

# Seismic ultimate bearing capacity of strip footings on slope

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**Abstract:** The influence of earthquake forces on ultimate bearing capacity of foundations on sloping ground was studied. A solution to seismic ultimate bearing capacity of strip footings on slope was obtained by utilizing pseudo-static analysis method and taking the effect of intermediate principal stress into consideration. Based on limit equilibrium theory, the formulae for computing static bearing capacity factors,  $N_q$ ,  $N_c$ ,  $N_\gamma$ , and dynamic bearing capacity factors,  $N_{qd}$ ,  $N_{cd}$ ,  $N_{\gamma d}$ , which are associated with surcharge, cohesion and self-weight of soils respectively, were presented. A great number of analysis calculations were carried out to obtain the relationship curves of the static and dynamic bearing capacity factors versus various calculation parameters. The curves can serve as the practical engineering design. The calculation results also show that when the values of horizontal and vertical seismic coefficients are 0.2, the dynamic bearing capacity factors  $N_{qd}$ ,  $N_{cd}$  and  $N_{\gamma d}$ , in which the effects of intermediate principal stress are taken into consideration, increase by 4%–42%, 3%–27% and 34%–57%, respectively.

**Key words:** slope foundation; bearing capacity; unified strength theory; limit equilibrium; earthquake effect

## 1 Introduction

The study on bearing capacity has been the hotspot of the geotechnical circle for ages. Among the limited literatures available on the seismic bearing capacity, MEYERHOF's<sup>[1-2]</sup> are perhaps the earliest, where the seismic forces were applied on the structure only as inclined pseudo-static loads. Effect of the seismic forces on the inertia of the supporting soil was not considered in these analyses. SARMA et al<sup>[3]</sup> and RICHARDS et al<sup>[4]</sup> considered the seismic forces both on the structure and on the supporting soil mass. Apart from these studies by the limit equilibrium method, DORMIEUX et al<sup>[5]</sup>, SOUBRA<sup>[6-7]</sup> and CHOUDHURY et al<sup>[8]</sup> used the upper bound limit method for analyzing the seismic bearing capacity of shallow strip footings. YU<sup>[9]</sup> and YANG<sup>[10]</sup> utilized numerical simulating method and energy dissipation method respectively for determination of the static bearing capacity of foundation on slope. However, for the seismic bearing capacity of strip footings founded on the slope in mountainous area, little investigations have been conducted. And the existing calculating formulae have not considered the effect of intermediate principal stress on ultimate bearing capacity. In addition, the results obtained by the existing calculating formulae are usually smaller than real values and the potential strength of soil is not fully mobilized. So in this paper, based on the unified strength theory that takes the effect of intermediate principal stress into consideration, by

supposing composite curved failure surface and employing pseudo-static approach and the method of limit equilibrium analysis, a new formula of seismic bearing capacity for slope foundation in mountainous areas was obtained.

## 2 Unified strength theory

Unified strength theory for geo-materials can be expressed as follows<sup>[11-12]</sup>:

$$F = \alpha\sigma_1 - \frac{b\sigma_2 + \sigma_3}{1+b} = \frac{2c_0 \cos \varphi_0}{1 + \sin \varphi_0},$$

$$\sigma_2 \leq \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{\sin \varphi_0}{2}(\sigma_1 - \sigma_3) \quad (1a)$$

$$F = \frac{\alpha}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3 = \frac{2c_0 \cos \varphi_0}{1 + \sin \varphi_0},$$

$$\sigma_2 \geq \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{\sin \varphi_0}{2}(\sigma_1 - \sigma_3) \quad (1b)$$

where  $\alpha = (1 - \sin \varphi_0) / (1 + \sin \varphi_0)$ ;  $\sigma_1$  is the major principal stress;  $\sigma_3$  is the minor principal stress;  $b$  is a coefficient that can reflect the influence of intermediate principal stress and normal stress on the failure degree of material;  $\varphi_0$  is the original angle of internal friction and  $c_0$  is the original cohesion of geo-material.

The normal stress  $\sigma_\omega$  and shear stress  $\tau_\omega$  acting on any inclined plane oriented  $\omega$  clockwise from the major principal plane can be determined:



respectively.

The values of  $c_m$  and  $\varphi_m$ , varying with the values of  $b$  and  $m$  can be obtained by computation (see Table 1). When  $b=0$ , Eqn.(9b) becomes Mohr-Coulomb criterion ( $\tau=c+\sigma\tan\varphi$ ), so Mohr-Coulomb criterion is just a special case of this method.

Similar to  $EK$ , the equation of  $EF$  can be given as follows:

$$r_{m\theta} = r_{m0} \exp(\theta_{m0} \tan \varphi_m) \tag{17}$$

where  $r_{m0}$  is the length of  $DE$ , calculated by

$$r_{m0} = \frac{B \sin \chi}{\sin(\chi + \chi_m)} \tag{18}$$

Hence, the length of  $DF$  can be written as follows:

$$r_{DF} = r_m = r_0 \exp(\theta_m \tan \varphi_m) \tag{19a}$$

Similar to  $AJK$ ,  $DFG$  is assumed as a Rankine passive states zone, and

$$\angle DFG = \pi/2 + \varphi_m \tag{19b}$$

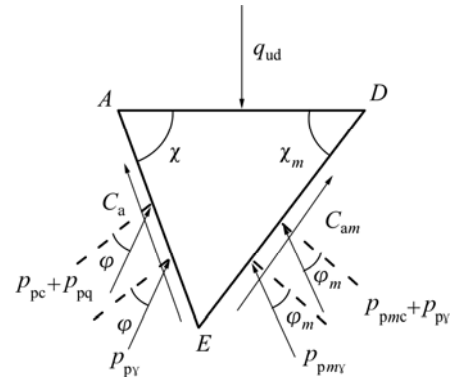
**Table 1** Values of  $c_m, \varphi_m$  varying with  $b$  and  $m$  under  $k=1.0, \varphi_0=20^\circ, c_0=20.0$  kPa

$b$	$m=1.0$		$m=0.9$		$m=0.8$		$m=0.7$	
	$c_m$ kPa	$\varphi_m$ ( $^\circ$ )	$c_m$ kPa	$\varphi_m$ ( $^\circ$ )	$c_m$ kPa	$\varphi_m$ ( $^\circ$ )	$c_m$ kPa	$\varphi_m$ ( $^\circ$ )
1.0	21.5	24.1	19.3	21.8	17.0	19.5	14.8	17.1
0.8	21.3	23.5	19.1	21.3	16.9	19.0	14.7	16.7
0.6	21.1	22.9	18.9	20.8	16.7	18.5	14.6	16.3
0.4	20.8	22.2	18.7	20.1	16.6	17.9	14.4	15.8
0.2	20.5	21.2	18.4	19.2	16.3	17.2	14.2	15.1
0.0	20.0	20.0	18.0	18.1	16.0	16.2	14.0	14.3

**3.2 Limit equilibrium analysis**

The free-body diagram of wedge  $ADE$  is shown in Fig.2. The forces acting on  $AE$  are  $p_{p\gamma}, p_{pq}, p_{pc}$  and  $C_a$ , where  $p_{p\gamma}$  is produced by the unit weight (the weight per unit volume),  $p_{pq}$  by the surcharge loading, and  $p_{pc}$  by the soil cohesion. Both  $p_{pc}$  and  $p_{pq}$  act at the midpoint of  $AE$ , and  $p_{p\gamma}$  is assumed to act at two-third position of  $AE$ . The directions of all these forces make the same angle of  $\varphi$  to the normal of  $AE$ . In addition, the adhesion  $C_a=c \cdot r_{DE}$ , where  $c$  is the unit cohesion.

In a similar manner, the forces acting on  $DE$  are shown in Fig.2. The subscript  $m$  of these forces indicates the mobilization degree of shear on virtual failure plane  $EF$ .  $p_{pm\gamma}, p_{pmq}$  and  $p_{pmc}$  are produced by the unit weight, surcharge and soil cohesion, respectively. The directions and positions of these forces are similar to those of the forces acting on  $AE$ .  $C_{am} = c_m \cdot r_{DE}$ , where  $c_m$  is the unit cohesion.



**Fig.2** Free-body diagram of wedge  $ADE$

The vertical seismic passive resistance can be computed as

$$P_p = (p_{p\gamma} + p_{pc} + p_{pq}) \cos(\chi - \varphi) \tag{20}$$

$$P_{pm} = (p_{pm\gamma} + p_{pmc} + p_{pmq}) \cos(\chi_m - \varphi_m) \tag{21}$$

From the vertical equilibrium of all the forces, we can obtain:

$$q_{ud} = P_p + P_{pm} + C_a \sin \chi + C_{am} \sin \chi_m \tag{22}$$

Determination of the values of all the passive resistances is as follows.

**Case 1:**  $c=c_m=0, q_2=q_m=0, \gamma \neq 0$

Considering the forces acting on the wedge  $AKE$ , as shown in Fig.3, then from the moment equilibrium of all the forces about the focus  $A$  we have

$$p_{p\gamma} = \frac{3}{2r_0 \cos \varphi} (M_1 - M_2 - M_3 - M_4) \tag{23}$$

where

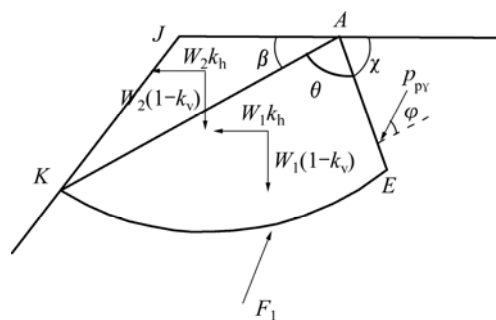
$$M_1 = \frac{1-k_v}{6} \gamma r_{AJ} \cdot r_{AK} \sin \beta (r_{AJ} + r_{AK} \cos \beta);$$

$$M_2 = \frac{k_h}{6} \gamma r_{AJ} \cdot (r_{AK} \sin \beta) (r_{AK} \sin \beta);$$

$$M_3 = \frac{\gamma r_0^3 (1-k_v)}{3} \int_0^\theta \exp(3\theta_0 \tan \varphi) \cos(\theta_0 + \chi) d\theta_0;$$

$$M_4 = \frac{\gamma r_0^3 k_h}{3} \int_0^\theta \exp(3\theta_0 \tan \varphi) \sin(\theta_0 + \chi) d\theta_0.$$

Angles  $\theta$  and  $\beta$  can be derived from the known conditions. The force  $F_1$  must act through the point  $A$ , so its moment for  $A$  is zero.



**Fig.3** Diagram of  $AKEJ$  only weight considered

Considering the forces acting on the wedge  $DEFN$ , as shown in Fig.4, then from the moment equilibrium of all the forces about the focus  $D$ , we have

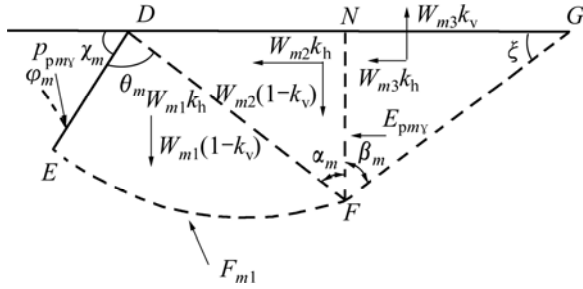


Fig.4 Diagram of  $DEFN$  only weight considered

$$P_{pm\gamma} = \frac{3}{2r_{m0} \cos \varphi_n} (-M_{\gamma m1} + M_{\gamma m2} + M_{\gamma m3} + M_{\gamma m4} + M_{\gamma m5} - M_{\gamma m6} + M_{\gamma m7}) \quad (24)$$

where

$$M_{m1} = \frac{\gamma r_{m0}^3 (1-k_v)}{3} \int_0^{\theta_m} \exp(3\theta_m \tan \varphi_m) \cos(\theta_m + \chi_m) d\theta_m;$$

$$M_{m2} = \frac{\gamma r_{m0}^3 k_h}{3} \int_0^{\theta_m} \exp(3\theta_m \tan \varphi_m) d\theta_m;$$

$$M_{m3} = \frac{1-k_v}{3} \gamma r_m^2 H_m \sin^2 \alpha_m;$$

$$M_{m4} = \frac{k_h}{6} \gamma r_m^2 H_m \sin \alpha_m \cos \alpha_m;$$

$$M_{m5} = \frac{2}{3} H_m E_{pm\gamma};$$

$$M_{m6} = \frac{k_v}{2} \gamma H_m^2 (r_m \sin \alpha_m + \frac{H_m \tan \beta_m}{3}) \tan \beta_m;$$

$$M_{m7} = \frac{k_h}{6} \gamma H_m^3 \tan \alpha_m.$$

$\xi$  is the included angle between  $GF$  and ground given by Choudhury and Subba Rao<sup>[8]</sup>:

$$\zeta = \frac{\pi}{4} - \frac{\varphi}{2} + \frac{1}{2} \arctan^{-1} \left( \frac{k_h}{1-k_v} \right) - \frac{1}{2} \arcsin^{-1} \frac{\sin[\arctan^{-1}(\frac{k_h}{1-k_v})]}{\sin \varphi}.$$

where  $k_h$  is horizontal acceleration under seismic action;  $k_v$  is vertical acceleration under seismic action;

$$\beta_m = \pi/2 - \xi; \quad \alpha_m = \varphi_m + \xi; \quad \angle DFG = \pi/2 + \varphi_m.$$

The passive pressure  $E_{pm\gamma}$  that acts on  $NF$  is horizontal and can be obtained as

$$E_{pm\gamma} = \frac{1}{2} H_m^2 \gamma K'_m$$

where  $K'_m = 1 + \sin \varphi_m / (1 - \sin \varphi_m)$ ;  $H_m = r_m \cos \alpha_m$ .

The force  $F_{m1}$  must act through the point  $D$ , so its moment for  $D$  is zero.

**Case 2:**  $q=q_m=0, \gamma=0, c, c_m \neq 0$

Considering the forces acting on the wedge  $AKE$ , as shown in Fig.5, then from the moment equilibrium of all the forces about the focus  $A$ , we have

$$P_{pc} = \frac{2M_{c1}}{r_0 \cos \varphi} \quad (25)$$

where  $M_{c1} = \frac{cr_0^2}{2 \tan \varphi} [\exp(2\theta) - 1].$

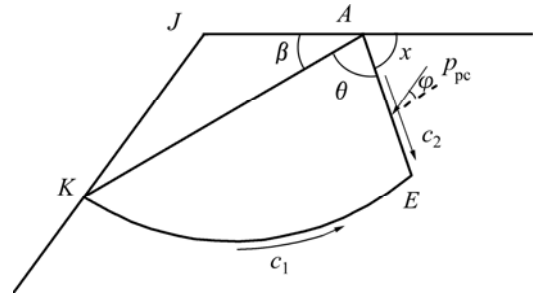


Fig.5 Diagram of  $AEJK$  applied by cohesion

Then considering the forces acting on the area  $DEFN$ , as shown in Fig.6, and the moment equilibrium of all the forces about the focus  $D$ , we can get

$$P_{pmc} = \frac{M_{cm1} + \frac{1}{2} H_m E_{pmc}}{\frac{1}{2} r_{m0} \cos \varphi_m} \quad (26)$$

where

$$M_{cm1} = \frac{c_m r_{m0}^2}{2 \tan \varphi_m} [\exp(2\theta_m \tan \varphi_m) - 1],$$

$$E_{pmc} = c_m H_m \frac{\cos \varphi_m}{\sin(\beta_m - \varphi_m) \cos \beta_m}.$$

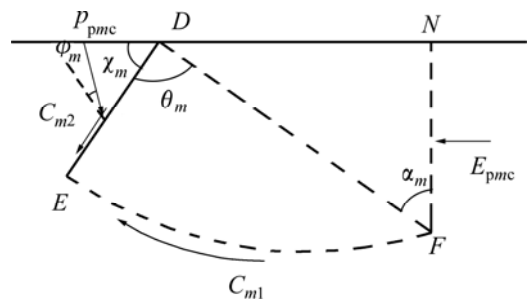


Fig.6 Diagram of  $DEFN$  applied by cohesion

**Case 3:**  $\gamma=0, c=c_m=0, q, q_m \neq 0,$

Considering the forces acting on the wedge  $AKE$ , as shown in Fig.7, and the moment equilibrium of all the forces about the focus  $A$ , we get

$$P_{pq} = \frac{\frac{1}{2}qr_{AJ}^2(1-k_v) + \frac{1}{2}k_h qhr_{AJ}}{\frac{1}{2}r_0 \cos \varphi} \quad (27)$$

where

$$q = \gamma \frac{lh \tan \varphi + \frac{1}{2}h^2}{l \tan \varphi + h}$$

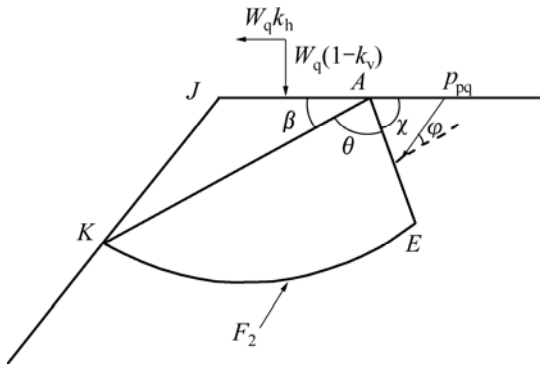


Fig.7 Diagram of AJEK applied with surcharge

The force  $F_2$  must act through the point  $A$ , so its moment for  $A$  is zero.

Then considering the forces acting on the wedge  $DEFN$ , as shown in Fig.8, and the moment equilibrium of all the forces about the focus  $D$ , we get

$$P_{pmq} = \frac{M_{qm1} - M_{qm2} + M_{qm3} - M_{qm4} - M_{qm5}}{\frac{1}{2}r_{m0} \cos \varphi_m} \quad (28)$$

where

$$M_{qm1} = \frac{1}{2}(1-k_v)\gamma h H_m^2 \tan^2 \alpha_m,$$

$$M_{qm2} = \frac{1}{2}k_h \gamma h^2 H_m \tan \alpha_m,$$

$$M_{qm3} = \frac{1}{2}E_{pmq} H_m,$$

$$M_{qm4} = k_v \gamma h H_m^2 (\tan \alpha_m + \frac{1}{3} \tan \beta_m) \tan \beta_m,$$

$$M_{qm5} = \frac{1}{2}k_h \gamma h^2 H_m^2 \tan^2 \beta_m,$$

$$q_m = \gamma h, E_{pmq} = \gamma H_m h K'_m.$$

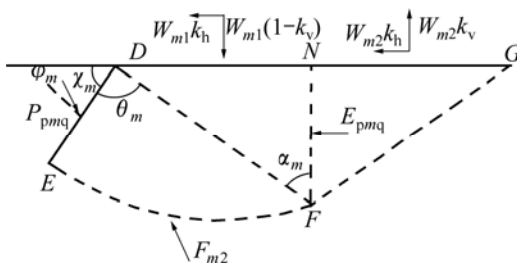


Fig.8 Diagram of DEFN applied with surcharge

The force  $F_{m2}$  must act through the point  $D$ , so its moment for  $D$  is zero.

### 3.3 Seismic bearing capacity

Substituting Eqn.(23)–(28) into the Eqn.(20), the ultimate seismic bearing capacity based on the unified strength theory can be expressed in the following form:

$$q_{ud} = cN_{cd} + qN_{qd} + \frac{1}{2}\gamma B N_{\gamma d} \quad (29)$$

where  $N_{cd} = \frac{P_{pc} + P_{pmc}}{cB} + \frac{(c + c_m) \sin \varphi \sin \varphi_m}{c \sin(\varphi + \varphi_m)}$ ;

$$N_{qd} = \frac{P_{pq} + P_{pmq}}{\gamma h B}; \quad N_{\gamma d} = \frac{2P_{p\lambda} + 2P_{p\mu\gamma}}{\gamma B^2}.$$

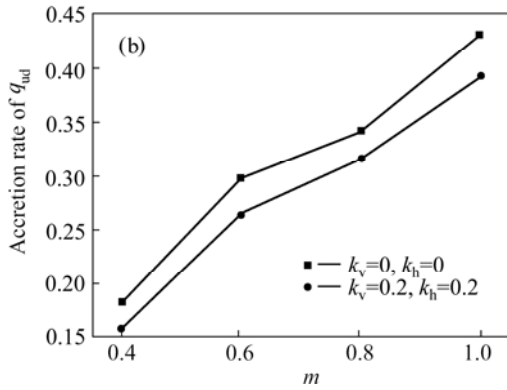
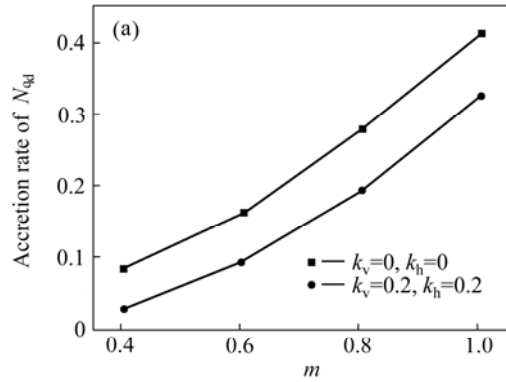
### 4 Examples and comments

The angle  $\chi$  should be assumed for the analysis of limit equilibrium. The trial and error method was applied to obtain the minimal coefficients of seismic bearing capacity.

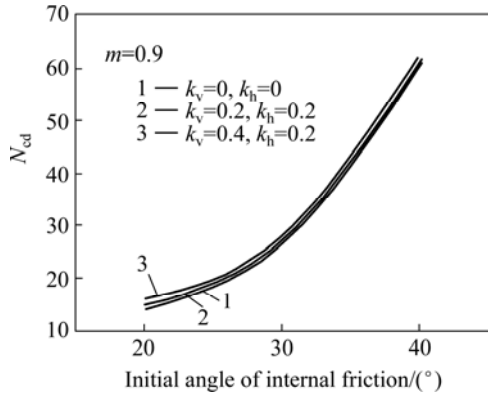
1) Comparison of bearing capacity based on unified strength theory with that based on Mohr-Coulomb Theory. For a sloping ground, given  $l/B=h/B=0.5$ ,  $\varphi=45^\circ$ ,  $\varphi_0=20^\circ$ ,  $c_0=20$  kPa,  $k=1.0$ . Analysis shows that if the intermediate principal stress is considered, the coefficients of seismic and static bearing capacity (given  $k_h=k_v=0.2$ ) are larger, for  $N_{cd}$  about 3%–27%, for  $N_{qd}$  about 4%–42%(see Fig.9(a)), and for  $N_{\gamma d}$  about 34%–57%. The static bearing capacity increases by 17%–43% and the seismic one increases by 16%–40%(see Fig.9(b)).

2) Coefficients of the seismic bearing capacity and coefficients of static bearing capacity. Figs.10–12 show the comparison between coefficients of seismic and that of static bearing capacity. Given parameters of a slope as:  $l/B=h/B=0.5$ ,  $\varphi=45^\circ$ ,  $\varphi_0=20^\circ, 30^\circ, 40^\circ$ ,  $c_0=20.0$  kPa,  $k=1.0$ ,  $b=1.0$ , considering seismic effect ( $k_h=0.2$ ,  $k_v=0.2$ ;  $k_h=0.2$ ,  $k_v=0.4$ ), through the variation of  $m(=\tau/\tau_m)$  and initial friction angle  $\varphi$ , we can see that under the seismic loading,  $N_{cd}$  changes very little, while both  $N_{qd}$  and  $N_{\gamma d}$  reduce to some extent. Meanwhile, the seismic bearing capacity is 10%–20% less than the static bearing capacity.

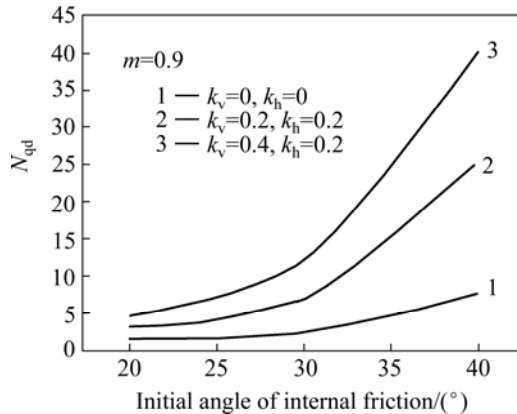
3) Effect of  $b$  on intermediate principal stress. Along with the increase of  $b$ , this reflects intermediate principal stress, the coefficients of seismic bearing capacity increases gradually. As far as the range of increase is concerned,  $N_{\gamma d}$  is the most obvious,  $N_{qd}$  is in the second place, and comparatively,  $N_{cd}$  is the most insensitive (see Fig.13).



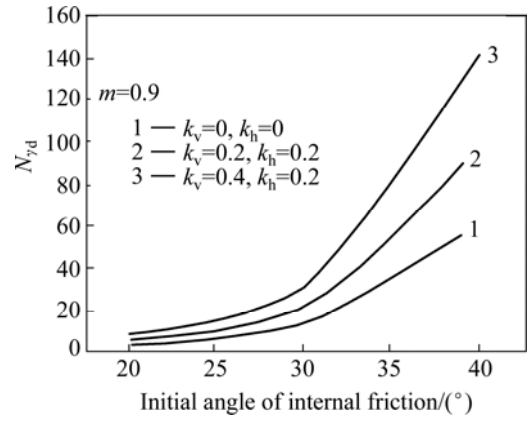
**Fig.9** Comparison between bearing capacity by unified theory and bearing capacity by non-unified theory  
Accretion rate: (a)  $N_{qd}$ ; (b)  $q_{ud}$



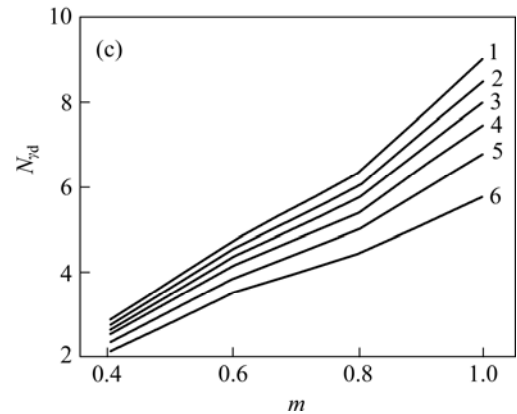
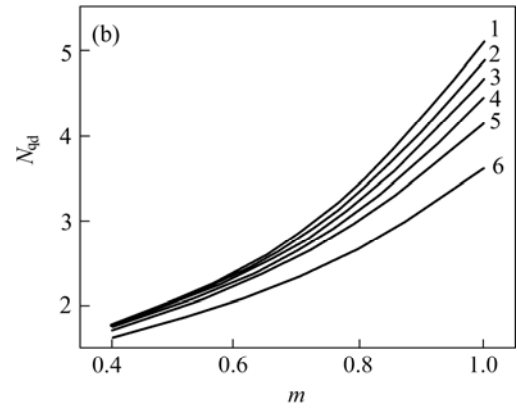
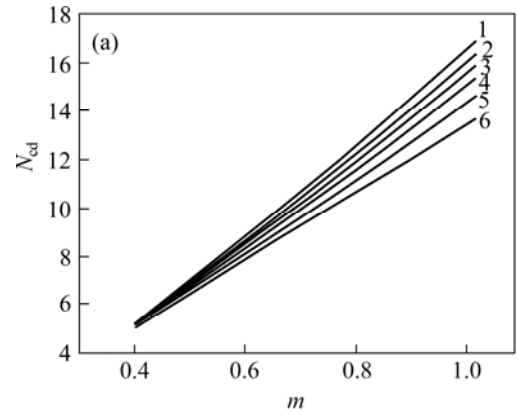
**Fig.10** Variation of seismic bearing capacity  $N_{cd}$  with  $k_h$  and  $k_v$



**Fig.11** Variation of surcharge seismic bearing capacity coefficients  $N_{qd}$  with  $k_h$  and  $k_v$



**Fig.12** Variation of bearing capacity coefficients  $N_{\gamma d}$  with  $k_h$  and  $k_v$  considering effect of self-weight



**Fig.13** Seismic bearing capacity coefficients: (a)  $N_{cd}$ , (b)  $N_{qd}$  and  $N_{\gamma d}$  affected by  $b$  at  $k_v=0.2$  and  $k_h=0.2$   
1— $b=1.0$ ; 2— $b=0.8$ ; 3— $b=0.6$ ; 4— $b=0.4$ ; 5— $b=0.2$ ; 6— $b=0$

## 5 Conclusions

1) Based on the unified strength theory that takes the effect of intermediate principal stress into consideration, by supposing composite curved failure planes and employing pseudo-static approach and the method of limit equilibrium analysis, a new formula of seismic bearing capacity of slope foundation was obtained.

2) The static and seismic bearing capacity that takes the effect of intermediate stress into account is larger than that without considering the effect of intermediate stress; the static bearing capacity increases by 17%–43% and the seismic one increases by 16%–40%.

3) Under the seismic loading,  $N_{cd}$  changes very little, while both  $N_{qd}$  and  $N_{\gamma d}$  reduce to some extent. Meanwhile, the seismic bearing capacity is 10%–20% less than the static bearing capacity.

4) Along with the increase of  $b$  that reflects intermediate principal stress, the coefficients of seismic bearing capacity increases gradually. As far as the range of increase lies concerned,  $N_{\gamma d}$  is the most obvious,  $N_{qd}$  in the second place, and comparatively,  $N_{cd}$  is the most insensitive.

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