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Fractal characteristics of rock fragmentation at strain rate of 10^0 - 10^2 s^{-1} ①

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Abstract: The fragmentation test of granite subjected to strain rate of 10^0 - 10^2 s^{-1} was carried out using split Hopkinson pressure bar(SHPB) whose diameter is 75 mm, where half-sine loading waveform was performed. The sieving statistics results of the fragments show that the distribution of the fragments is a fractal, and the fractal dimension values fall into the range of 1.2 - 2.4. The correlation analysis between the fractal dimension and the logarithm of the energy density shows that they have approximately linear relation. Finally, based on damage theory and scale invariant principle, the fragmentation model with renormalization method was put forward, and the fractal dimension value predicted with the model was compared with the test results. It is found that the fractal dimension value obtained from the improved fragmentation model is more reasonable.

Key words: SHPB test; rock fragmentation; fractal; damage; renormalization
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1 INTRODUCTION

The behavior of rock under impact loading is widely involved in many aspects such as drilling, earthquakes, accidental impacts and explosions. A detailed understanding of rock fragmentation is very important in blasting, mining and beneficiation. Many researchers have carried out lots of experiments with pendulum tests, drop weight tests, SHPB(split Hopkinson pressure bar) tests, etc, and have gotten plenty of experimental data[1-5]. However, because of the heterogeneous nature of rock, unavoidable imprecision of the loading and measuring devices, there are still few generally accepted results, especially in rock fragmentation aspect.

Fractal geometry, originally defined by Mandelbrot, supplies a different view from traditional Euclidian geometry, and has been used to gain a series of progresses in rock mechanics and other fields[6-11]. In the paper, the sieving statistics of rock fragments from SHPB test were carried out to show the fractal characteristics of rock fragmentation under impact loading with the strain rate of 10^0 - 10^2 s^{-1}. And a corresponding theoretical fragmentation mode was established to describe the test results.

2 FRAGMENTATION TEST OF GRANITE AT STRAIN RATE OF 10^0 - 10^2 s^{-1}

The technique using SHPB equipment, which offers a relatively accurate result under impact loading, has been widely applied to the dynamic test of rock and rock-like materials. The tests determining the rate sensitivity of rock have been done[12-14]. The results show that it is essential to obtain dynamic characteristics and to understand the rock dynamic properties at different strain rates, at least in different rate regions.

2.1 SHPB equipment

In order to overcome the size effect of testing rock-like materials, the conventional SHPB equipment, usually with diameter of 20 mm, must be changed so as to get the test results that can represent the properties of the in-situ rock as far as possible.

A new type of SHPB whose diameter of the input and output bars is 75 mm, has been developed in Singapore and in Central South University in China[4,13,14].

At the same time, in order to eliminate the P-C (Pochhamer and Chree) oscillation and to get constant strain rate, a novel projectile that can produce half-sine wave-form is fabricated with characteristic lines and FEM methods[15-18]. The profile of the projectile is shown in Fig. 1.

2.2 Specimens for testing

The specimens used in the impact testing were obtained from the same granite block elaborately. The diameter of the specimens is 70 mm and the ratio of length to diameter (L/D) equals 0.5. Before the test, all the specimens were labeled, weighed

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Fig. 1 Profile of projectile producing half-sine wave-form

and the dimensions were measured. The sonic wave velocities through the specimens were obtained with ultrasonic pulse velocity measurement device.

2.3 Fragments sieving statistics

After each test, the smashed fragments of specimens were collected. Then the classification of different size fractions were carried out with sieve mesh diameter of 0.6, 2.5, 6.0, 9.0 and 12.0 mm. Based on the fact that the fragmentation has aggregative characteristic, the test results with few broken pieces weren't included. Nor were the specimens that were over smashed (the diameter of most fragments are smaller than 0.6 mm).

3 FRACTAL CHARACTERISTICS OF ROCK FRAGMENTATION AT STRAIN RATE OF 10^0 - 10^2 s^{-1}

3.1 Fractal and fractal dimension

Fractal is an effective tool to describe the irregular phenomena[6-11]. Fractal dimension value, which usually is a fraction, is an important character to indicate the fractal. The similarity dimension, which is the most acceptable one, has the following form:

$$D=\frac{\ln N_r(F)}{\ln r} \tag{1}$$

where r is scaling factor; F is fractal aggregate; N is number of elements in F measured with the scale of r, that is,

$$N \propto r^{-D} \tag{2}$$

3.2 Description of fractal of rock fragmentation

The rock fragmentation is a complicate process, which is affected by loading modes, strain rates and inherent constituent of rock. Many researchers have been devoted to understanding the mechanism of rock fragmentation, and lots of distribution functions of fragments have been offered to describe the size distribution of fragmentation. Among them, the R - R(Rosin - Rammler) distribution and G - G - S(Gate - Gaudin - Schuhmann) distribution are two of the typical ones.

The R - R distribution has the following form:

$$y=1-\exp[-(r/r_0)^a] \tag{3}$$

where r_0 is character size; a is distribution parameter.

The G - G - S distribution has the following form:

$$y=(r/r_m)^b \tag{4}$$

where r_m is the maximum size of the fragment; b is regression coefficient.

Comparing formulae (3) and (4), the same results can be gotten by truncating the high order items of the respective series expression. If $m(r)$ is used to represent the passing percentage of sieve with the mesh size of r, and M is used to represent the gross mass of the fragments, then

$$\frac{m(r)}{M}=(\frac{r}{r_m})^b \tag{5}$$

According to formula (5), there is:

$$dm \propto r^{b-1}dr \tag{6}$$

The relation between the quantity and the mass of fragments is:

$$dm \propto r^3 dN \tag{7}$$

From formula (2), the following formula can be gotten:

$$dN \propto r^{-D-1}dr \tag{8}$$

Combining formulae (6)-(8), then

$$D=3-b \tag{9}$$

In this way, the fractal dimension value can be gotten using the sieving statistics.

3.3 Calculation of the fractal dimension values of rock fragmentation

The tests were carried out on SHPB with the shaped projectile which can produce half-sine stress wave. By increasing the impact velocity of projectile gradually, 10 specimens were tested and recorded. Fig. 2 shows the scatter diagram for the logarithm of the particle size and the passing percentage of sieve, the 6 lines represent the regression lines of the scatters. The regression coefficient b and the calculated fractal dimension value D are listed in Table 1, where e_s represents the energy density that specimen absorbed.

From Fig. 2 and Table 1, it can be seen that there exists good relevance between the logarithm of the particle size and the logarithm of the passing percentage of sieve, and all of the regression coefficients are close to 1. The results show that the fragmentation of granite under impact loads of strain rate of 10^0 - 10^2 s^{-1} is a fractal, and the fractal dimension values fall in the range of 1.2 to 2.4.

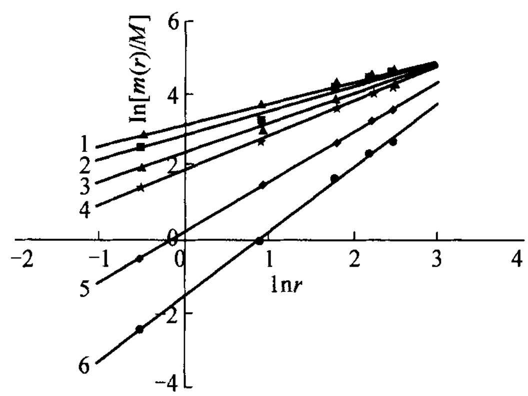

Fig. 2 Natural logarithm relation between particle size and passing percentage of sieve

1—Specimen Ⅷ; 2—Specimen Ⅶ; 3—Specimen Ⅵ; 4—Specimen Ⅴ; 5—Specimen Ⅲ; 6—Specimen Ⅱ

Table 1 Testing result sets

Specimen ID	Energy Density/ (J • cm^{-3})	b	Correlation coefficient	Fractal dimension
Ⅱ	0.46	1.766	0.999	1.234
Ⅲ	0.64	1.371	0.999	1.629
Ⅴ	1.20	0.981	0.998	2.019
Ⅵ	1.27	0.834	0.997	2.166
Ⅶ	1.56	0.695	0.995	2.305
Ⅷ	1.87	0.603	0.997	2.397

3.4 Relation between the fractal dimension and the energy density

The energy density is often used as a parameter to study the effect of dynamic loadings. The rock fragmentation fractal and the energy density have the relation as follows:

$$D \propto \log e_s \qquad (10)$$

Fig. 3 gives out the logarithm graph between the fractal dimension and the energy density of the tests in this paper. It can be seen that the fractal dimension has linear relation with the logarithm of the energy density for granite at strain rate of $10^0 - 10^2\ s^{-1}$, and the correlation coefficient is 0.985.

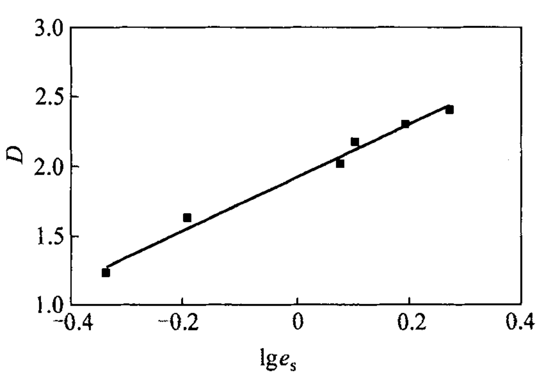

Fig. 3 Relationship between the fractal dimension and logarithm of energy density

4 AN IMPROVED DAMAGE FRAGMENTATION MODEL WITH RENORMALIZATION GROUP THEORY

4.1 Fractal of rock fragmentation with invariant failure probability

Turcotte et al[6-8] studied the fractal model for the fragmentation of solid materials with invariant failure probabilities. An element with side length h, named as 0 level cell, is divided into 1 level elements with the number is b^3 and its side length of $1/b$, and then the 1 level elements are divided into 2 level elements in the same way, where b is the scaling factor. In the process, the element failure probability is assumed to be invariant. The model that b equals 2 is shown in Fig. 4.

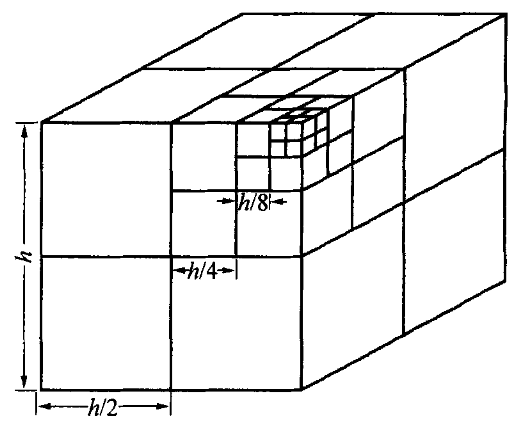

Fig. 4 Fractal fragmentation model when $b=2$

The relation between the fractal dimension value and the failure probability can be expressed as[8]:

$$D=3+\log[P(1/b^i)]/\log b \qquad (11)$$

where the allowed range for the failure probability is $b^{-3}<P(1/b^j)<1$.

4.2 Critical element failure probability leading to overall fragmentation

Renormalization theory is mainly applied to analyze the scale-invariance behavior by changing scales in a model to produce a set of renormalization group equations to get the critical failure probability of a system. Fig. 5 shows a simple procedure of renormalization approach, where scaling factor equals 2 and every four smaller elements are represented by one larger element at the higher level.

For the model shown in Fig. 4, the 0 level ele-

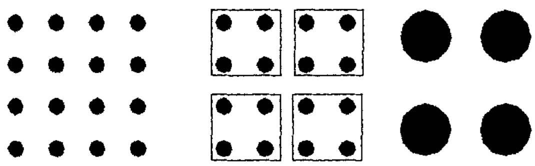

Fig. 5 Diagram of renormalization approach

ment is divided into 1 level elements, the number of element changes from 1 to 8. The failure of any element is possible. If we want to know the overall failure probability, 256 kinds of possibilities must be considered. In the view of topology, there are only 22 possibilities, as shown in Fig. 6, the black dots at the corners of the cube represent the failed elements; the number before the bracket is the number of failed element; the number in the bracket is the times that the situation repeats.

From the point of view of Turcotte[6], at level 1, considering the 8 elements as a whole, if there are intact elements forming a path from side to side vertically, then the 0 level element is intact (shown in Fig. 7). Then the 4f(2), 5c(8), 6b (12), 6c(4), 7(8), 8(1) in Fig. 6 are all fragmental as a whole.

However, based on damage theory, for the 0 level element in Fig. 7(a), the vertical section in $A-B$ direction is completely damaged(Fig. 8), so the 0 level as a whole is fragmental. Then the 6a (12), 5a(24), 5b(24), 4a(6), 4d(24), 4e(6) in Fig. 6 are also fragmental as a whole.

Considering all the possibilities, the relation between the 0 level element fragmental probability p_0 and 1 level element fragmental probability p_1 can be expressed as

$$p_1=p_0^8+8p_0^7(1-p_0)+28p_0^6(1-p_0)^2+56p_0^5(1-p_0)^3+38p_0^4(1-p_0)^4 \qquad (12)$$

After renormalization, the iteration function can be gotten:

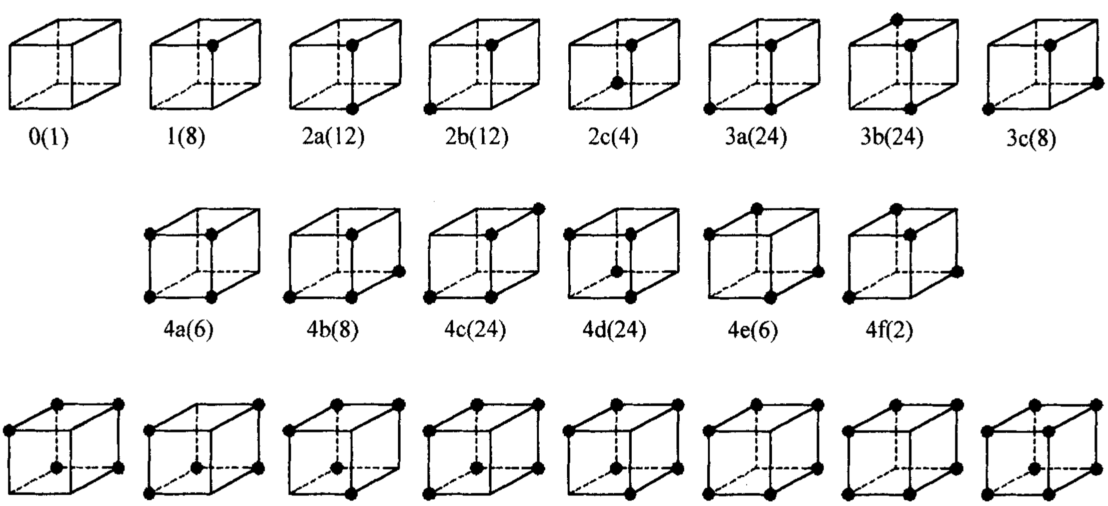

Fig. 6 22 possibilities of 8 elements combination with 0 to 8 failed elements

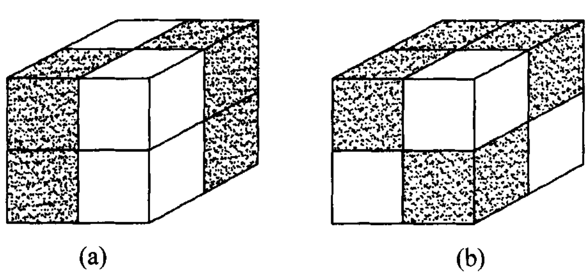

Fig. 7 Two instances from turcotte
(a)—Intact as a whole; (b)—Fragmental as a whole

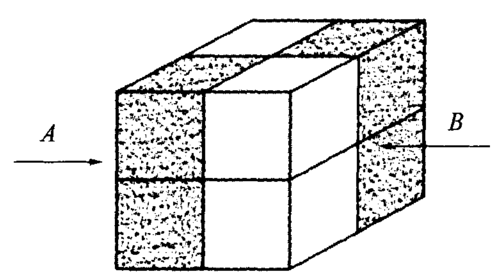

Fig. 8 Fragmental as a whole for total damage in vertical section through $A-B$

$$p_{n+1}=p_n^8+8p_n^7(1-p_n)+28p_n^6(1-p_n)^2+56p_n^5(1-p_n)^3+38p_n^4(1-p_n)^4 \quad (13)$$

Based on the assumption of invariant element fragmental probability, p_{n+1} equals p_n. So the critical element fragmental probability p^* can be gotten with equation (13):

$$p^*=0.4901 \quad (14)$$

Combining formulae (13), (14) and (11), the fractal dimension value of the fragmentation can be gotten:

$$D=3+\log 0.4901/\log 2=1.9711 \quad (15)$$

However, the fractal dimension value calculated with model from Turcotte et al is 2.842[6].

4.3 Discussion

From the test results shown in Table 1, the fractal dimension value of rock fragmentation under impact and explosive loading usually falls in the range of 1.1 to 2.4. The model established by Turcotte et al gives a large value of 2.842, which seldom appears in reality. The main reason is that it minifies the effect of damage in rock fragmentation.

5 CONCLUSIONS

1) The fragmentation of granite at strain rate of $10^0-10^2\ s^{-1}$ is a fractal, and the fractal dimension values fall into the range of 1.2 - 2.4.

2) The fractal dimension and the logarithm of the energy density have approximately linear relation.

3) An improved fragmentation model with damage theory and renormalization method is put forward, and the fractal dimension value obtained with the model is more reasonable.

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