m^{-3})
1	Miscellaneous fill	2.8	18.5
2	Silt clay	11.0	20.1
3	Silt	2.2	20.7
4	Circular-gravel	6.3	25.3
5	Micro-weathered dolomite	8.0	26.3
Thickness of rock soil layer		30.3	

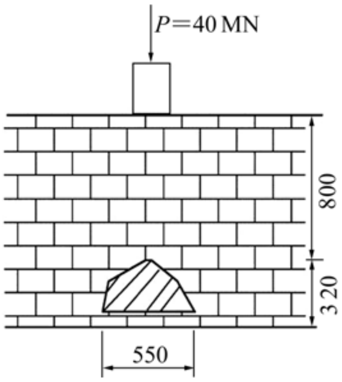

Fig.4 Section of rock mass under tip of pile 40 (Unit: cm)

The limited load of pile is 41.16×10^3 kN according to the calculation method in the work, so roof of pile is steady, the project has finished and run well for many years.

7 Influence factor analysis of cave roof stability

7.1 Influence of load size

In order to consider the stability of subgrade cave roof in karst region, suppose load size increased from 552.6 kPa to 2 210.5 kPa, which can get the relation curve between load and the stability of subgrade cave roof in karst region, as shown in Fig.5.

It can be seen from Fig.5 that as the load increases, S increases from 0.5 to 9, therefore the load has obvious influence on the subgrade cave roof.

7.2 Influence of subgrade cave roof span

Suppose the subgrade cave roof span is 1.0−2.5 m, we can get the relation curve between the subgrade cave roof span and the stability of subgrade cave roof in karst region, as shown in Fig.6.

From Fig.6, it can be seen that as subgrade cave

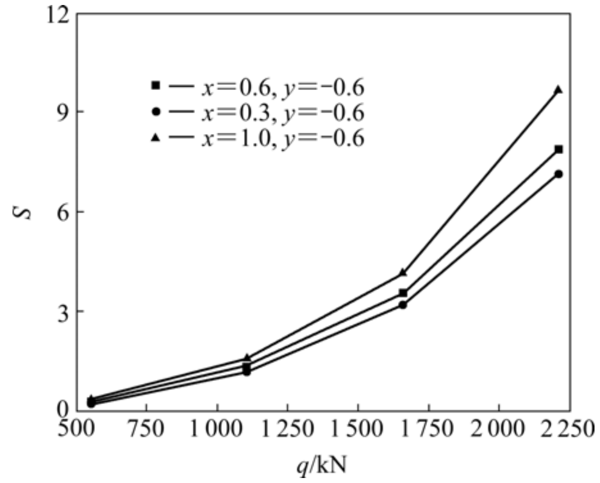

Fig.5 Relation curve between load (q) and stability of subgrade cave roof (S)

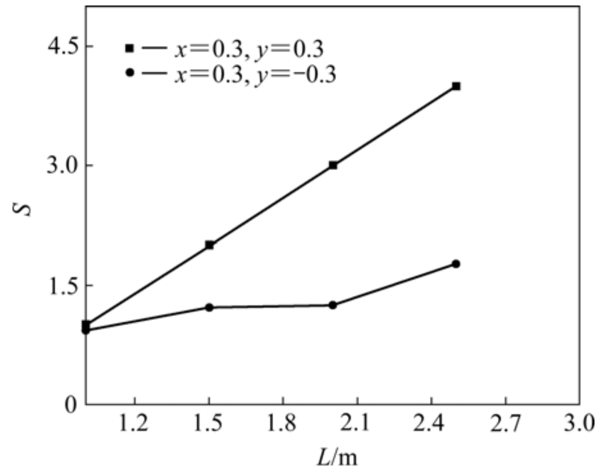

Fig.6 Relation curve between subgrade cave roof span (L) and stability of subgrade cave roof (S)

roof span changes, S increase from 0.9 to 5, therefore the subgrade cave roof span is also big to the stable influence of subgrade cave roof, the load is higher, it is more disadvantageous to the subgrade cave roof stability.

7.3 Influence of subgrade cave roof thickness

Suppose subgrade cave roof thickness is 0.6−2.0 m, simultaneously, we can obtain the curve between subgrade cave roof thickness and its stability criterion S (Fig.7).

When the thickness of subgrade cave roof increases from 0.5 m to 2.5 m, S drops from 3.6 to about 1.8, which indicates that increasing thickness of subgrade cave roof has advantageous to the subgrade cave roof stability.

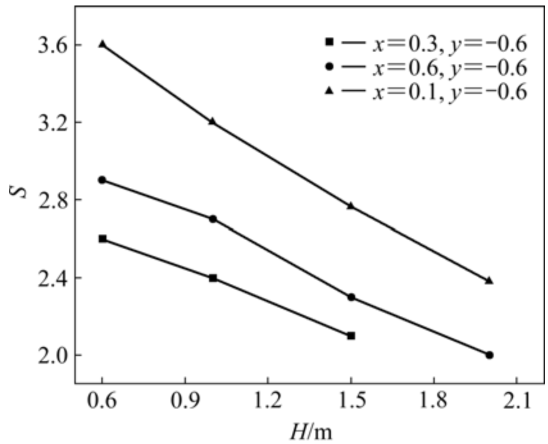

Fig.7 Relation curve between subgrade cave roof thickness (h) and stability of subgrade cave roof (S)

8 Conclusions

1) On the basis of clamped-clamped beam mechanical model, introducing catastrophe theory, a cusp catastrophe model of subgrade cave roof bearing capacity and safe depth of the underlying karst cave roof was established, and the necessary condition of subgrade cave roof catastrophe instability is obtained.

2) A new method of subgrade cave roof bearing capacity and safe depth of the underlying karst cave roof is proposed, the example analyses show the rationality of the methods.

3) The major effect factors of stability on cave roof under pile in karst region are deeply discussed and some results in quality are acquired.

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